EXPERIMENTAL EVALUATION OF STATIC AND DYNAMIC P-Y CURVES IN DENSE SAND

Masoud HAJIALILUE-BONAB¹, Daniel LEVACHER², Jean-Louis CHAZELAS³

ABSTRACT

Flexible piles with pile caps of different masses and instrumented with 20 strain gauges on the length of the pile, displacement, acceleration and force sensor which were impacted at variable levels of force with the help of a new ball gun designed for this purpose. The equation of the movement of a comparable beam to the pile under dynamic loading has been established and the various terms of this equation was evaluated using the experimental results. It is shown that the value of internal damping of pile compared to other terms in the equation is negligible. The term of inertia was divided into two parts, one related to the mass of the pile and the other related to the mass of the associated soil. The contribution of each term to the equation in different instant of vibration was shown. The distribution of displacements and the reactions of the soil at various depths and various steps of time were evaluated starting from the profiles of the bending moments by a double integration and a double derivation respectively. The dynamic P-y curves or loops were built starting from these results. A static test on the same pile under same soil condition was undertaken and static P-y curves were established. The procedure of experimental tests and P-y curves preparation procedures are explained and a comparison between static and dynamic P-y curves is also discussed.

Keywords: P-y curves, piles, centrifuge, soil-pile interaction

INTRODUCTION

Lateral loads on piles are usually developed either from the superstructure which they support or from the soil in which they are located or the both. Dynamic loads are due to the horizontal movement of these superstructures generated by wind effects, machine vibrations, impact of vehicles or boats, or coming from the ground motion such as earthquakes. These types of action shows clearly that the piles are subjected to two kinds of interaction, i.e., an inertial one from the movement of the superstructure and a kinematical one from the soil motion.

Apart the FEM and BEM methods which require important computing facilities, the use of the Winkler model for non linear support of a beam is considered as the most adapted method for design of laterally loaded piles. From the early developments static loads to the most of recent publications for dynamic loadings, the modeling has consisted in taking into account of different aspects of the interactions such as the non linear behavior of the soil, rate of strain effects, the phenomena of compaction, of damping, of gapping and the slippage. All these interactions are finally concentrated along the depth given by P-y relationships between the pile and the soil. Mainly, the consideration of the dynamic aspects in the Winkler model has consisted in complementing the P–y curves by rheological elements such as springs and dash-pots. The latest research results clearly show that to get the representative dynamic P-y curves or P-y loops is difficult.

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Experimental approaches are required to establish these rheological elements. Centrifuge modeling is an inexpensive and now an established experimental technique alternative to field tests. The behavior of soils can be correctly modeled with a low cost test by means for running parametric studies. The aim of this research is to study the inertial interaction of soil-pile system and establish static and dynamic P-y curves.

**TEST SET-UP**

The Fontainebleau sand which is a fine siliceous, clean and homogeneous sand is used for this study. The ratio between the diameter of pile (18mm) and grain size is 90, which is higher than the value of 30 above which there is no important scale effect on the bearing pressures of foundation (Ovesen, 1979). Method applied for sand reconstitution was sand pluviation. The average density was 16.05 kN/m³ which gives a relative density of 85.3%.

The prototype pile has a diameter B of 0.72 m, an embedded length L of 12m with a stiffness EI equal to 505.4 N.m² which is considered a flexible pile. In this study the inertial effect produced by adding different pile caps. The free height H where the impact applied is 2.2 m (Figure 1). The pile model is 1/40 of prototype and all the tests were carried out under 40g.

The pile caps were designed with a minimal thickness in the direction of the impact (closer to the diameter of the pile) to decrease the effect of the rocking mode. Four pile caps were manufactured for parametric studies relating to the mass of the superstructure. The total masses of pile caps include the instrumentations (force sensor and accelerometer) are shown in Table 1. All the installation of piles, as reported in this paper, were carried out under 1g condition.

