SOME NEW STRENGTH CRITERIA FOR FRP TEST METHODS
SUBSTANTIATION

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ABSTRACT
The four well-known test methods for FRP (fiber-reinforced-plastics) are analyzed in this report and using new simple fracture criteria it’s possible to understand the experimental data more correctly.

Introduction

The FRP with weak polymeric matrix and fiber-matrix interface demonstrate some specific modes of failure behavior (delamination – p. 1, strip buckling – p.2, splitting – p.3, shear failure along the fibers – p. 4, 5, kink, etc.) and to understand and to describe numerically these failure modes it is necessary to use some non-traditional approaches and new strength criteria taking into account the normal-shear stresses interaction (see p. 1.1, 4, 5) the energy balance condition (see p. 1.2, 2, 5) and some specific models of fracture processes (see 2, 3, 5). It’s shown in the report that the application of new fracture criteria makes it possible to estimate experimentally some fracture resistance parameters for FRP being used in more correct design of composite elements of constructions.

1. Short Beam Bending

The short beam bending is the standard method for interlaminar shear strength determination. This strength is usually considered as constant being equal to the maximum value of shear stresses in the middle of the beam under the critical load \( P_c \) at the condition \( l/h = 5-7 \). But really this conventional shear strength \( \tau_0 \) depends on \( l/h \) ratio and on scale factor and these effects may be explained by using two different strength criteria: linear stress criterion – see p.1.1, and energy balance criterion – see p.1.2.

1.1. The Linear Criterion of Composite Beam Delamination

Let us consider a short-beam-bend-test (Figure 1), which is widely used for interlaminar shear strength determination. According to ASTM D 2344-76 and other standards the conventional shear strength \( \tau_0 \) is equal to maximum shear stress in the middle section of the beam \( \tau_0 = \frac{3}{2} P_c/\ell h \) at the critical load \( P_c \). In such a way the \( \tau_0 \) is proportional to critical load and does not depend on length to height \( l/h \) ratio. This assumption, unfortunately, is in contrast to experimental results and to explain the real strength on \( l/h \) dependence we proposed the linear criterion [1, 3]:

\[
f(\sigma, \tau) = \sigma_x + m \tau_{xy} = c \quad (1.1)
\]

where \( \sigma_x, \tau_{xy} \) - normal and shear stresses, \( m, c \) are the experimentally determined material constants. We consider that delamination starts when linear combination of normal and shear stresses will reach the critical value. The stress distributions along the critical section under the load \( P \) have the following expressions:

\[
\sigma_x = \frac{3Ply}{\ell h^3} \quad \tau_{xy} = \frac{3P}{\ell h^3} \left( \frac{h^2}{4} - y^2 \right) \quad (1.2)
\]

\[
f = \frac{3P}{\ell h^3} \left( ly + m \left( \frac{h^2}{4} - y^2 \right) \right)
\]
The condition \( df/dy=0 \) from Eqs. (1.1), (1.2) gives us the critical coordinate \( y^*=l/2m \) where the function \( f \) has a maximum value, and so – the delamination should start at \( y=y^* \). Putting \( y^* \) into Eqs. (1.1), (1.2) one can easily get the following relation between critical load \( P_c=4/3 \sqrt{IJ} \) and the length to height ratio \( l/h \) in a good accordance to experimental data, in contrast to traditional maximum stress criteria:

\[
\tau_0 = \frac{cm}{m^2 + (l/h)^2}
\]  

(1.3)

Of course, delamination must start inside the beam, not outside, and so \( y^*<h/2 \) and the criterion Eq. (1.1) is valid when \( l/h>m \).

When \( l/h=m \) the usual normal stress criterion is valid

\[
\sigma_{\text{max}} = \frac{3}{2} \frac{P_c}{bt^2} = \frac{2\tau_0 l}{h}, \quad \tau_0 = \frac{ch}{2l}
\]  

(1.4)

The point \( l/h=m \) corresponds to change of fracture mode. It is interesting that in the point \( l/h=m \) the Eqs. (1.3) and (1.4) gives the same results and slopes of these two curves in this point (the derivatives \( d\tau_0/dh \)) are equal to each other too. So these two proposed criteria Eq. (1.3) and Eq. (1.4) give the smooth united curve in good accordance with experimental data. The point \( m=l/h \) is the point of tangency of the curve Eq. (1.3) and the hyperbole Eq. (1.4) (Figure 1).

