Opening and Mixed Mode Fatigue Crack Growth Simulation Under Random Loading

P. C. Gope¹, S. P. Sharma², B. Kumar²
¹ Mechanical Engineering Department, College of Technology, G. B. Pant University of Agriculture & Technology, Pantnagar-263145, India
E MAIL: pcgope@rediffmail.com
² Mechanical Engineering Department, National Institute of Technology, Jamshedpur-831014, India

ABSTRACT
A model for the crack growth analysis under mixed mode random loading that includes the loading sequence effect is presented. The model was applied for the analysis of the crack growth life under different random loading on the sheets of two different aluminum alloys, 2024-T3 and 2199-T851. Analyses were carried out on the simulated loading histories obtained from different spectral densities of the nominal stress and various loading history lengths for each case.

INTRODUCTION
Engineering structure and component contains cracks of varying size, shape, and orientation. The randomly oriented crack produces mixed mode loading. In the mixed mode loading condition a crack will propagate in a non-self similar manner. Fracture mechanics under mixed mode loading condition as a part of the failure analysis can be studied in different steps out of which the study of crack growth direction and crack propagation rate are most important. Several criteria have been proposed so far for the prediction of the crack growth direction under mixed mode condition. Some of these are, maximum tangential stress criterion (MTS), strain energy density criterion (SED), or Griffith-Irwin Energy criterion [1-3].

Several parameters have been suggested by different authors to correlate the fatigue crack growth rate under mixed mode condition. Most of these are in form of Paris Equation. The parameters which have been used to correlate fatigue crack growth rate under mixed mode condition are effective Stress Intensity Factor, SED factor, J-integral, Equivalent strain intensity factor etc. A good correlation of mixed mode crack growth rate data have been seen by some of the aforementioned parameters, but there is no single parameter which gives satisfactory correlation under all loading conditions.

The load non-proportionality, over load, crack closure and T-term has influences on the fatigue crack growth behavior under mixed mode loading. Extensive studies of the effect of these factors on mixed mode crack growth rate have not been conducted yet now. Only a few studies on crack growth characteristics are available in the literature. A few of them are the study of Nayes-Hashemi, Hwang et.al [4] and, Nayes-Hashemi [5] in which they studied the effect of the mode II over load on subsequent mode I crack growth using four point and three point bending and shear specimens made of AISI 4340 steel. The number of studies of the crack growth dealing with the factor like over load, and T-stress term is very limited, and the roles and contributions of these factors in mixed mode crack growth are far from clear yet.

In this paper the effect of the over load on the crack growth characteristics has been studied. The different type of random loading has been considered for mixed mode fatigue crack growth studies. In the present paper different random loading pattern has been generated and used in the crack growth studies. The simulated results are also compared with the available experimental results.

SPECTRUM LOADING GENERATION
There are two basic ways of obtaining random loading histories, namely (a) directly from the system under service condition (b) by computerized simulation. In the present study all the load history used in the crack growth analysis are simulated through computer. Different loading histories defined by their spectral density function and length (represented by number of the peaks) were generated. Six different form of the spectral density function of normal stresses, S(w) have been used. Table 1 lists the characteristic values of the spectral densities considered. The meaning of each symbols are illustrated in Fig. 1

<table>
<thead>
<tr>
<th>Type</th>
<th>W₁</th>
<th>W₂</th>
<th>W₃</th>
<th>W₄</th>
<th>H₁/H₂</th>
<th>α*</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>18</td>
<td>20</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.9982</td>
</tr>
<tr>
<td>E</td>
<td>7</td>
<td>27</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.85</td>
</tr>
<tr>
<td>F</td>
<td>5</td>
<td>40</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.795</td>
</tr>
<tr>
<td>G</td>
<td>5</td>
<td>15</td>
<td>80</td>
<td>130</td>
<td>2.0</td>
<td>0.64</td>
</tr>
<tr>
<td>H</td>
<td>5</td>
<td>25</td>
<td>30</td>
<td>60</td>
<td>2.0</td>
<td>0.705</td>
</tr>
<tr>
<td>P</td>
<td>5</td>
<td>15</td>
<td>25</td>
<td>75</td>
<td>6.67</td>
<td>0.756</td>
</tr>
</tbody>
</table>

Fig. 1 Spectral densities
\( \alpha \) is a parameter descriptive of the load irregularities and indicative of band width. It is defined as

\[
\alpha = \frac{M_2}{\sqrt{M_0 M_4}}
\]

where \( M_n = \int_{-\infty}^{\infty} w^n S(w) dw \) is the \( n^\text{th} \) order moment. Six loading history length, \( L \) of 300, 500, 1000, 2000, 3000 and 5000 peaks were taken. For each type of spectrum density and number of peaks, 25 loading histories were generated. The simulation was done by superimposing the sinusoidal function of the random phase with uniform distribution. The mean square stress of each loading histories were kept at constant value of 1947 MPa \(^2\). Each group were designated by letter L followed by number of peaks and type of spectra. Example L300G means G type of loading spectra with 300 numbers of peaks. Some of the results are shown in Fig. 2.

