DETERMINATION OF THE STRESS INTENSITY FACTOR BY MEANS OF THE ESPI TECHNIQUE

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ABSTRACT

Results from Electronic-Speckle-Pattern Interferometry, ESPI, was used to calculate the stress intensity factor, \( K \), on four points bending specimens with single edge crack. A custom software for complete phase stepping analysis and automated calculation of fracture parameters was realized. An iterative complex mean phase filter was developed and successfully used to increase displacement measurement and accuracy of fracture parameters. The fracture parameters, which are calculated through measurements of displacement along crack direction have shown a negligible error (< 2%) respect theoretical parameters. Instead measurements of displacement perpendicular to crack direction, have shown a relevant error due to low Signal to Noise ratio.

Introduction

The evaluation of the fracture behavior is one of most important research activity in the field of experimental mechanics. Although, the experimental computation of the stress intensity factor in non-transparent material could be considered difficult; personal computers are convenient to use for several optical full-field techniques needing preliminary calibration procedures. The stress intensity factor is usually obtained through their application upon standard materials having a known stress-strain behavior [1]. Among those techniques, which measure surface displacements, are to mention: holographic interferometry [2], Moiré interferometry [3] and speckle techniques [4,5]. Among the above-mentioned techniques, Electronic Speckle Pattern Interferometry, ESPI, is less sensitive to environmental disturbances; it is simple to use with respect to Moiré interferometry; because, it does not require the application of a surface grating and it is a contact less full field technique.

Main features of ESPI technique enable to evaluate the fracture and mechanical characterization of several materials, i.e. composite structures, wood components, and so on, usually difficult to analyze with traditional ER techniques because of their low Young modulus or heterogeneous behavior. Indeed, the application of strain gauges or other devices on specimen of such materials can significantly change the mechanical response thus preventing the possibility to observe the real behavior of the tested material [6]. The main drawback of ESPI technique is the phase noise introduced by the use of speckle pattern. In this way, an effective noise reduction strategy is necessary to perform a correct measurement.

This paper presents an application of a phase shifting ESPI technique to determine the stress intensity factor that implements an original iterative mean phase filter. The technique was applied on a single edge cracked homogeneous specimen loaded by a four points bending test machine. Recording and analysis of the experimental fringes has been made by means of software “ad hoc” for simple and accurate evaluation of the stress intensity factor.

Phase Error and filtering

Main error sources correlated with the ESPI techniques are speckle decorrelation and electronic noise [7]. Decorrelation can be significantly reduced by applying the technique between two specimen load conditions presenting low displacement, i.e. partitioning load application and recording every intermediate state. In this paper it has been considered only the reduction of electronic noise, presenting an automatic technique that significantly increments the signal to noise ratio of the recorded interferogram.

A four frame method applied to the setup of the in plane interferometer (Fig. 1) was considered. Four images \( I_1, \ldots, I_4 \) were acquired with \( \pi/2 \) phase-step:

\[
I_i(x, y) = I_0(x, y) + I_m(x, y) \cdot \cos(\phi(x, y) + \varphi_i)
\]

\[
\varphi_i = (i - 1)\frac{\pi}{2}, \quad i = 1, 2, 3, 4
\]
where: $x,y$ is the coordinates of a generic point of the image  
$I_0$ is the background intensity  
$I_m$ is the modulation intensity  
$\phi$ is the phase of a generic point of the image and,  
$\varphi$ is the introduced stepped phase.

Phase step, $\varphi$, was generated with a piezoelectric mirror translator, PZT. The in-plane displacement value, (which is assumed monotonically related to the phase $\phi$ [8]), was obtained by means of the phase difference information $\Delta \phi$ between two different specimen conditions and it can be expressed by the following equation:

$$d = \frac{\Delta \phi}{2} \cdot \frac{\lambda}{2 \cdot \sin \alpha}$$

(3)

where:

- $d$ is the displacement lying in the plane of the illumination and reference beams
- $\lambda$ is the laser light wavelength and,
- $\alpha$ is the direction of the illumination incident beams

The three unknowns in Eq. (1): $I_0, I_m, \phi$ were calculated solving a set of four simultaneous equations $I_1,...,I_4$. Analyzing two consecutive different load conditions one can obtain the wrapped phase distribution over the observed area by means of the following equation:

$$\Delta \phi_w(x, y) = \tan^{-1} \left[ \frac{I_4(x, y) - I_2(x, y)}{I_1(x, y) - I_3(x, y)} \right]$$

(4)

The wrapped phase value $\Delta \phi_w$, i.e. lying in the range $[-\pi, \pi]$, due to the presence of random and systematic errors is an estimator for the true phase $\Delta \phi$.