In the case of cyclic loads of steady amplitude [1] the parameters \( m \) and \( c \) in Eq. (1.3) can be represented in the form of functions of the number of cycles \( N \). For an analytical description of these functions, we plotted experimental data on cyclic strength in \( 1/\tau_0 - (l/h)^2 \) coordinates in which, according to Eq. (1.3), they should lie on the straight lines \( 1/\tau_0 = 1/[c(N) m(N)] (l/h)^2 + m(N)/c(N) \) for any fixed number of cycles. It’s very interesting that the slopes of these straight lines were almost equal and so \( m(1)=m(N)/c(N)=\text{const} \), irrespective of the number of cycles. For \( c \) we can consider relationship \( c(1)/c(1)=1.04 - 0.035 \cdot \lg N \) which is known from independent fatigue tensile experiments. And then we make it possible to determine the \( m(N) \) relationship and to derive a very simple expression for fatigue convenient interlaminar shear strength

\[
\tau_0(N) = \frac{c(1) \cdot m(1)}{m(1)^2 (1.04 - 0.35 \cdot \lg N)^2 + (l/h)^2}
\]

The experimental results for both static and cyclic loadings agree well with results of computation of this relationship.

Bending composite elements carrying capacity may be estimated by using these criteria with higher degree of precision than by using traditional approach.

1.2. Energy criterion of delamination

For critical load estimation at whole delamination of composite beam with height \( h \) (Figure 1) it is possible to use Griffith-type energy criterion. The initial elastic energy \( U_0 \) stored in the beam is equal to the work of the force \( P \) on the deflection \( v \):

\[
U_0 = \frac{1}{2} P v = \frac{P^2 v^3}{8 Eth^3} = \frac{2v^2 Eth^3}{l^3}
\]  

(1.5)
The two originated after delamination beams with heights $ah$ and $(1-α)h$ have the minimum elastic energy $U_1$ for the same deflection $v$ when $α=1/2$.

$$U_1 = U_0\left(α^3 + (1-α)^3\right); \frac{∂U_1}{∂α} = 0 \Rightarrow \alpha^2 - (1-α)^2 \Rightarrow α = \frac{1}{2}; \ U_1 = \frac{1}{4}U_0$$

(1.6)

As usually we assume that during the fast process of delamination the deflection $v$ is not change and that the whole difference in elastic energies $U_0-U_1$ before and after delamination is equal to the work of fracture being proportional to the square of fracture surface. So we have $\frac{1}{2}U_0γL$, where $L=LA2a$ – the whole beam length, $t$ – beam thickness, $γ$ – critical energy release rate (the specific fracture work per unit of surface), $E$ – Young's modulus along $x$-axes, $l$ – the base of bending.

$$P_c = \frac{32Et^2h^4L}{3l^3},$$

$$τ_0 = \frac{h}{l} \frac{6Ey}{h} \left(1 + \frac{2a}{l}\right)$$

(1.7)


Using Griffith-type energy fracture criterion we can describe the critical load dependence on material properties and sizes (link length, radius) of unidirectional (pultrusion) composite tubes [2, 4].

The compression failure of composite unidirectional tubes and rods may have three main different modes (Figure 2):

1) A local microwave buckling at a critical stress

$$σ = σ_c$$

(2.1)

not depending on the length of tube.

