MECHANICAL AND THERMAL BEHAVIOUR IDENTIFICATION FOR GLASS WOOL

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ABSTRACT
A Mineral wool is a cellular solid made of fibres with micrometric diameter and millimetric or centimetric length sprayed with a binder, and cured in an oven for freezing the arrangement of the fibres and hence providing some elasticity. To increase the mechanical performances, the fibre mat is processed before the curing stage to produce a structure endowing the material with a higher mechanical strength. This step is called “crimping” and it produces a better mechanical behaviour due to a more favorable fibre orientation. However, the increase of stiffness goes together with an enhanced thermal conductivity, so that depending on applications, a compromise between mechanical and thermal properties has to be obtained. The aim of the present study is to relate the thermomechanical behaviour of a crimped glass wool product with its initial texture.

To characterise the texture of a finished product, a specific software has been developed. Through the analysis of the autocorrelation function of small zones, it provides, from a textured product image, a local anisotropy field, i.e., the dominant orientation and its associated amplitude [1].

For the mechanical part, Digital Image Correlation (based on Q4 finite element shape functions [2]) is used to measure the displacement field of mineral wool samples subjected to mechanical tests. A linear elastic anisotropic Finite Element code has been designed in order to compute the mechanical response of the material as a function of the initial texture of a mineral wool sample and the loading. The anisotropic elasticity properties are estimated via an inverse identification procedure.

For thermal properties, the temperature field of mineral wool sample subjected to a stationary uniaxial conduction heat flux is obtained by using a middle wave Infra Red camera. A linear stationary anisotropic Finite Element code has been designed in order to compute the temperature response of the material as a function of the initial texture and the loading conditions. The anisotropic conductivity properties are estimated via an inverse identification procedure.

Introduction
The crimping process shown schematically in Figure 1 (top) performs a compression of the fibre mat both across its thickness and along the line direction. This process modifies significantly the fibre orientation as shown through some examples in Figure 1 (bottom). As a result, the elastic response of the finished product and its load bearing capacities are very sensitive to this crimping process.

Figure 1. Crimping process and different textures
Identification method

The identification procedure developed herein is presented in Fig. 2. The fibre mat is first characterized locally in terms of its texture, i.e. the local dominant orientation of the fibres. Then it is assumed that at the same scale, the elastic properties or the thermal conductivity can be considered as constant but anisotropic with principal axes imposed from the texture analysis. The next step is to obtain full field temperature or displacement maps from experimental measurement. The former is obtained from a IR camera, while the latter is the result of a digital correlation analysis from pictures taken on the sample under different stages of loading. A final identification procedure is proposed to quantify the elastic constants or the thermal conductivity from a finite element computation of the temperature, or of the displacement, based on the texture field, which aims at reproducing the measured fields.

![Figure 2. Mechanical (left) and thermal (right) procedure of identification.](image)

Anisotropy analysis

In order to analyse the texture of the map, digital images of a free side of the sample are exploited. The first step is to split the pictures into several zones of interest, ZOI (Fig. 3) where the anisotropy parameters are considered as constant in each ZOI. The anisotropy analysis consists in finding the main orientation $\Theta$ and anisotropy amplitude $A$ [1]. They are obtained from the auto-correlation function of the grey levels. Basically, the curvature tensor of the autocorrelation function is computed and the principal axis of least curvature defines the anisotropy axis, and the contrast between both curvatures gives access to the amplitude of anisotropy. Figure 3 illustrates these two values regarding the autocorrelation analysis of each ZOI.

![Figure 3. Illustration of an anisotropy analysis.](image)

Experimental tests
Regarding experimental thermal characterization, different tests are conducted. All of them are carried out in the steady state regime of stationary temperatures. Two types of boundary conditions are considered:

- uniaxial conduction,
- hotwire conduction.

These experimentations are not done yet, the evaluation of the temperature field will be obtained by thermography using a CEDIP Middle Wave Infra Red camera with 320x240 pixel, 14 bits resolution, and noise equivalent thermal difference equal to 20 mK.

For mechanical tests, imposed displacement are imposed either in

- uniaxial compression (Fig. 4),
- biaxial compression / shear (Fig. 5).

Some uniaxial tests have been performed [3] on different types of crimped materials (Fig. 1) that lead to very different mechanical responses (Fig. 4). Figure 4 shows that the texture of a product has a strong impact on the compressive response.