The software “ad hoc” developed provides the wrapped phase difference with pixel intensity within the range [0-255]. Figure 2 illustrates a typical fringe pattern obtained by the setup implemented of Figure 1.

![Figure 1. Experimental Speckle Interferometer setup](image)
Fringes shown in Figure 2(a) are clearly affected by an appreciable noise component. As written above, the CCD camera is the main noise source; therefore, the noise components are [9]: 1) photon shot noise; 2) thermal noise; 3) read out noise; 4) fixed pattern noise. It should be noticed that those noises are normally distributed and can be filtered in the complex plane by means of a convolution technique. Figure 2(b) shows the pixel intensity along the straight line $AB$ depicted in Figure 2(a).

Figure 2(a). Typical phase fringe pattern

Figure 2(b). Intensity variation along the straight line $AB$.

Figure 3 displays the theoretical intensity variation, obtained considering the displacement of the point lying along the straight line $AB$ depicted in Figure 2(a). The general behavior is recognizable even in Figure 2(b) but the noise dramatically affects any following quantitative evaluation.

By subtracting the theoretical intensity value from equivalent experimental one, the intensity variation, which is due to the noise, can be obtained, Figure 4.

Figure 3. Theoretical Intensity variation along the straight line $AB$. 
As reported by Huntley [7], the noise has a Gaussian behavior; therefore, it can be filtered out by convolution technique. The generic pixel centered in \( n \times n \) convolution matrix can be expressed as in the following equation:

\[
Z(x, y) = \sum_{r=-n}^{n} \sum_{s=-n}^{n} e^{i\phi(x+r, y+s)}
\]  \( (5) \)

where \( Z \) is a complex number strictly depending on the phase and \( \phi \) is the resulting phase at a generic point of the specimen surface, which is expressed as follows:

\[
\phi = \Delta \phi_n + \phi_n
\]  \( (6) \)

being \( \phi_n \) the contribution from the above cited noise sources.

If the convolution matrix is sufficiently small; then, \( \Delta \phi_n \) can be assumed constant. Moreover, being \( \phi_n \) with null mean value, Eq. (5) can be rewritten as follows:

\[
Z(x, y) \equiv K e^{i \Delta \phi_n(x, y)}
\]  \( (7) \)

The parameter \( \Delta \phi_n(x,y) \) in the above equation, that conveys the desired information after application of the mean filtering algorithm based on Eq. (5) (which is the kernel of the applied filter) is therefore a better available estimator compared to that calculated by Eq. 4; it can be obtained by means of the following equation:

\[
\Delta \phi_w(x, y) \equiv \arctan \left\{ \frac{\text{Im} \left[ Z(x, y) \right]}{\text{Re} \left[ Z(x, y) \right]} \right\} = \arctan \left\{ \frac{\sum_{r=-n}^{n} \sum_{s=-n}^{n} \sin \left[ \phi(x+r, y+s) \right]}{\sum_{r=-n}^{n} \sum_{s=-n}^{n} \cos \left[ \phi(x+r, y+s) \right]} \right\}
\]  \( (8) \)

The proposed approach, rather than a single application of the filter to a fixed \( n \times n \) mobile window onto the image, is based on an iterative application with variable \( n \times n \) dimensions. The effectiveness of the filtering operation, whose kernel is described by
Eq. (5), lies; therefore, in the proposed strategy to automatically optimize both the number of iterations and the dimensions of the convolution window. In practice starting with 3x3 convolution matrix after each iteration the software evaluates total intensity variation between image before and after matrix application that has been defined as:

\[
\text{Diff} = \sum_{i=0}^{H \times W} |I_{\text{before}}(i) - I_{\text{after}}(i)|
\]

\[H = \text{image height}, \ W = \text{image width}; \quad (9)\]

If this parameter is less than reference one, that has been chosen as a compromise between optimum result and time to reach it, then algorithm increase convolution matrix dimension i.e. 5x5. and so on. Practical experience showed that max dimension of convolution matrix should be 9x9 or 11x11 to preserve the global information conveyed by the investigated fringe pattern and to appreciably increase the Signal to Noise ratio. Figures 5(a) and 5(b) shows respectively the result of the described filter and the evaluation of the intensity variation along the same line of Figure 2. The fringe pattern in Figure 5(a) shows a good signal to noise ratio. Figure 5(b) highlights that the curve is quite similar to theoretical one obtained in Figure 3; it asserts the validity of proposed filter.