2) “Chinese lantern” mode of a tube fracture with multiply splitting and buckling of originated strips. It's necessary to calculate the critical stress $σ$ numerically, but for thin-walled tubes we can receive the formula (Figure 2):

$$P_c = \frac{32Et^2h^4L}{3l^3},$$

$$τ_0 = \frac{h}{l} \frac{6Ey}{h} \left(1 + \frac{2a}{l}\right)$$

(1.7)

Figure 2. Three failure modes of composite tubes under compression:

1 – micro-buckling, Eq. (2.1); 2 – Chinese lantern, Eq. (2.2); 3 – macro-buckling, Eq. (2.3).
\[
\sigma_i = \frac{\pi^2 E \alpha}{F_a L^2} + \sqrt{\frac{EY}{\alpha R}} \approx 12 \left( \frac{\sqrt{4E^5}}{R^2 L^2} \right) \quad \text{for } \alpha^* = 3.21 \left( \frac{\sqrt{L^4 E R^5}}{\alpha} \right) \tag{2.2}
\]

where \(Y\) - specific work of splitting per unit of surface (see App. 2).

3) Euler-type macro-buckling of a tube as a column at the critical stress, depending on the length \(L\):

\[
\sigma_e = \frac{\pi^2 ER^2}{2L^2} \tag{2.3}
\]

If we set equal 1) to 2) and 3): Eq. (2.1)=Eq. (2.2)=Eq. (2.3), we can find the optimum sizes of tubes (for the known properties of the material), it’s ensure the initiation of different modes of compression failure simultaneously, i.e. in such a way the weight of the tube construction tends to the minimum.

It is important to note, that in this case the “equal strength” approach allows us to find not only the optimum ratio of the tube sizes (radius of a tube (\(R\)) to it’s length (\(L\) )or to distance between fittings), but also the optimum absolute sizes (in mm). The condition of simultaneous start of all fracture modes gives us the optimum sizes \(L, R\) of the composite tube structure.

\[
R_0 = 1.01 \frac{EY}{\sigma^* c}, \quad L_0 = 2.24 \frac{E^{3/4}Y}{\sigma^{5/2} c} \tag{2.4}
\]

Appendix 2

Let’s estimate the critical value of compressive stress at which the multiple splitting and buckling start. The energy of compressive strain has the formula: \(U_0 = \frac{1}{2} \frac{\pi^2}{\sigma^* c} \frac{EY}{c^2} \). The critical stress for buckling of strips with circle section expresses as following:

\[
\sigma_e = \frac{\pi^2 E I}{F L^2}
\]

where moment of inertia of the circle segment

\[
l_c = \left( \alpha + \sin \alpha \cos \alpha \right) \frac{R_1^4 - R_2^4}{4} - \frac{4 \sin^2 \alpha}{9\alpha} \left( \frac{R_1^3 - R_2^3}{R_1^2 - R_2^2} \right)^2
\]

in case \(H/R<<1\), \(\alpha<<1\), wall thickness \(H=R_1-R_2\), \(R_1+R_2=2R\), cross section area of the strip \(F=2\pi RH\). The energy of compressive strain after buckling \(U_i=\frac{\pi^2}{\sigma^* c} nF L/E\), the bending energy \(U_i=\frac{\pi^2}{\sigma^* c} nF L/E\), the work of splitting \(Y=nY H L\), where \(Y\) – is the specific energy of longitudinal fracture. According to energy criterion of fracture (Griffith type) \(U_i=U_i+U_f+Y\), and this energy balance condition leads to the following expression for critical stress: \(\sigma = \sigma e+\sqrt{(EY/\alpha R)}\).

For example, let us consider that \(H/R<<\alpha^2\). The minimum strength \(\sigma_e\) corresponds to \(\alpha^*=[2025\sqrt{L^2/(64\pi^2 ER^2)}]^{1/9}\), this angle being computed from equation \(da/d\alpha=0\):

\[
\sigma_e = 1.2 \left( \frac{\sqrt{4E^5}}{R^2 L^2} \right)^{1/9}
\]

It is a roughly approximate formula for illustration only, but we have a lot of exact numerical results for many kinds of pultrusion formed rods with different cross-section. It’s important to note that unidirectional composite tubes or rods have a specific form of fracture&bucking which goes lower than classic curves on Figure 2.