![Figure 4. Schematic view of a compression test on mineral wool and typical stress / strain responses](image)

Biaxial tests have been performed on a hexapod to induce compressive and shear loadings (Fig. 5). This device offers the possibility to follow complex stress path and open the way to characterize the mat under loadings close to service conditions. Although the present analysis is limited to the elastic regime, extensions to non-linearities can easily be accessed experimentally with this set-up.

![Figure 5. Schematic view of the hexapod in its initial and final position for a shear test](image)
The texture of the mat after crimping can be strongly heterogeneous, and hence our analysis calls for a local characterization. For this purpose, a full field analysis of the displacement under different loadings is necessary. A digital image correlation (DIC) technique [2] is applied to a series of pictures taken at different stages of the loading, from a side view. The specificity of the latter is that it provides a continuous displacement field based on finite element shape functions (quadratic elements, polynomials of order 1 in both space directions). The latter techniques allows one to obtain both a good resolution (16x16 pixels per element) and good accuracy (typically $5 \times 10^{-2}$ pixel) in the measurement.

Subsequently, one performs an identification procedure to obtain the optimized thermal and mechanical parameters.

**Temperature and displacements fields computation**

**Finite Element Method**
To obtain the computed displacement and temperature fields, a Finite Element code was written. It takes into account the local anisotropy of the material. The material is modelled as a locally orthotropic medium. The local orientation and anisotropy amplitude are included in the model from the direct texture analysis of the studied sample.

**Homogenised conductivity**
Locally the material is assumed to be characterized by an anisotropic conductivity tensor characterized by its two eigen conductivities, $k_n$ and $k_t$. If the anisotropy amplitude $A = 1$, the material is locally non-crimped, and thus, the original conductivity tensor is simply rotated to align to the determined direction of anisotropy from the texture analysis. Otherwise (i.e., $A < 1$), it is assumed that the crimping takes place at a smaller length scale than the element size, and an equivalent conductivity tensor, $\langle K \rangle$, is assumed to result from a homogenization procedure. In the present case, a Reuss’ approximation (all directions are “equally loaded”) is made.

$$\langle K \rangle^{-1} = \int_{-\pi}^{\pi} K^{-1} (\theta) p(\theta) d\theta$$

(7)

where $K(\theta)$ denotes the intrinsic conductivity tensor rotated by an angle $\theta$, and $p(\theta)$ is a Gaussian distribution adjusted so that its mean and width match the determined local orientation and anisotropy amplitude. The effect of the anisotropy amplitude is thus such that in its principal axis, the conductivity tensor has the following form eigenvalues

$$\langle k_1 \rangle^{-1} = \frac{1}{2} \left[ k_n^{-1} + k_t^{-1} \right] + \left[ k_n^{-1} - k_t^{-1} \right] A,$$

$$\langle k_2 \rangle^{-1} = \frac{1}{2} \left[ k_n^{-1} + k_t^{-1} \right] - \left[ k_n^{-1} - k_t^{-1} \right] A.$$  

(9)

As a consequence, the entire conductivity map is obtained from the texture analysis of the real material, and is parameterized by the two conductivities of an ideal uncrimped material. The same strategy is used to describe the anisotropic elastic
behaviour and rotation and Reuss homogenization are used. Four parameters are now required to describe the elastic
problem.

Using experimentally measured boundary conditions (either temperature from IR measurements, or displacement from DIC), a
standard finite element computation can be performed to compute the entire temperature or displacement fields.

**Identification method**

The identification process is based upon the comparison between the computed and measured temperature or displacement
fields. For the thermal problem, the following equations are to be satisfied where $K$ is the conductivity field computed from the
above described anisotropic texture and two initial guessed values for the conductivity, $k_n$ and $k_t$.

$$\text{div}(- [K] \text{grad} T) = 0 \quad ; \quad T_{1 \Gamma} = T_i \quad ; \quad -[K] \text{grad} T_{2 \Gamma} = \Phi_i$$  \hspace{1cm} (11)

The solution is computed by using the Finite Element code. The global conductivity matrix is assembled and following linear
system is solved

$$[K_{FEM}]_g T_0 = F$$ \hspace{1cm} (12)

Once a first estimate of the temperature field is computed, one evaluates additional fields, $T_n$ and $T_t$, which give the evolution
of the temperature field subjected to an infinitesimal change of any of the constitutive parameters, $k_n$ and $k_t$.