![Figure 1. Experimental set-up for dynamic test](image)

**Table 1. Specification of pile caps**

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<thead>
<tr>
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<th>M₁</th>
<th>M₂</th>
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<tr>
<td>1: 20 Strain gauges</td>
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<td>2: Model pile</td>
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<td>3: Laser displacement sensor</td>
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<td>4: Acceleration transducer</td>
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<td>5: Pile cap</td>
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<td>6: Force transducer</td>
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<td>7: Electromagnetic impact device</td>
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<td>8: Power supply to the ball gun.</td>
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<td>9: Steel ball</td>
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<td>10: Diode for evaluation of ball velocity</td>
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To measure the deformations and to deduce the bending moments from them, the model piles are instrumented with 20 strain gauges assembled in half-bridge. In static tests 20 pair of gauges were used but in dynamic test 7 pair of these gauges were used because of channel limitation. To evaluate the applied force, a piezoelectric sensor is fixed on the pile cap. An analogical laser sensor with integrated amplifier used for displacement measurement. A piezoelectric accelerometer was also fixed on the pile cap.

The impact is generated with an impact device. This device consists of a tubular guide wounded by a coil. The coil is fed by a short high driving current pulse from the in flight capacitors and triggered by a signal generator from the command room. The exit is just near the impact point (~ 1 cm) so that the ball bounces back in the tube. A light slope in the tubular guide enables the gravity to draw back the ball to the initial position, ready for the next impact. This system is able to repeat impacts in flight either to reproduce the same pulses one by one or to cumulate impacts for other applications. Changing the capacitor tension and the current pulse duration in flight markedly by varying the amplitude of the impact force. The time duration of the impact is typically of $t = 0.01$ s (prototype scale) which corresponds to a Dirac impulse. (Hajialilue-Bonab et al, 2007).

In static tests the pile is the same but the lateral loading of the pile is performed by a hydraulic servo-actuator (Figure 2a). Instead of laser displacement transducer, two LVDTs at the heights of 20 mm and 65 mm (model scale) measure the pile head deflections and thus give the rotation. The load is applied in about 20 steps of 80 N each.

**STATIC P-Y CURVES**

The strain gauges fixed on the piles give the bending moment profiles during static lateral loading. The study is based on the analysis of these experimental data. For a given lateral load, the measured distribution of bending moment $M$, with depth $z$, is used to derive the reaction profile of the soil $P$, by double derivation, and the displacement profile $y$, by double integration (Figure 2b).

$$
E_p l_p \frac{\partial^4 y}{\partial z^4} + c_p I \frac{\partial^5 y}{\partial z^4 \partial t} + m \frac{\partial^2 y}{\partial t^2} + c \frac{\partial y}{\partial t} - p = 0
$$

| Prototype (kg) | 6554 | 13360 | 32019 | 54502 |

\[\text{Figure 2. a) Experimental set-up for static test, b) Static P-y curves}\]
The first term corresponds to the force resulting from the deformation of the pile. It is of static nature, i.e., independent, the velocity or the frequency of vibration. The second term is the internal force of damping of the pile which depends on the frequency of the movement. The third term is the inertia produced by the vibration of the pile which depends on acceleration. In this term (m) is the modal mass of the soil-pile system which includes the mass of the pile and a part of the associated mass of the soil which vibrates with the pile. The acceleration and velocity of the soil around the pile vary between a maximum value in contact with the pile and a very low value beyond which the effects of inertia become negligible. This volume corresponding to this mass remains difficult to evaluate and constitutes a question in the analysis of the Winkler type. The forth term corresponds to an equivalent force of viscous damping in the soil. It cumulates the various forms of damping in the soil including the radiation damping of the soil-pile system. There is no analytical solution for this differential equation but the experimental approach makes it possible to simplify this equation. Using the strain gauges it can deduce the bending moment profile of the pile with each step of the time. According to the theory of the beams, the relation between displacement and the bending moment is expressed as follows:

\[ y(z, t) = \int \left( \int \frac{M(z, t)}{E_p I_p} \, dz \right) \, dz + C_1 z + C_2 \]  

(2)

This equation is valid under any condition, whether the stiffness is variable or constant. Using this equation it is possible to calculate the displacement profile of the pile with each step of time. The equation (1) can then be rewritten as follow:

\[ \frac{\partial^2 M(z, t)}{\partial z^2} + \frac{E_p}{c_p} \frac{\partial}{\partial t} \left( \frac{\partial^2 M(z, t)}{\partial z^2} \right) + m \frac{\partial^2 y(z, t)}{\partial t^2} = p - c \frac{\partial y(z, t)}{\partial t} \]  