### 3. Critical Stress Intensity Factor for Splitting of Edge-notch-beam

It was proposed the experimental method for composite materials fracture toughness estimation in similar way as for metals, using the edge-notch specimens for four-point bending (Figure 3). The stress field near the crack tip in linear elastic plate subjected to symmetrical tension (mode I) is determined by a single parameter – stress intensity factor (SIF) \(K_i\) and the critical value of \(K_i\) may be used as a criterion of any crack extension mode starts. In unidirectional FRP stretched along the fibers the
transverse notch (crack) does not propagate across the fibers but it turns and the longitudinal (splitting) crack starts from the tip of initial transverse notch. We may assume that this splitting crack starts when the SIF for the initial notch reaches some critical value \( K_{IJ} \) - the co-called splitting toughness. Some experimental results for three kinds of unidirectional carbon FRP (CFRP) are shown in Figure 3. The load values at the start of longitudinal crack were determined by two methods: visually and as the load which corresponds to the onset of the nonlinearity (5%) on the load-deflection curve. The critical load \( P \) at the splitting onset is determined as the intersection of this curve and the straight line with the slope being 5% smaller than the initial slope of load-deflection curve. The results of the two methods for three kinds of tested CFRP were similar. The critical value of SIF may be calculated using the usual for isotropic metal techniques:

for three-point bending

\[
K_{II} = \frac{3P}{2th^2} \cdot \sqrt{I_0} \cdot Y_1(\lambda)
\]

\( \lambda = l_0/h; \; Y_1(\lambda) = 1.96 - 2.75\lambda + 13.66\lambda^2 - 23.98\lambda^3 + 25.22\lambda^4; \; 0.1 < \lambda < 0.6; \; Y_1(\lambda) = 1.85 \) for \( \lambda = 0.25 \)

for four point bending

\[
K_{II} = \frac{3Pc}{2th^2} \cdot \sqrt{I_0} \cdot Y_2(\lambda)
\]

\( Y_2(\lambda) = 1.99 - 2.47\lambda + 12.97\lambda^2 - 23.17\lambda^3 + 24.80\lambda^4; \; Y_2(\lambda) = 1.85 \) for \( \lambda = 0.25 \)

The specimens sizes were: \( h = 12 \) mm, \( h = 8h = 96 \) mm, \( L = 10h = 120 \) mm, \( c = 3h = 36 \) mm, \( l_0 = 1 - 7 \) mm.

The value of \( K_{II0} \), corresponding to a very short initial notch \( l_0 \neq 0 \) may be used for the estimation of the potential strength of unidirectional FRP (UFRP) with the fixed interface strength or splitting resistance. There are always some broken fiber bundles in any UFRP which have been broken during loading process. According to well-known LEFM (linear elastic fracture mechanics) formula we can express the UFRP potential strength controlled by splitting at the broken bundles with diameter \( d \) in the following form

\[
\sigma_p = \frac{K_{II}}{\sqrt{dY(l_0/S)}}
\]

where \( S \) – width of the plate. We can assume \( l_0 \) to be equal to bundle diameter \( d \) (about 90 microns for CFRP) and \( Y = n^{1/2} \) (since \( d \) to \( S \) tends to zero) and calculate from Eq. (3.3) the potential strength for \( K_{II} = 16 \) MPa\(\cdot\)m\(^{1/2} \) (50 kg/mm\(^{3/2} \)); \( \sigma_p = 950 \) MPa (94 kg/mm\(^2 \)). This value is almost equal to the actual strength of the unidirectional CFRP (UCFRP). Thus, the potential strength may be estimated in terms of splitting toughness \( K_{II} \) value and bundle (not single monofiber) diameter, and it does not depend on the mean fiber strength. It should be noted that for the fixed splitting toughness \( K_{II} \) (or interface strength) there is no point in excessive fiber strength since the UFRP strength is governed by the multiple splitting near the inevitable fiber breakages at weak points.

Figure 3. Four-point bend specimen with edge notch. Critical SIF versus notch length \( l_0 \) for splitting start.

