FFT active contour model in grout propagation study in granular soils

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ABSTRACT

This article presents a FFT active contour model image analysis technique applied to grout flow detection during soil injection process. Among model based techniques, these models also known as deformable models are powerful image segmentation techniques that combine geometry, physics, and approximation theory. Two distinct formulations exist to employ these techniques, parametric or geometric curves. These models have proven to be effective in segmenting and tracking no rigid structures by exploiting constraints derived from the image data together with a priori knowledge about the location, size, and shape of these structures. We focus in parametric approach for deformable models because they support highly intuitive interaction mechanisms that, when necessary, allow scientists to bring their expertise to bear on the model based image interpretation task. In this paper, the development of spectral methods which are an extension of classical deformable model and its application to one-dimensional grout injection problems are described.

Introduction

DEFORMABLE models are object-delineating curves that move within two-dimensional (2D) digital images under the influence of both internal and external forces and user defined constraints. Since their introduction by Kass et al. [1], these algorithms have been at the heart of one of the most active and successful research areas in edge detection, image segmentation, shape modelling, and visual tracking. There are two general types of deformable models in the literature today: parametric deformable models and geometric deformable models. In particular, we focus on parametric deformable models which are represented explicitly as parameterized contours (i.e., curves) in a Lagrangian framework.

Segmenting structures from images and reconstructing a compact geometric representation of these structures is difficult due to the small size of the datasets and the complexity and variability of the studied medium properties i.e.: soil. Furthermore, the shortcomings typical of sampled data, such as sampling artifacts, spatial aliasing, and noise, may cause the boundaries of structures to be indistinct and disconnected.

The challenge is to extract boundary elements belonging to the same structure and integrate these elements into a coherent and consistent model of the structure.

This article presents a particular deformable model, a spectral method based on computer assisted image analysis. As the other technique deformable models stems from their ability to segment, match, and track images of various structures by exploiting constraints derived from the image data together with a priori knowledge about the location, size, and shape of these structures. Deformable models are capable of accommodating the often significant variability of soils structures over time. Furthermore, deformable models support highly intuitive interaction mechanisms. We will review the basic formulation of deformable models and their application to fundamental image analysis problems, including segmentation, and matching.

As this approach does not have high convergence rate, spectral methods have been introduced. They allow creating real time detection application. The cement grout transport evolution will be shown as an application of this technique.

Indeed, fluid displacement studies are important tools in soil physics for understanding transport of adsorbed and no adsorbed solutes through soil. Such experiments provide valuable insight about the porous media, the behavior of chemicals, and associated processes such as diffusion, dispersion, sorption during transport [1]. The mathematical conceptualization of these processes at an individual pore scale is available (e.g., Navier- Stokes equation). However, it is not possible to characterize completely the variability or heterogeneity of the pore structure. Even if it were, in most practical applications the transport domain is much too great for computer simulation on the basis of a solution using the Navier-Stokes equation. Therefore, most models provide the description of solute transport at a macroscopic scale.

The convective dispersion transport equation remains the foundation on which most analyses of solute transport in porous media have been based [2]. In this model, fluid velocities and concentration must be correctly evaluated. Therefore we develop an application which allows tracking fluid motion respect to time. Figure 1 shows a scheme of the one dimensional laboratory cement grout injection experiment.
For this study, dry Loire sand was packed into PMMA plastic cylinders having length of 90 cm and diameter of 10 cm at different defined densities. Care was taken to follow exactly the same procedure for packing all of the soil columns. Each soil column was first fully saturated by water from the column bottom using a point source. Later, cement grout was slowly injected through the column. Cement grout transport evolution was controlled by monitoring the weight of the column, the pressure at different heights and effluent volumes with respect to time (Figure 1). Cement grout position is tracked using active contour model.

Energy minimizing deformable models

The basic premise of the energy minimizing formulation of deformable contours is to find a parameterized curve that minimizes the weighted sum of internal energy and potential energy. The internal energy specifies the tension or the smoothness of the contour. The potential energy is defined over the image domain and typically possesses local minima at the image intensity edges occurring at object boundaries. Minimizing the total energy yields internal forces and potential forces. Internal forces hold the curve together (elasticity forces) and keep it from bending too much (bending forces). External forces attract the curve toward the desired object boundaries. To find the object boundary, parametric curves are initialized within the image domain, and are forced to move toward the potential energy minima under the influence of both these forces.