$$[K_{FEM}]_g T_n + \frac{\partial [K]}{\partial k_n} T_0 = \Phi_{\Delta k_n}$$
$$[K_{FEM}]_g T_t + \frac{\partial [K]}{\partial k_t} T_0 = \Phi_{\Delta k_t}$$ \hspace{1cm} (13)

where $\Phi_{\Delta k_n}$ and $\Phi_{\Delta k_t}$ are the corresponding change in boundary conditions due to a modification of the conductivity tensor
(they are zero for a Dirichlet condition). As the temperature field $T_0$ was computed from arbitrary values of the conductivity, it
does not generally match the measured field. Thus, corrections to $k_n$ and $k_t$, $\Delta k_n$ and $\Delta k_t$, are obtained by minimization of the
following function

$$J(\Delta k_n, \Delta k_t) = \int (T_0 - T_m - \Delta k_n T_n - \Delta k_t T_t)^2 \, dx$$ \hspace{1cm} (14)

which projects the difference between measured and computed temperatures onto the two influence fields. As the problem is
not linear, it is necessary to iterate to find the optimum solution. An error indicator is computed to evaluate the quality of the
identification process

$$e = \frac{(T_0 - T_m)^2}{T_m}$$ \hspace{1cm} (15)

The problem is solved with the same ingredients for the mechanical part except that four parameters are to be identified and
displacements are to be substituted to the temperature.
Identification results

Thermal case
In this first part, the method is validated on artificial cases. The boundary conditions used are a temperature equal to zero on the right edge, a temperature equal to 19°C in the middle of the left edge and zero heat fluxes elsewhere (Fig. 7). It is representative of the hot-wire experiment. The initial conductivity ratio $K_n/k_t$ is equal to 2.5.

![Figure 7. Direct computation using an anisotropy analysis of a real image](image)

The identification is performed using the “measured” temperature field on the boundaries and an initial conductivity ratio equal to 5. An additional white noise over 5 percent of variation of temperature is applied to the “measured” temperature to mimic the noise in the IR camera image acquisition.

![Figure 8. Identification results: The final identified temperature field is shown on the left, as well as its difference with the referenced one (center). On the right, the rate of convergence is plotted (ratio of conductivities versus iteration number).](image)

A very good agreement is obtained between measured (Fig. 7) and identified (Fig. 8) temperature fields. The conductivity ratio error is equal to $1.2 \times 10^{-7}$ and the relative temperature error is equal to $2.1 \times 10^{-10}$, in spite of the added noise. The key to this precision is the large amount of data of the temperature field which is used to determine the two conductivity constants. The feasibility of the analysis is thus assessed. However, actual measurements are not yet available.

Mechanical case
A validation of the procedure based on artificial cases, similar to the previous example has been performed successfully. In this case however, experimental data were obtained from digital image correlations. The mechanical identification has been tested on both uniaxial compression and compression/shear tests. Table 1 gives the optimised elastic properties, and Fig. 9 shows that a reasonable agreement is observed between measured and predicted displacement fields.

![Figure 9. Uniaxial compression test and identification results](image)

A global error in terms of displacements is less than 6.5%.

<table>
<thead>
<tr>
<th>Table 1. Identified local elastic properties of compression specimen</th>
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<tr>
<td>Identified value</td>
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The same of analysis is performed for the biaxial experiment on another sample. The global displacement error is now larger (of order 7.4%). However, the identification is deemed reasonable (Fig. 10 and Table 1.).

<table>
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<tr>
<th>Table 2. Optimised local elastic properties obtained from a shear test</th>
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<tr>
<td>Identified value</td>
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Figure 10. Biaxial compression / shear test and identification results

Identified value

| Identified value | $S_{uu} = 0.87$ MPa | $S_{dd} = 0.51$ MPa | $S_{tt} = 2.03$ MPa | $S_{uu} = 1.9$ MPa |
Conclusion and perspectives

This study presents a global identification procedure for thermal and mechanical properties which is based on a similar procedure. From an experimentally observed texture of a heterogeneous material, it allows one to have access to the local transport or elastic properties. Being based on full field data measurements, it allows for a full benefit of a rich amount of data and not only global macroscopic measurements. For thermal measurement, the procedure has only been tested against artificial data to establish its feasibility. For mechanical measurements, the complete procedure (mechanical test with image acquisition, DIC analysis of the displacement field, texture analysis and identification), has been followed up to the quantification of the elastic constants.

On going work consists in temperature map measurements, and more complex loading path to resolve the large variability observed on the elastic constants depending on the test.
References