ASSESSMENT OF DIGITAL IMAGE CORRELATION PERFORMANCES

Workgroup “Metrology” of CNRS research network 2519a “MCIMS”,
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ABSTRACT

Optical full-field measurement methods such as Digital Image Correlation (DIC) are increasingly used in the field of experimental mechanics, but they still suffer from a lack of information about their metrological performances. In order to assess the performance of DIC techniques, some collaborative work has been carried out by the Workgroup “Metrology” of the French CNRS research network 2519 “MCIMS”. Basically, the study is based on displacement error assessment from synthetic speckle images. First, some series of synthetic images with random patterns and submitted to sinusoidal displacements with various frequencies and amplitudes have been generated. Then displacements are evaluated by several DIC packages that are based on various formulations. Different correlation window sizes and speckle pattern sizes are tested. Displacements found are finally compared with the exact imposed values. Results show that the overall RMS error seems to be mainly controlled by the first order difference between the real transformation and the local transformation of the correlation window assumed by the DIC algorithm. Two limiting situations have been observed: on one hand, displacements varying at a low spatial frequencies for which the classical error observed for rigid motions is recovered. On the other hand, displacement fields with high spatial frequencies for which the error is equal to the RMS of the displacement itself (this situation does not allow DIC evaluation).

Introduction

Optical full-field measurement techniques are very promising tools for the experimental analysis of the mechanical properties of materials and structures. While they are more and more widely used, they still suffer from the lack of a complete metrological characterisation. Such techniques rely on complex measurement chains, and the error sources of each of its elements require proper evaluations before a global assessment of the measurement. The collaborative work carried out by the members of the workgroup “Metrology” of the French CNRS research network 2519, coordinated by Y. Surrel (Visuol Technologies) and F. Brémand (Univ. Poitiers), aims at contributing to a systematic and quantitative approach to these questions [1].

Digital Image Correlation (DIC) technique is among the most popular optical methods, because of the availability of commercial packages, the constantly shrinking cost of digital cameras and computers, and the general apparent simplicity of sample preparation and optical setup. An important, but not sole, element of the measurement procedure is the image analysis software package supposed to provide an apparent 2-D displacement field that maps a so-called “reference image” to a “deformed image”, provided by an appropriate optical system, at a discrete set of positions, according to some principle of optical flow conservation.

The analysis of rigid body motions (both on real or simulated images) shows that the ultimate resolution of this mapping, in pixels, is bounded by the actual dynamic range of the digital images (noise divided by the full grey levels range). This can be justified by the fact that a motion smaller than this limit does not generate any significant modification in the images. In practice, resolutions close to 1 over 100 pixels can be achieved with noiseless 8 bits images. Note however that such values can only be reached when the speckle pattern used by the matching algorithm exhibits appropriate characteristics in terms of

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grey level histograms, typical size with respect to pixel and correlation window size, spectral content, etc. Issues relative to the definition of an optimal pattern are not addressed in this presentation [2], even if they are part of the research group discussions.

Such quantitative evaluations of the errors of DIC measurements are usually limited to situations with homogeneous mechanical transformations, namely, simple uniform translation, in plane rotations, or out of plane rigid body motions, which result in apparent (almost) affine transformations of the 2-D image [3,4]. Very few studies [2,4,5] address situations with spatially fluctuating displacement fields, which need to be investigated for a quantitative assessment of the spatial resolution of such techniques. Since it is very difficult, and in practice impossible, to experimentally generate non uniform deformation fields, with precisely prescribed strains (some authors have investigated this way, e.g. [6]), it is necessary to perform the analysis on simulated images, obtained with algorithms that mimic as closely as possible the generation of images in a real camera. In Ref. [5] quadratic displacement fields have been considered; the present approach extends the analysis to in-plane sinusoidal displacements exhibiting varying spatial frequencies.

The proposed methodology is similar to that leading to the Modulation Transfer Function classically used to characterize optical devices. The RMS errors of the displacements obtained with various DIC softwares are evaluated as functions of the spatial frequency and the amplitude of the displacement field, for various correlation window sizes and DIC formulations.