Mathematically, a deformable contour is a curve \( v(s) = (x(s); y(s)) \), which moves through the spatial domain of an image to minimize the following energy functional:

\[
E(v) = S(v) + P(v)
\]  

(1)

The first term is the internal energy functional and is defined to be

\[
S(v) = \int_0^1 w_1(s) \left( \frac{\partial v}{\partial s} \right)^2 + w_2(s) \left( \frac{\partial^2 v}{\partial s^2} \right)^2 ds
\]  

(2)

The first-order derivative discourages stretching and makes the model behave like an elastic string. The second-order derivative discourages bending and makes the model behave like a rigid rod. The weighting parameters \( w_1 \) and \( w_2 \) can be used to control the strength of the model's tension and rigidity, respectively. In practice, \( w_1 \) and \( w_2 \) are often chosen to be constants.

The second term is the potential energy functional and is computed by integrating a potential energy function \( P(x,y) \) along the contour \( v(s) \):

\[
P(v) = \int_0^1 P(v(s)) ds
\]  

(3)

The potential energy function \( P(x,y) \) is derived from the image data and takes smaller values at object boundaries as well as other features of interest. Given a gray-level image \( I(x,y) \) viewed as a function of continuous position variables \( (x,y) \), a typical potential energy function designed to lead a deformable contour toward step edges is
\[ P(x, y) = -w_l \left| G_{\sigma}(x, y) * I(x, y) \right|^2, \tag{4} \]

where \( w_l \) is a positive weighting parameter, \( G_{\sigma}(x, y) \) is a two-dimensional Gaussian function with standard deviation \( \sigma \), \( \nabla \) is the gradient operator, and \( * \) is the 2D image convolution operator. If the desired image features are lines, then the appropriate potential energy function can be defined as follows:

\[ P(x, y) = w_l \left[ G_{\sigma}(x, y) * I(x, y) \right], \tag{5} \]

where \( w_l \) is a weighting parameter. Positive \( w_l \) is used to find black lines on a white background, while negative \( w_l \) is used to find white lines on a black background.

For both edge and line potential energies, increasing \( \sigma \) can broaden its attraction range. However, larger \( \sigma \) can also cause a shift in the boundary location, resulting in a less accurate result.

Regardless of the selection of the exact potential energy function, the procedure for minimizing the energy functional is the same. The problem of finding a curve \( v(s) \) that minimizes the energy functional \( E \) is known as a variational problem \([3]\). It has been shown that the curve that minimizes \( E \) must satisfy the following Euler-Lagrange equation \([1,4]\):

\[ -\frac{\partial}{\partial s} \left( w_l(s) \frac{\partial P}{\partial s} \right) + \frac{\partial^2}{\partial s^2} \left( w_2(s) \frac{\partial^2 P}{\partial s^2} \right) + \nabla P(v(s, t)) = 0 \tag{6} \]

To gain some insight about the physical behavior of deformable contours, we can view Eq. (6) as a force balance equation

\[ F_{\text{int}}(v) + F_{\text{pot}}(v) = 0 \tag{7} \]

where the internal force is given by

\[ F_{\text{int}}(v) = \frac{\partial}{\partial s} \left( w_l(s) \frac{\partial v}{\partial s} \right) - \frac{\partial^2}{\partial s^2} \left( w_2(s) \frac{\partial^2 v}{\partial s^2} \right) \tag{8} \]

and the potential force is given by

\[ F_{\text{pot}} = -\nabla P(v) \tag{9} \]

The internal force \( F_{\text{int}} \) discourages stretching and bending while the potential force \( F_{\text{pot}} \) pulls the contour toward the desired object boundaries. In this chapter, we define the forces, derived from the potential energy function \( P(x, y) \) given in either Eq. (4) or Eq. (5), as Gaussian potential forces.

To find a solution to Eq. (6), the deformable contour is made dynamic by treating \( v(s) \) as a function of time \( t \) as well as \( s \)—i.e., \( v(s, t) \). The partial derivative of \( v \) with respect to \( t \) is then set equal to the left-hand side of Eq. (6) as follows:

\[ \gamma \frac{\partial v}{\partial t} = \frac{\partial}{\partial s} \left( w_l(s) \frac{\partial v}{\partial s} \right) - \frac{\partial^2}{\partial s^2} \left( w_2(s) \frac{\partial^2 v}{\partial s^2} \right) - \nabla P(v) \tag{10} \]

The coefficient \( \gamma \) is introduced to make the units on the left side consistent with the right side. When the solution \( v(s, t) \) stabilizes, the left side vanishes and we achieve a solution of Eq. (6).

We note that this approach of making the time derivative term vanish is equivalent to applying a gradient descent algorithm to find the local minimum of Eq. (1) \([5]\). Thus, the minimization is solved by placing an initial contour on the image domain and allowing it to deform according to Eq. (10).