![Figure 5(a). Fringe pattern of the Figure 2 after filter application](image)

![Figure 5(b). Experimental intensity variation along straight line AB after filtering process.](image)

**Experimental**

The above described system was tested to measure the SIF on a specimen subjected to a four point fracture test. A Plexiglas specimen (E = 3200 MPa, ν = 0.39), which presents a edge crack located on the transversal axis of symmetry, was loaded by a four point bending machine at five different load levels, Figure 6. The crack was realized with a jeweler cutter machine with a very thin blade (thickness < 0.3 mm), and it was refined by means of sharp razor blade to obtain a crack tip with negligible radius.
The speckle visibility of the specimen was improved by spraying an opaque white paint on the specimen surface. A solid state green (532 nm) laser and a 768x576 CCD camera have been used. An in plane interferometer with horizontal sensibility vector was implemented. In order to evaluate $K_I$ with both $u$ and $v$ displacement component two different tests were performed at the same load step. The maximum applied, $P$, load was 82.5 N; it was reached by four incremental steps of about 15 N with the last step of 22.5 N. A Newton-Rapson algorithm [10] was implemented for the evaluation of the fracture parameters to interpolate displacement data obtained by speckle analysis of a great number (> 400) of point lying along straight lines radially and symmetrically distributed respect to the crack, Figure 7. The experimental displacement data were interpolated through the following expressions comprising six coefficients:

\[
\begin{align*}
  u &= \frac{1}{E} \sum_{j=0}^{N} C_{2j} \frac{r_{j+1}}{j+1/2} \left[ (1-v) \cos(j+1/2)\theta - (1+v)(j+1/2)\sin\theta \sin(j-1/2)\theta \right] + \\
  &+ \frac{1}{E} \sum_{j=0}^{N} C_{2j+1} \frac{r_{j+1}}{j+1/2} \left[ 2 \cos(j+1)\theta - (1+v)(j+1)\sin\theta \sin(j+1/2)\theta \right] \\
  v &= \frac{1}{E} \sum_{j=0}^{N} C_{2j} \frac{r_{j+1}}{j+1/2} \left[ 2 \sin(j+1/2)\theta - (1+v)(j+1/2)\cos\theta \cos(j-1/2)\theta \right] + \\
  &+ \frac{1}{E} \sum_{j=0}^{N} C_{2j+1} \frac{r_{j+1}}{j+1/2} \left[ (1-v) \sin(j+1)\theta - (1+v)(j+1)\sin\theta \cos(j+1/2)\theta \right]
\end{align*}
\]  

(10)

(11)

The positions of points on the lines rising from crack tip were chosen considering both the plastic altered zone at crack tip and the max distance from the crack tip where the interpolation functions are no longer valid. It has also been considered that for chosen load condition and specimen geometry, a plane strain condition is verified. Experimental results have been compared with those theoretically obtained using the following equation [11]:

\[
K_I = \frac{1}{K_I} - 1.39 \left( \frac{a}{b} \right) + 7.32 \left( \frac{a}{b} \right)^2 - 13.10 \left( \frac{a}{b} \right)^3 + 14.00 \left( \frac{a}{b} \right)^4
\]

(12)

where:

- $a$ is the crack length
- $b$ is the specimen’s width and,
- $K_I$ is expressed as below

\[
K_I = \frac{6M \sqrt{a}}{b^2} \left[ \frac{N}{\text{mm}^2} \right]
\]

(13)
Results

Experimental results are summarized in Table 1 where experimental $K_I$ were evaluated by analyzing the phase pattern obtained by software addition of the filtered patterns generated by each load steps. Figure 8 shows the experimental vs. theoretical $K_I$ profiles.

The analysis of charts reported in Figure 8 allows to assert that for $u$ displacements the experimental results is praticly the same of those theoretically predicted except for the first point where the error is about 18.8%. This error is probably caused by the low signal to noise ratio present at this step. For $v$ displacement, the error over $K_I$ increases with the load step reaching 40% at the last load condition. The same problem is reported in [5] and [10] and it could be due to the evaluation of the $C_I$ parameter and its relative error that have a strong influence on the calculation of $K_I$. In fact, for direction perpendicular to the crack $C_I$ parameter for the adopted experimental layout, have a negligible value, consequently its correct estimation has been affected by a relevant error invalidating the $K_I$ evaluation.

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<th>$K_I$ EXP.</th>
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Table 1: Experimental results a): $u$ displacement and b) $v$ displacement

Figure 7. Filtered Phase Map for $P= 82.5$ N with analyzed lines of (a): $u$ displacement and (b) $v$ displacement
Conclusion

A complete ESPI system for the analysis of fracture parameters has been presented. The developed software implements the four step phase shifting algorithm. An original version of mean phase filter to improve signal to noise ratio of the speckle phase image has been presented showing a better performance of the classical version [12]. The use of the filter developed enables a more accurate analysis of fracture parameters using a Newton-Rapson approach; especially, when considering the displacement along direction parallel to the crack. A comparison with theoretical $K_I$ parameter showed a negligible error (< 2%) for $u$ displacements whereas the interpolation of $v$ displacements value present an increasing error probably due to a bad $C_1$ evaluation.

References