**Methodology**

Two sets of synthetic speckle painting images have been generated. In the first set, Wima (provided by S. Coudert, ENSAM Paris and S. Mistou, ENIT) the images are defined as random distributions of circular spots with a Gaussian local variation of the grey levels. The second set of synthetic images (provided by L. Robert and J.J. Orteu, EMAC) has been obtained using the TexGen software [7]. The method is based on successive transformations of Perlin's coherent noise function to produce the desired pattern aspect. It is worth noting that no bias or interpolation errors are induced by the image generation because the speckle is deformed in the real space before being mapped in the object space. In both sets, parameters were selected such that the radius $r$ at half height of the autocorrelation function of the reference image is 3 pixels. Deformed images are obtained assuming an in-plane sinusoidal displacement. In a first approach, only tensile/compressive displacements along the x direction with a zero y displacement are considered. The displacement along x is given by:

$$u_x(X, Y) = \alpha \sin \left( \frac{2\pi X}{p} \right)$$  \hspace{1cm} (1)

where $p$ is the period in pixels and $\alpha$ is the amplitude of the fluctuation of the xx component of the displacement gradient. The xx component of the first and second displacement gradients are given by Eq. (2) and (3) respectively:

$$u_{xx}(X, Y) = 2\pi\alpha \cos \left( \frac{2\pi X}{p} \right)$$  \hspace{1cm} (2)

$$u_{xy}(X, Y) = -\frac{4\pi^2\alpha}{p} \sin \left( \frac{2\pi X}{p} \right)$$  \hspace{1cm} (3)

Grey levels of the pixels in the deformed images are obtained according to the optical flow conservation rule:

$$g(x) = f(\phi^{-1}(x))$$  \hspace{1cm} (4)

where $\phi(X) = X + u(X)$ is the transformation map and $g$ (resp. $f$) is the grey level in the deformed (resp. reference) image. Note that appropriate procedures have been used in the TexGen software in order to mimic the (perfect or imperfect) spatial integration of light performed by a real image sensor. All generated sets contain 1024 x 1024 or 512 x 512 pixel image and various deformed configurations ($\alpha \in \{0.1, 0.05, 0.02, 0.01, 0.005, 0.001\}$ and $p \in \{10, 20, 30, 60, 130, 260, 510\}$ pixels). The set considered in the present work have been generated using the TexGen software. It contains one reference 512 x 512 pixel image and 28 deformed configurations obtained with $\alpha \in \{0.02, 0.01, 0.005, 0.001\}$ and $p \in \{10, 20, 30, 60, 130, 260, 510\}$ pixels. Note that values of the maximum of the displacement gradient are $u_{xx}^{max} = 2\pi\alpha \epsilon \{12\%, 6.3\%, 3.1\%, 0.63\%\}$ respectively. At this stage only the 8 bit (256 grey levels) images have been processed so far.

Figure 1 presents, for the considered set of images, a sub-image (512 x 100 pixel image) of the reference image and deformed images obtained for $p = 130$ pixels (four periods) and $\alpha = 0.02, 0.05$ and 0.1 respectively.
In order to study the influence of the speckle pattern size on the displacement ultimate resolution (resolution for low frequency deformation field), the speckle pattern mean size (radius \( r \) at half height of the autocorrelation function of the ref. image) can be adjusted in order to produce fine (\( r/2 \)), medium (standard, \( r = 2.2 \) pixels) or coarse (\( 2r \)) patterns, as presented in Figure 2.

Images have been processed with seven DIC packages, namely 7D (P. Vacher, Univ. Savoie) [8], Aramis 2D (S. Mistou, ENIT) [9], Correla (J.C. Dupré/F. Brémand, Univ. Poitiers) [10], Correli (F. Hild, ENS Cachan) [11], CorrelManuV (M. Bornert, EP) [12], KelKins (B. Wattrisse, Univ. Montpellier) [4] and Vic-2D (L. Robert/J.J. Orteu, EMAC) [13].

Packages are based on the minimisation of a correlation function (e.g. cross-correlation coefficient) \( C(\phi) \) [14] in the real or Fourier space. Several definitions of \( C(\phi) \) exist, depending on the software, as well as descriptions of the local transformation of the subset \( \phi \) (the shape functions that relate image coordinates in the reference image to coordinate in the deformed image), mainly translation (two coefficients), rigid transformation (3 coefficients), affine (6 coefficients), bi-linear (8 coefficients), quadratic (12 coefficients) and bi-quadratic (18 coefficients) transformations. Full local optimisation is carried out directly
(Aramis 2D, Correla, Vic-2D), otherwise the higher order terms are computed from the neighbourhood (CorrelManuV, KelKins, 7D). The sub-pixel evaluation is classically done by direct interpolation of the deformed image. Image interpolation includes bilinear (Correla, CMV, KelKins), bi-cubic (Correla, 7D) and higher order interpolations (e.g. bi-quintic splines, Vic-2D) as well as interpolation in the Fourier space (Correli). Optimization of the correlation coefficient $C(\Phi)$ is performed by minimization of gradient (7D, Aramis, Correla, CorrelManuV, Vic-2D) or by bi-parabolic interpolation (Correli Q1, KelKins).