1,2,3 – three kinds of UCFRP.
4. Strength Criteria for Monolayer and the Optimum Angle of Misalignment

A lot of authors formulated the strength criteria for composite monolayer in terms of symmetric invariants, being convolutions of the stress tensor and a number of even order tensors, characterizing the strength properties of composites. Using any one of these criteria we have to determine experimentally a large number of material constants, and the calculation of these values using a rather restricted amount of the experimental data may be unstable. When the strength criterion is formulated in term of symmetric invariants, nothing can be said about the character of rupture, but in fiber-reinforced plastic there are two main modes of rupture connected with fibers and matrix or weak fiber-matrix interface [3]. Taking into account these very specific kinds of rupture of unidirectional reinforced plastics it seems reasonable to formulate the failure criteria as relationships connecting the values of the normal and shear stresses in one of the two planes of possible failure

\[ F(\sigma_{n1}, \tau_{n1}) = c \]

This criterion resembles the well known Mohr strength condition, according to which rupture occurs when this condition is satisfied for any one direction. In our case the directions of vectors \( n \) are fixed and there are only two possible cases. The simplest assumption concerning the function \( F \) being a linear one.

There are two fracture modes of unidirectional FRP or prepreg (the traditional semi-finished product for polymeric composites):

1) rupture of fibers (a normal \( n_1 \) on Figure 4)

\[ \sigma_{n1} + m_1 \tau_{n1} = c_1, \quad (4.1) \]

where \( m_1, c_1 \) - experimentally determined phenomenological material parameters,

\( c_1 = \sigma(0) \) - tensile strength along the fibers for \( \alpha = 0^\circ \).

In some papers the fibers rupture condition takes the form

\[ \sigma_{n1} = \sigma(0) \Rightarrow \sigma(\alpha) = \frac{\sigma(0)}{\cos^2 \alpha} \quad (4.2) \]

This condition means the growth of strength with growth of misalignment angle \( \alpha \), but it’s in contrast with experimental data.

2) splitting, failure along fibers on the plane with a normal \( n_2 \) on a scheme (Figure 4):

\[ \sigma_{n2} + m_2 \tau_{n2} = c_2 \quad (4.3) \]

Where \( c_2 = \sigma(90) \) - tensile strength across the fibers at \( \alpha = 90^\circ \).

From Figure 4 one can see, that \( \sigma_{n1} = \sigma(\alpha) \cos^2 \alpha; \sigma_{n2} = \sigma(\alpha) \sin^2 \alpha; \tau_{n1} = \tau_{n2} = \sigma(\alpha) \sin \cos \alpha \). We substitute these values into Eqs. (4.1), (4.3) and have the following expressions for strength versus angle functions:

\[ \sigma(\alpha) = \frac{c_1}{\cos^2 \alpha + m_1 \sin \alpha \cos \alpha} \quad \text{if} \quad \alpha \leq \alpha^* \quad (4.4) \]

\[ \sigma(\alpha) = \frac{c_2}{\sin^2 \alpha + m_2 \sin \alpha \cos \alpha} \quad \text{if} \quad \alpha > \alpha^* \quad (4.5) \]

Angle \( \alpha^* \) (Eq. (4.6)) is a transition point from one mode of fracture (fibers rupture) to another mode (splitting along the fibers). We can find angle \( \alpha^* \) from an equality Eq. (4.5) = Eq. (4.4):

\[ \alpha^* = \arctg \frac{1}{2} \left( m_1 \frac{\sigma(90)}{\sigma(0)} - m_2 + \sqrt{(m_2 - m_1 \frac{\sigma(90)}{\sigma(0)})^2 + 4 \frac{\sigma(90)}{\sigma(0)}} \right) = \arctg \frac{\sigma(90)}{m_2 \sigma(0)} \quad (4.6) \]

The composite material with this misalignment angle \( \alpha^* \) has near the same strength as the strength of unidirectional FRP, but its splitting resistance is much more.
It would be possible to present the generalized criterion in the form:

\[
\frac{\sigma_{nt}}{\sigma(0)} + \frac{\sigma_{n2}}{\sigma(90)} + \frac{T_n}{T} = 1
\]  \hspace{1cm} (4.7)

Calculation with this criterion is shown on Figure 4, but a transition point \(\sigma^*\) is indeterminable.