**Fourier Spectral Algorithm for the active contour model equation**

In this section, the principles behind the Fourier spectral method will be discussed \([6,7]\) and an algorithm for implementation of the snake evolution equation using Fourier spectral method will be presented.
The basic idea of spectral methods for solving partial differential equations is to assume that \( u(x) \), the solution to the equation, can be approximated by a sum of \( N+1 \) basis functions \( \phi_n(x) \)

\[
\sum_{n=0}^{N} a_n \phi_n(x) \approx u(x)
\]  

(11)

When this series is substituted into the differential equation

\[
Lu = f(x)
\]  

(12)

where \( L \) is the operator of the differential, the result is the so-called residual function defined by

\[
R(x_i, a_0, a_1, \ldots, a_N) = Lu_N - f
\]  

(13)

If the basis function individually satisfy the homogeneous boundary conditions on \( u(x) \), then their sum will too. Therefore, the only error in \( u_N(x) \) is that it does not exactly satisfy the differential equation. Since the residual function is identically 0 for the exact solution, the challenge is to invent systematic methods for choosing the series coefficients \( a_N \) in such way that the residual function is made as small as possible. The different spectral methods differ in their method of minimizing the residual.

When the boundary conditions require the solution to be spatially periodic, the solution generation procedure is normally much simpler then for non-periodic boundaries. In the periodic case, the sines and cosines of a Fourier series automatically and individually satisfy the boundary conditions. Consequently, our only remaining task is to choose the coefficients of the Fourier series to minimize the residual function.

Consider a set of points

\[
x_j = \frac{2j\pi}{N}, \quad j \in \mathbb{N} \cap [0, N-1]
\]  

(14)

which are called collocation points. The discrete Fourier coefficients are

\[
\tilde{u}_k = \frac{1}{N} \sum_{j=0}^{N-1} u(x_j) e^{-i k x_j}, \quad -\frac{N}{2} \leq k \leq \frac{N}{2} - 1
\]  

(15)

Due to the orthogonality relation

\[
\frac{1}{N} \sum_{j=0}^{N-1} e^{ipx_j} = \begin{cases} 1 & \text{if } p = Nm, m \in \mathbb{Z} \\ 0 & \text{otherwise} \end{cases}
\]  

(16)

we have the inversion formula

\[
u(x_j) = \sum_{k=0}^{N-1} \tilde{u}_k e^{-i k x_j}, \quad j \in \mathbb{N} \cap [0, N-1]
\]  

(17)

So, the polynomial

\[
I_N u(x) = \sum_{k=0}^{N-1} \tilde{u}_k e^{-i k x_j}
\]  

(18)

is the N-degree trigonometric interpolant of \( u \) at the collocation points.

**Fourier Spectral Algorithm for the active contour model equation**

Transforming the internal energy terms should, therefore, not be a problem. The image energy, on the other hand, is a non-linear function.

Transforming such a function and integrating it into the resulting differential equation in the Fourier space may cause some computational problems. An alternative algorithm was therefore developed. Each iteration of the snake evolution equation was split into two steps. The internal energy term was iterated in the Fourier space, transformed back to the physical space where the image energy term was added.
We start by transforming the equation Eq.3.10 into Fourier domain

\[
F\left[ \frac{\partial \mathbf{v}}{\partial t} \right] = w_1 F\left[ \frac{\partial^2 \mathbf{v}}{\partial s^2} \right] + w_2 F\left[ \frac{\partial^4 \mathbf{v}}{\partial s^4} \right]
\] (19)

which gives

\[
\frac{\partial \mathbf{v}}{\partial t}(\omega, t) = w_1 (i\omega)^2 \mathbf{v}(\omega, t) - w_2 (i\omega)^4 \mathbf{v}(\omega, t)
\]

\[
\frac{\partial \mathbf{v}}{\partial t}(\omega, t) = -w_1 \omega^2 \mathbf{v}(\omega, t) - w_2 \omega^4 \mathbf{v}(\omega, t)
\]

\[
\frac{\partial \mathbf{v}}{\partial t}(\omega, t) = -(w_1 \omega^2 + w_2 \omega^4) \mathbf{v}(\omega, t)
\] (20)

This is a first-order differential equation with respect to t and its general solution is

\[
\mathbf{v}(\omega, t) = F[v(s, 0)]e^{-[(w_1 \omega^2 + w_2 \omega^4)t]}
\] (21)

where \( F[v(s; 0)] \) is evaluated using FFT.

We iterate this explicit solution with a time step and after each step transform it back to the physical space where the image energy term is added. In the \( k^{th} \) step the iteration equation is

\[
v(s, t_k) = F^{-1}[\mathbf{v}(\omega, t_k)] - \nabla \mathbf{E}_{\text{image}}(v(s, t_k))
\] (22)

**Results**

The experiments with synthetic and preprocessed grout captured images are performed using Matlab based functions for its discrete implementation. The active contour was initialized on the image extracted from captured video in case of grout injection and generated by user in the other cases. The initialization was followed by temporal and spatial snake development using an iterative procedure.