(3)

The left part of this equation (3) represents the force exerted by the pile and a portion of inertial force produced by the associated soil in vibration. The right part is the reaction of the soil which has a component related to the pressure exerted by the deformation of the ground and a component which is produced by the damping of the soil. In this equation the bending moment profile in each step of the time is known and in order to find other terms we need to double integrate these profiles to find displacement time histories \( y(z, t) \) and double derivate to find the first and second terms. To find the remaining terms derivation of the displacements with respect to time should be carried out. The procedures of double integration and of double derivation with respect to depth in each step of time are explained as follow.

**Procedure of double integration**

For each step of time, a polynomial is adjusted on the diagram of the experimental bending moments using the method of least squares. The polynomials of odd degree always give more satisfactory results than those of even degree. For the constants of integration, two following conditions were chosen, namely:

-a. The displacement at the level of the point of impact (\( Z = 2.2 \) m) was obtained by the laser incremental position sensor. It was found that the displacement of the pile must be measured more close possible to the surface of the ground for more accuracy.

-b. The displacement at pile tip was considered nil.

**Procedure of double derivation**

The profiles of experimental bending moments are treated with program SLIVALIC5 (Degny, 1985). This software was especially conceived for the calculation of the profiles of pressure. Initially, the program smoothes the groups of data by quintic splines, and then, it carries out two successive derivations. A disadvantage of the method is met on the pile head and on pile tip, i.e., at the
boundaries of the interval of smoothing. For the last two points of measurement at the ends there is no sufficient information. The calculated spline is degree three in these points; double derivation thus carries out on a line at the ends of the profiles. To overcome this problem, the data are added at the head of the pile. In static tests three points are added which correspond to the bending moments at \( Z = +0.6 \text{m}, Z = +1.2 \text{m} \) and \( Z = +1.8 \text{m} \), calculated from the lateral force applied to \( Z = +2.2 \text{m} \) multiply by distance from the load. In dynamic tests (case with 7 gauges), two points are added corresponding to the bending moments at \( Z = +0.6 \text{m}, Z= +1.8 \text{m} \), calculated from the estimated force at \( Z = +2.2 \text{m} \). In free vibration there is no external force and the pile vibrates under the inertial effect. It can be suppose that this force is concentrated in the centre of gravity of a part of the pile out of soil. However, it is possible to calculate this force, either by multiplication of the modal mass by acceleration or by the division of the bending moment obtained by the first gauge of deformation (located at \( Z = -0.6 \text{m} \)) to the distance between this gauge and the center of gravity (\( L = 2.8 \text{m} \)). The inertial forces at various times of vibration were evaluated by two methods and a very good agreement is observed.

**Evaluation of pile internal damping**

To evaluate internal damping of the pile, the pile was fixed on a support and put in free vibration. the damping ratio (\( \xi \)) is calculated by the method of the decrement logarithmic curve. The average value obtained for damping ratio (\( \xi \)) is equal to 0.9%.

**ESTABLISHMENT OF EQUATION OF MOTION**

The weight of each parameter in the equation of motion (3) can be evaluated. For a given test and a depth \( Z \) equal to \( Z= -1.8 \text{m} \) which is close to the maximum reaction, various terms of the equation were calculated. Figure 3 shows the whole of the curves for the pile with pile cap M3 which corresponds to a vibration frequency of 2.3 Hz (\( \omega = 14.45 \text{ rad/s} \)). The other parameters which were used for calculation are as follows:

- \( E_p = 7.4x10^{10} \text{ Pa (N/m²)} \)
- \( m_p = 530 \text{ kg/m (pile including wire of connections and coating)} \)
- \( C_p = 2.M_p.\omega.\xi = 2 \times 530 \times 14.45 \times 0.009 = 138 \text{ (N.s/m)/m} \)