### 5. Strength Criteria for Balanced Couple (\(\pm\varphi\)) Pair of Layers

For symmetric balanced pairs of layers the new criterion was proposed and it explains rather well the experimental results for biaxial tension of composite tube with lay-up structure (\(\pm\varphi\)). In \(\sigma_{\varphi}\sigma_{\varphi}\) plane (Figure 5) this criterion has the form of two straight lines 1 [4, 5] (see App. 5)

\[
|\sigma_{z} / \tan \varphi - \sigma_{\varphi} / \cot \varphi| = \sigma(45) \hspace{1cm} (5.1)
\]

and one cross-line 2 corresponds to strength along the fibers \(\sigma(0)\) and the set of experimental points may be approximated also by ellipse (5.2), where, \(\sigma(0), \sigma(45)\) - uniaxial strengths for tubes with \(0\), and \(\pm45\) wound structure, correspondently:

\[
\sigma_{z}^2\tan^2\varphi + \sigma_{\varphi}^2\cot^2\varphi - \sigma_{z}\sigma_{\varphi}\left(2 - \sigma(45)^2\right)\left(1 + \tan^2\varphi\right)^2 \frac{\cot^2\varphi}{\sigma(0)^2} = \sigma(45) \hspace{1cm} (5.2)
\]

**Appendix 5. The Rhombic Model and Failure Criteria.**

Let us consider the rhomb made of non-stretched rod of length \(L\) with the matrix inside which prevents their rotation under biaxial tension (in Figure 5). When the angle \(\varphi\) have an increment \(\Delta\varphi\), the force \(P_z=2L\sin\varphi\Delta\varphi\), and the force \(P_{\varphi}=2L\cos\varphi\Delta\varphi\) provides the work – \(P_z2L\sin\varphi\Delta\varphi\) accordingly (but with the opposite sign). The moment \(M\) acts on rods due to matrix and prevents their rotation, \(M\) is proportional to square of rhomb and is equal to \(\mu L^2\sin\varphi\cos\varphi\). Thus from the equality of works of forces to works of the moment we can derive the following criterion of rupture:

\[
\left|\frac{\sigma_{z}}{\tan \varphi} - \frac{\sigma_{\varphi}}{\cot \varphi}\right| = \mu = \sigma_z(45) \hspace{1cm} (5.1)
\]

The constant \(\mu=\sigma_z(45)\) is the tensile uniaxial strength of the tube with lay-up – \((\pm45)\), which can be determined by direct experiment.

That Eq. (5.1) corresponds to straight lines 1 on Figure 5. The line 2 on limit surface corresponds to fiber rupture. Depending on ratio \(tg\beta=\sigma_\varphi/\sigma_z\) between \(\sigma_\varphi\) and \(\sigma_z\) the vector of loading in \(\sigma_\varphi-\sigma_z\) space can touch lines 1 (matrix rupture) or line 2 (fiber rupture). In each pair of layers the ratio will be different, and the condition of final rupture will be different too. Note, that \(tg\beta=\tan^2\varphi\). The best case is when the vector of load has an angle \(\beta\). For cylindrical section of tank \(\sigma_\varphi=2\sigma_z\) and the best angle of wound \(\varphi=\arctg(\sqrt{2})=54^\circ44\).

For convenience of calculation rectangular 1–2–1 may be substituted by ellipse 3, which intersection the same points on the axes Eq (1.5).
The laminate failure criterion models the composite structure as a homogeneous, anisotropic solid, making no distinction for its layered construction and the possibility of different stress states within the individual layers.

For balanced off-axis lay-up of thin-walled tubular specimens it was shown from experimental data and from analytical predictions that the shear strength at torsion tests has a very simple form of dependence on off-axis angle: $\tau(\varphi) = \tau(45) \sin^2 \varphi$, (where $\tau(45)$ – shear strength of (±45) laminate tube under torsion) or taking into account the non-zero shear strength $\tau(0) = \tau(90)$ of unidirectional and circular reinforced tubes: $\tau(\varphi) = (\tau(45) - \tau(90)) \sin^2 \varphi + \tau(0)$.

Conclusion

For more correct interpretation of experimental results it is necessary to develop strength criteria taking into account the specific fracture modes of composite materials.

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