The power spectral density for other random loading types E-P described in Table 1 are shown in Fig. 3. Fig. 4 shows simulated load histories for different spectral density function.
Fig. 3 Simulated PSD for assumed spectral density function (Refer Table 1)

(a) Load history for L100P

(b) Load history for L100E

Fig. 4 Simulated load histories for different spectral density functions
CRACK GROWTH MODEL

Most of the crack growth rate equations proposed for mixed mode loading are based on either strain energy density approach or stress intensity factor approach. Sih & Bathelemy [6] have proposed the following relation

\[
\frac{da}{dN} = 0.02(\Delta S)^{1.58}
\]  

(3)

where \( \frac{da}{dN} \) is in m/cycle and \( \Delta S \) is in MJ/m².

Patel & Pandey [7] proposed the following relation to study the crack growth rate

\[
\frac{da}{dN} = C \left( \frac{4\mu}{1-2\nu} \Delta S \right)^{n/2}
\]  

(4)

where \( C \) and \( n \) are mode I Paris constants. The comparison of the above two equations was made with the experimental data of 2024-T3 aluminum alloy. It is found that the experimental data points are not in agreement with the predicted values obtained from Eqns. 3 and 4 for given \( \Delta S \). Equation given by Sih & Bathelemy predicts the higher crack growth rate whereas Eqn. 4 predicts lower rate as compared to the experimental crack growth data [10].

The crack opening stress is not included in any one of the above SED based models. Hence, a closure parameter based on strain energy density factor has been introduced in the present crack growth model. From the experimental results, the following type of equation has been obtained.

\[
\frac{da}{dN} = (8.2186) \times 10^{-07} (\Delta U_s)^{1.0394}
\]  

(5)

where \( \frac{da}{dN} \) is in m/cycle and

\[
\Delta U_s = \frac{\Delta S_{\text{min}} - \Delta S_{\text{op}}}{\Delta S_{\text{cri}} - \Delta S_{\text{op}}}
\]  

(6)

where \( \Delta S_{\text{min}} = S_{\text{max}}^\text{min} - S_{\text{min}}^\text{min} \)  

(7)

and \( S_{\text{max}}^\text{op} = S(\theta_0, \sigma_{\text{max}}) \), is the maximum strain energy density factor, \( S_{\text{min}}^\text{op} = S(\theta_0, \sigma_{\text{min}}) \), is the minimum strain energy density factor, \( \Delta S_{\text{op}} = S(\theta_0, \sigma_{\text{op}}) \), is the opening strain energy density factor, \( \Delta S_{\text{cri}} = S(\theta_0, \sigma_{\text{cri}}) \), is the critical strain energy density factor. \( S \) is the strain energy density factor and given elsewhere [6]

CRACK CLOSURE MODEL

Newman [8] calculated the crack opening stress for CCT specimen subjected to uniaxial constant amplitude loading, and obtained the following equation fitted to the numerical results.

\[
\frac{S_{\text{op}}}{S_{\text{max}}} = A_0 + A_1 R + A_2 R^2 + A_3 R^3 \quad \text{, } R \geq 0
\]  

(8)

\[
\frac{S_{\text{op}}}{S_{\text{max}}} = A_0 + A_1 R \quad \text{, } -1 \leq R < 0
\]  

(9)

\( S_{\text{op}} \) is the opening stress, \( S_{\text{max}} \) is the maximum stress. The coefficients are given as,

\[
A_0 = (0.825 - 0.3\alpha + 0.05\alpha^2)^\frac{1}{\alpha} \left( \cos \left( \frac{\pi}{2} \frac{S_{\text{max}}}{\sigma_0} \right) \right) \quad \text{, } A_1 = (0.415 - 0.071\alpha) \frac{S_{\text{max}}}{\sigma_0} \quad \text{, } A_2 = 1 - A_0 - A_1 - A_3 \quad \text{, } A_3 = 2 A_0 + A_1 - 1
\]

The above equation is a function of stress ratio, \( R \), stress level, \( S_{\text{max}} \), and three dimensional constraint factor \( \alpha_c \). Generally, plane stress or plane strain conditions are simulated with \( \alpha_c = 1 \) or 3, respectively. Recently McMaster and Smith [9] presented a third order polynomial equation for \( \alpha_c \) in the range of 0.27 < \( \alpha_c < 1.15 \) for 2024-T351 Al alloy. The variation of constraint factor as given by them is:

\[
\alpha_c = \begin{cases} 
1.0 & \text{if } \phi > 1.15 \\
3.0 & \text{if } 0.27 \leq \phi < 1.15 \\
-0.754\phi^3 + 5.179\phi^2 - 8.219\phi + 4.864 & \text{if } 0.27 < \phi < 1.15 
\end{cases}
\]  