The second term in the equation which corresponds to internal damping of the pile is:

\[
\frac{138}{7.8 \times 10^{10}} \frac{\partial}{\partial t} \left( \frac{d^2 M}{dz^2} \right) \quad \text{or} \quad 1.77 \times 10^{-6} \frac{\partial}{\partial t} \left( \frac{d^2 M}{dz^2} \right)
\]

The maximum value of this term is equal to \( 3.5 \times 10^{-4} \text{ N/m} \). It can be easily observed that the value of internal damping compared to other terms in the equation is negligible. The third term in the equation which corresponds to the inertial force can be divided into two parts, one related to the mass of the pile and the other related to the mass of the associated soil. As the term of inertia is in phase with that obtained by double derivation, the importance of the effect of pile mass can be compared with that obtained by double derivation in each step of time. For example, at the time of the first maximum of the signal (\( T = 0.75 \text{ S} \)), the value given by double derivation (first term) is equal to:

\[
\frac{\partial^2 M}{\partial z^2} (t=0.75s) = 1.4 \times 10^{5} \text{ N/m},
\]

However, the term of the inertial force of pile mass in the same step of time is:

\[
m_p \frac{\partial^2 y}{\partial t^2} (t=0.75s) = 530 \times 3 = 1590 \text{ N/m},
\]

The ratio between them is equal to 1590/1.4x10^{5} =1.1%

The inertial force of the pile is 1% of the force represented by the term resulting from the double derivation, which is completely negligible. This ratio can be changed with depth. Its maximum is close to surface since it is here that the term resulting from double derivation is minimal, but in no case this value exceeds 3%, which is negligible. Thus the equation of motion (3) can be simplified as follows:

\[
\frac{\partial^2 M(z,t)}{\partial z^2} = p - c_s \frac{\partial y(z,t)}{\partial t} - m_s \frac{\partial^2 y(z,t)}{\partial t^2}
\]

\( p \)
In this equation $c_S$ is the equivalent damping ratio of the soil and $m_S$ is the equivalent mass of the soil associated with the vibration. It thus appears that the double derivation of the profile of the bending moments gives the dynamic reaction of the ground in which, there is the term of the reaction due to the deformation of soil, the effect of damping and the effect of inertial force due to soil.

**Figure 3. Different terms of the equation of motion obtained experimentally**

**DYNAMIC P-Y CURVES**

With the procedure of double integration and double derivation, the displacement and the dynamic reaction of the pile, in various depths and various steps of time, can be calculated. In static tests, the relation between the reaction of the soil and displacement are called “P-y curves”. In dynamics tests, these relations are not any more simple curves but are presented in the form of series of hysteretic loops. The profiles of displacements, of the experimental bending moments, the shear force and the dynamic reactions of the soil are represented in Figure 4. The dynamic P-y loops are shown in Figure 5 at various depths until $Z = -3.6$ m beyond this depth the displacement of the pile is almost negligible.

The first observation is the increase in the stiffness with respect to the depth which appears by an increase in the slope of the P-y curves. This has already been observed in the static behavior. An agreement can be seen between the static P-y curves and the first cycle of dynamic P-y loops. A hysteretic reduction of damping with respect to depth is also another result of these curves. In fact, as the amplitude of the deformation is reduced, the nonlinear behavior of the soil is reduced. The same general observations are obtained for three other pile caps. One of very important results in this part is
the form of the P-y loops. This can be very useful to characterize the dynamic springs in the numerical analysis based on the assumption of Winkler.

**CONCLUSION**

A simplified differential equation of motion of a pile in vibration is established. Different terms of this equation are evaluated using experimental data. It was observed that the value of pile internal damping compared to other terms in the equation is negligible. In the same way, the effect of pile inertial force was found negligible comparing other terms.
From the profiles of the bending moments of the pile, we calculated the profiles of displacement by double integration and the profiles of dynamic reaction of the soil by double derivation in each step of time. The static and dynamic P-y curves or loops are thus established for various depths.

The increase in the stiffness compared to the depth which appears by an increase in the slope of P-y curves is well observed. This had already been observed in the static behavior. The dynamic P-y loops for the first cycle of vibration are very close to the static P-y curves. These curves highlighted the hysteretic characteristics of the soil. These curves will be very useful for the characterization of the nonlinear springs and dashpots in a numerical analysis of Winkler type.

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