Square correlation windows (or subsets) of different sizes have been used: 9 or 10, 15 or 16, 21 or 22, 31 or 32 and 63 or 64 pixels. They are denoted $CWS$ in the following. Displacements are evaluated at all intersection points of a regular square grid in the initial image, with a pitch such that correlation windows at adjacent positions do not overlap (i.e., statistical independence of the corresponding errors).

**Results**

Differences between the evaluated and prescribed displacements along $x$ are analysed statistically in terms of RMS errors:

$$\sigma_{\text{global}} = \sqrt{\frac{1}{\text{number of points}} \sum_{i=1}^{\text{number of points}} (u_{\text{measured}} - u_{\text{imposed}})^2}$$

This analysis can be performed globally for all points in the image but also more locally, with error averages computed over points undergoing the same displacement and the same deformation gradient, i.e. points with same $x$ coordinate. As a general point of view, it is shown that the RMS error is described as $\sigma_x = \text{function}(p, r, CWS, interp, ...)$ where $p$ and $\alpha$ describe the imposed transformation, $r$ characterises the speckle size, and $CWS$ and $interp.$ are relative to the main DIC packages options (shape function, subset size and grey level interpolation). In the following, for a better clarity, the results are presented in three subsections corresponding to the three types of local transformation $\Phi$ of the DIC packages used for the correlation.

**Rigid transformation (translation):**

Figure 3 presents the RMS error normalised by $\alpha$ as a function of the period $p$. Results are given for various strain amplitudes $\alpha$ and local rigid transformations. The whole set of seven DIC packages described above was used.

![Figure 3](image-url)
The following conclusions can be drawn. First, there are very small differences between DIC packages and one master curve can be obtained, as clearly seen in Figure 3 where each curve is obtained with one of the DIC packages described above.

All these results obtained with the DIC packages can be gathered in a schematic view that presents the different regimes for the RMS error normalised by the maximum first derivative of the displacement, as a function of the period $p$ (see Figure 4). For periods smaller than the subset size, it is found that the RMS error is equal to the RMS of the displacement itself, that is $\rho_{\Delta x} / \sqrt{2}$, and the DIC is not able to evaluate any displacement (several periods in a subset). For periods between $CWS$ and about 15 $CWS$, there is a transition regime mainly described by the relationship between the subset $CWS$ and the speckle size $r$ (the asymptotic regime is obtained faster for small $CWS$ and $r$). The displacement resolution can be defined somewhere in this area, and the ultimate spatial resolution could be $CWS$ itself. Work is going on for a more precise description of the transition regime. For periods greater than 15 $CWS$ (low frequency strain field), an asymptote is reached and the error is mainly independent of the period and of the subset size. The behaviour is close to those observed for rigid body motions because the strain is quasi uniform inside the subset: the RMS error is proportional to the strain amplitude thus the error is essentially controlled by the first displacement gradient.

Figure 4. Schematic observation of the different regimes for the RMS error normalised by the maximum first derivative of the displacement, as a function of period $p$ for a local rigid transformation.

It should be noted, firstly, that for large subsets ($CWS = 32$ or 64), the convergence is not completely reached even for a period of 510 pixels. Secondly, the asymptotic error depends highly on the speckle size: the ratio $\sigma_{\Delta x}^{global} / u_{\Delta x}^{Max}$ was found to be about 0.35, 0.6 and lies between 0.8 and 1 for respectively the fine ($r/2$), medium ($r$) and coarse ($2r$) speckle pattern presented in Figure 2. Two limitations to the general observations reported in Figure 4 are observed: (i) for small strains and small subsets, the resolution is close to the resolution for a pure translation. The error is $\sigma_{\Delta x}^{trans}(N,CWS,interp,...) \approx \text{funct}(CWS,interp,...)/N$ where $N$ is the image bit depth, typically less than 0.005 pixel; (ii) for large strains and large subsets, it is observed that the asymptotic value increases slightly with the subset.