(10)

\( \phi = \Delta K / (\sigma_0 \sqrt{f}) \), \( t \) is the specimen thickness, \( \sigma_0 = \sigma_y + \sigma_u / 2 \)

Incorporating this in Eqn. 8 we can derive as [10-11]

\[
\frac{S_{\text{op}}}{S_{\text{max}}} = 0.3725 + 0.6275(A_0 + A_1 R + A_2 R^2 + A_3 R^3)
\]  

(11)
The variation of above equation for different constraint factors is shown in Fig. 5 along with some of the experimental results. Figure shows that $S_{op}/S_{max}$ can be well approximated by properly selecting the constraint factor.

MATERIALS & METHOD
The materials on which the simulations were carried out is 2024-T3 Al alloy. The material properties are $E=67850.0$ MPa, $\sigma_Y = 370$ MPa, $
u = 0.334$, $K_I = 2.75$ MPa $\sqrt{m}$, $\Delta K_{IC} = 50.22$ MPa $\sqrt{m}$. Cycle-by-cycle simulation of crack growth from specified crack length $a_I = 10$ mm to final crack length $a_f = 25$ mm has been carried out. The growth lifetimes obtained for different loading history and different groups are presented in Figs 7-11. The procedure is explained in Fig. 6. The specimen geometry is described in reference [10-11].

RESULTS & DISCUSSION

Mode I Fatigue life under random loading
To prove the validity of the presented model, another group of random loads (M81,...) available in literature [12] are considered. The growth lifetimes were predicted using the presented model. Table 3 shows the summary of the predicted ratio and for different loading along with the predicted results given in reference (ASTM STP 748). The average ratio and standard deviation estimated from the results of present method are 1.118 and 0.259 where as these values are 0.846 and 0.213 obtained from JC(1) model. JC(1) is based on Walker Crack growth rate equation, compressive stress set to zero, and tensile load interaction not accounted for.

Table 3. Absolute comparison of the test and predicted life of 2219-T851 Al alloy under mode I condition

<table>
<thead>
<tr>
<th>Load</th>
<th>$a_I$ (mm)</th>
<th>$a_f$ (mm)</th>
<th>Test</th>
<th>Present</th>
<th>Life ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>M81</td>
<td>4.06</td>
<td>13.0</td>
<td>115700</td>
<td>179875</td>
<td>1.554</td>
</tr>
<tr>
<td>M82</td>
<td>3.81</td>
<td>35.4</td>
<td>58585</td>
<td>59957</td>
<td>1.023</td>
</tr>
<tr>
<td>M83</td>
<td>3.81</td>
<td>23.3</td>
<td>18612</td>
<td>15000</td>
<td>0.806</td>
</tr>
<tr>
<td>M84</td>
<td>4.00</td>
<td>55.9</td>
<td>268908</td>
<td>329850</td>
<td>1.227</td>
</tr>
<tr>
<td>M85</td>
<td>3.87</td>
<td>32.8</td>
<td>36397</td>
<td>35314</td>
<td>0.970</td>
</tr>
<tr>
<td>M88</td>
<td>3.81</td>
<td>45.8</td>
<td>380443</td>
<td>357101</td>
<td>0.939</td>
</tr>
<tr>
<td>M89</td>
<td>3.81</td>
<td>38.4</td>
<td>164738</td>
<td>184149</td>
<td>1.118</td>
</tr>
<tr>
<td>M90</td>
<td>3.81</td>
<td>51.6</td>
<td>218151</td>
<td>331834</td>
<td>1.521</td>
</tr>
<tr>
<td>M91</td>
<td>3.81</td>
<td>36.1</td>
<td>65627</td>
<td>78269</td>
<td>1.193</td>
</tr>
<tr>
<td>M92</td>
<td>3.81</td>
<td>29.5</td>
<td>22187</td>
<td>18315</td>
<td>0.826</td>
</tr>
</tbody>
</table>

Mixed Mode Fatigue life under Random loading
Simulations of crack growth are carried out considering the generated load histories described above. The fatigue lives obtained are presented below in Figs. 7-11.
Input initial, final crack length, loading history and constants of crack growth model used in the analysis, crack angle etc.

Enter over load factor, $X_{OL}$

Calculate over load $S_{OL} = X_{OL} \cdot S_{max}$

Initialization $\text{i cycle}=1$

$I=1$
$J_{ol}=0$

Calculate stress intensity factors, stress ratio

Compute opening stress $S_{op}(I)$ from empirical relation
$\text{Sigop} = S_{op}(I)$

$S_{max}(I) \geq S_{OL}$

No
Yes
If Iol < 1:

- Search next maximum stress which is greater than S_{OL}. Find position j where S_{max} ≥ S_{OL}.
- Jol = j

Find rms of S_{max} & S_{min} between two consecutive overloads S_{max}(Iol) & S_{max}(Jol). Rms values are S_{opx1} & S_{opx2}.