With theses two examples (figures 2.a-h), we can examine the effect of the image force -\( \nabla \mathbf{P} \) the direction of this force implies steepest descent in \( \mathbf{P} \), which is natural since we want to get a minimum of \( \mathbf{P} \). Equilibrium is achieved when \( \mathbf{P} \) is minimum in the direction normal to the curve. Although, the initial guess contour is close to an edge, instability can occur due to the discrimination of the evolution problem (figures 2.e-h).
Figure 2. Synthetic images to test robustness of the algorithm

Fortunately, this phenomenon does not occur when we realize a grout injection. To extend this application to microscopy analysis, we test a SEM image of grouted sand. The initial snake contour is shown on Figure 3.a). The Figure 3.b) shows a developed contour at the next level after 50 iterations. It illustrates that the gradient forces "pulled" the snake toward the pore. The Figure 3.c) gives the final result of the snake, fully enclosing the pore after 100 iterations.

The active contour performance is still dependent on the local maxima from the image if we only use this feature as potential function. The high gradient local values, yielded from reflection, although smoothed from the filtering, still have important contribution as a local potential energy maximum. The performance of the active contour depends of the image preprocessing, and edge map or Gradient vector Flow (GVF) computation.

To reduce time consuming due to edge map and GVF field, we implemented FFT formulation to transform 2D computation to 1D computation which allows converging 4 times faster than the initial algorithm presented in [8] on condition that users have knowledge on image processing analysis to set relevant parameters of \( w_1 \) and \( w_2 \).

We analyze one-dimensional grout injection process to target different possible shape of the grout front. The FFT active contour improves the performance of the ordinary contour by minimizing memory consuming. This technique is relevant when the size of the analyzed images is significant (more than 256×256).

Figure 4a-e) presents a first set of frames acquired during the water saturation of a dry Loire Sand column. It constitutes the reference to compare slightly the algorithm performances. This case is equivalent to a threshold segmentation which can be reduced to a binary image. Then it is easy to extract front and compare it.

The contour detected by the FFT active contour model is shown on Figure 4.f-j) for water saturation process and Figure 4 k-o) for grout injection process. In all cases, the contour encloses the correct front position. Little iterations at each step, less than 10 are needed to converge towards the final shape. This type of algorithm uses spectral method and it is well recommended to close contour. Fortunately, this technique still gives best results in grout injection image analysis. We show that active contour model and in particular parametric deformable model can be easily employed to track grout propagation in laboratory columns. However, other kind of active contour models may be used to realize image segmentation and motion tracking.
Initial flow tracking during water saturation of the Loire Sand column

Detection of water front during saturation using active contour model

Detection of grout injection front using active contour model

Figure 4. a-e) initial frame from water saturation, f-j) contour from water saturation, k-o) contour from grout injection.

Conclusions

We will start by examining properties of the FFT and active contour model approach. The use of FFT for accelerating the energy minimizing procedure in the active contour model was recently proposed by many authors working in image processing segmentation. They pointed out some of the weaknesses of the classical variational approach and suggested spectral method as a way of overcoming these obstacles. They pointed out that the Euler condition for a functional minimum is a necessary but a sufficient condition. Compared with finding the minimum of a one dimensional function, it corresponds to the requirement that the first derivative is equal zero. If it is satisfied, the point is a function extremum, yet we do not know if it is minimum or a maximum. A way to ensure that the solution is a minimum is to show that other stronger necessary conditions are satisfied. Such conditions are Jacobi condition, Legendre condition or Weierstrass condition for strong relative minimality. These conditions are often difficult to test. The dynamic approach in deformable model, on the other hand, ensures that the Legendre, Weierstrass and also the Euler condition are satisfied. Another important observation is that the evolution equation is a dynamic model searching the equilibrium which is the lowest energy position. When using the variational approach, there is a need for estimates of higher order derivatives of discrete data. This process may lead to numerical instabilities. The issue of numerical instabilities is a concern one should pay attention to if for example the explicit finite difference method is used to iterate the evolution equation. If the Fourier spectral method is used, stability is not an issue. this method is stable. Furthermore, the Fourier spectral methods are significantly more time efficient and its implementation algorithm is simpler and easier to debug. Generally, the spectral methods are much more accurate than the finite difference methods mainly employed in image processing analysis. The Fourier spectral method has proved to be more time efficient in our simulations even if they required more image processing skills.
References