Affine transformation:

Figure 5 presents the RMS error normalised by the maximum second derivative of the displacement $4\pi^2 \omega p$ as a function of the period $p$, for various strain amplitudes $\alpha$ and for various DIC packages in the case of local affine transformations. In this case, various curves are observed for the subset of 10 pixels, and for small strain level ($\alpha = 0.001$) except for large subsets. Under that consideration, a master curve could be also considered for almost all DIC packages only for sufficiently large strain and subset. All these results can be gathered in a schematic view that presents the different regimes for the RMS error normalised by the maximum second derivative of the displacement, as a function of the period $p$ (see Figure 6). For periods smaller that the subset size, it is always found that the RMS error is equal to the RMS of the displacement itself and DIC is not able to evaluate any displacement. For periods between $CWS$ and about 5 $CWS$, there is a transition regime shorter than for a rigid transformation. The displacement resolution can be defined somewhere in this area, and the ultimate spatial resolution could be also $CWS$. Work is going on for a more precise description of the transition regime. For periods larger than 5 $CWS$ (low frequency strain field), an asymptote is reached which increases with the subset size, as illustrated in Figure 6.
7 on the left, and not with the speckle size \( r \). The error is thus essentially controlled by the second displacement gradient and CWS.

For small subsets and small strains, as it was previously seen for rigid shape function, the asymptotic regime differs and the resolution is close to the resolution for a pure translation. In case of affine transformation however, it is observed that the asymptotic value increases as the subset size decreases (see Figure 7, right). The asymptotic behaviour can be expressed as follows:

\[
\sigma_u \leq \sup[K(CWS)\|u_{x,\alpha}\|^{\text{max}}(N,CWS,\text{interp},...)]
\]

The first term is highest for larger strains. It increases with CWS. The second term is the highest for smaller strains. It decreases with CWS, showing that an optimal CWS exists.

Figure 5. RMS error normalised by \( 4\pi^2\alpha/p \) as a function of period \( p \) for various strain amplitudes \( \alpha \) for a local affine transformation and for 6 DIC packages. Subset window size CWS is equal to 10, 16, 22 and 32 pixels respectively.

Figure 6. Schematic observation of the different regimes for the RMS error normalised by the maximum second derivative of the displacement as a function of period \( p \). Case of a local affine transformation \( \phi \).
Transformation of higher order:

In the case of quadratic or bi-quadratic shape functions, the trends are as follows: for \( p < \text{CWS} \), no measurement is possible. Then the transition is shorter and the asymptotic regime is quickly obtained (error similar to those of pure translation), but in this case the RMS error is independent of \( p \) and \( \alpha \). It is found that the errors are slightly dependent on the window size \( \text{CWS} \), on the degree of grey level interpolation.

**Conclusion**

RMS errors of the displacements obtained with 7 DIC softwares on synthetic speckle pattern images have been evaluated as functions of the spatial frequency and the amplitude of the displacement field, for various correlation window sizes, speckle size and DIC formulations. Preliminary results show that the overall RMS error is mainly controlled by the first order difference between the real transformation and the local transformation \( \Phi \) of the subset. Limiting situations are, on the one hand, displacements varying at low spatial frequencies for which the classical error observed for rigid motions is recovered, and on the other hand, displacement fields with small periods (lower than the subset size \( \delta \)), for which the RMS error is equal to the RMS of the displacement itself (this situation does not allow DIC evaluation). The transition between these two regimes depends on \( \text{CWS} \) and \( \Phi \): an asymptotic regime is obtained faster if \( \text{CWS} \) is smaller and \( \Phi \) is of higher degree. When \( \Phi \) is assumed to be a rigid (resp. affine) transformation, the asymptotic error is proportional to the first (resp. second) derivative of the displacement. Moreover, this asymptotic error is independent of \( \text{CWS} \) but increases with the speckle size \( r \) in the case of a rigid transformation \( \Phi \), or increases with \( \text{CWS} \) in the case of an affine transformation \( \Phi \) for sufficiently large \( \text{CWS} \) and strains. For small subsets and small strains, the asymptotic error decreases with \( \text{CWS} \) as it is observed for pure translation. For \( \Phi \) of high order (e.g., quadratic), it is shown that the asymptotic error is independent of \( p \) and \( \alpha \) and slightly dependent on \( \text{CWS} \). It is not governed by the third derivative of the displacement, which can be a second order phenomenon. Additional investigations, including the analysis of the transition and the influence of the image noise, are the subject of ongoing collaborative work and will be addressed during the presentation. Works focusing on assessment of DIC packages on images submitted to shear strain fields, and assessment of strain measurements by DIC, are also in prospect.

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**References**