If Iol < 1:

- Compute crack opening stress taking maximum rms
  \[ S_{op}(I) = S_{opx} + \frac{Jol - I - 1}{Jol - Iol - 1} (S_{op}(Iol) - S_{opx}) \]

If S_{op}(I) ≤ 0:

- Yes: S_{op}(I) = S_{opx}
- No: Sigop = S_{op}(I)

If Sigop ≥ S_{max}:

- Is: Sigop ≥ S_{max}
- No: da = 0

Evaluate opening stress intensity factors:

- Sigop = S_{op}(I)
- K_{top} = K_{topi}

Evaluate Stress Intensity Factors:

- B

If K_{top} ≤ K_{min}:

- Yes: K_{top} = K_{min}

Compute crack extension angle:

- Compute ΔS_{min}
- Compute ΔS_{Cri}, ΔS_{OP}

Compute critical condition:

- Compute ΔS_{Cri}, ΔS_{OP}

- R
Fig. 6 Flow chart for the crack growth studies including sequence effect
**Probability distribution:** Fig. 7(a-b) shows the probability distribution of fatigue life for $\alpha = 60^\circ$ for bandwidth $\alpha^* = 0.756$ and history length of 1000 peaks of stress on normal and Weibull probability paper. It is seen that normal distribution is well fitted to the data than Weibull distribution. Fig. 8 shows the effect of the bandwidth on the probability distribution of the life. The probability distribution of two different type of random loading obtained for bandwidth 0.795 and 0.705 and same history length of 1000 stress peaks are shown for crack position $\alpha = 30^\circ$ on normal probability paper. From the figures it can be seen that in case of life obtained from the high bandwidth the fit is excellent on normal and Weibull probability paper whereas for low bandwidth normal distribution can be best approximated. It is clear from the figure 8 that all bandwidths except the narrowest bandwidth $\alpha^* = 0.9982$ can be grouped into one category whereas life data with bandwidth 0.9982 shows different behavior and shows almost constant life for all 25 tests although the loading groups are generated randomly. This may be due to extreme narrow bandwidth where sequence effect can be considered as small due to quick repetition of the load causing the sequence effect. This loading history shows similar effect as that of constant amplitude loading. Fig. 9 shows the effect of history length on the probability distribution of the fatigue life. The presented results show that, between the two distributions, the normal distribution produces a better overall fit to the fatigue life data. The similar results are presented by Domigenez et al [12] for horizontal crack (mode I) under random loading.

**Mean life, $\mu$:** The effect of bandwidth and history length on mean fatigue life is shown in Fig. 10 for $\alpha = 30^\circ$. It is seen that mean life remains constant for random loads having history lengths more than 1000 peaks. The history length less than 1000 shows major influence in the mean life. The bandwidth has also major influence on the mean fatigue life. It is seen that mean life increases with decrease in bandwidth up to certain bandwidth approximately equal to the bandwidth of white noise (0.795), there after, it decreases. These types of variations have been observed for all history length.

![Fig. 7 Probability distribution of fatigue life for $\alpha = 60^\circ$, L100P type load on (a) Normal probability paper (b) Weibull probability paper](image)

![Fig. 8 Probability distribution of Fatigue life for $\alpha = 30^\circ$, Bandwidth=0.64, crack angle =30 deg](image)

![Fig. 9 Probability distribution of fatigue life for different HL](image)
The effect of highest peak stress generally known as over load in the load history on fatigue life is shown in Fig. 11 for different bandwidths and history lengths. It is seen that as the peak stress increases the resulting mean fatigue life also increases. This is due to retardation mechanism occurring due to overload.

CONCLUSIONS
In the present work a crack growth model derived considering experimental crack opening and growth results of aluminum alloys was used. The results reported here are concluded as follows:
1. The history length to be used is a decisive element in making a test or simulation of crack growth process under random loading.
2. The use of too short histories and their repetition until failure may produce non conservative results and provide much longer lives than those obtained with longer loading histories, which will be closer to facts.
3. In estimating fatigue lives under random loading it is required to generate or test with different representative random loading histories and to determine the life distribution.
4. Life obtained for bandwidth between 0.705 to 0.795 shows excellent fit both on Normal and Weibull Probability paper. Narrowest or broadest bandwidth shows poor fit on Weibull probability paper
5. Peak stresses in the loading cycles has significant effect on opening stress and hence on the crack growth.
6. Mean life increases with decrease in Bandwidth up to certain Bandwidth equals to Bandwidth of white noise, there after it decreases

REFERENCES