AN EXPERIMENTAL STUDY ON THE ELASTIC-PLASTIC FRACTURE IN A DUCTILE MATERIAL UNDER MIXED MODE LOADING

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ABSTRACT
A stereo vision is used to measure the crack tip parameters, such as J integral, plastic mixity, and elastic mixity, of mixed mode fracture specimens, and to study the applicability of the Shih's plane strain solution to the mixed mode crack tip fields. The fracture specimen used in the paper is a compact tension shear (CTS) specimen made of 2024-O aluminum. To conduct mixed mode fracture experiments at the loading angles of 75° and 45°, a special loading device is used. The in-plane strain and stress fields near the mixed mode crack tip are determined using the deformation field measured by the stereo vision. Then the J integral along rectangular contours surrounding the mixed mode crack tip can be evaluated. It can be seen that the computed J integral values approach constant after \( r/h > 0.5 \). In the paper the in-plane strains determined experimentally at several points near the crack tip are compared with the values calculated using Shih’s plane strain solution. It is found that the measured values follow the trends of the Shih’s plane strain solution. The elastic mixity evaluated using the measured crack tip stress fields are close to those obtained from analytical solution. However the evaluated plastic mixity deviates from the analytical solution.

Introduction
For cracks in elastic-plastic material, strain hardening plasticity solutions have been presented by Hutchinson [1, 2] and Rice and Rosengren [3] to characterize the behavior at the crack tip in the plastic zone. They are referred as HRR solution and are restricted to problems in which the stress distribution is either symmetric (Mode I) or antisymmetric (Mode II) with respect to the crack tip. For a mixed-mode crack problem, Shih [4] used the J-integral [5] which is path independent and defined the plastic mixity to modify the HRR solution and obtained the asymptotic crack-tip stress and strain fields around the crack tip. For plane strain mixed-mode fields, Shih [6] presented tables of the plane strain dimensionless functions for several mixities ranging from Mode I to Mode II.

Pawliska et al. [7] used finite element analysis to determine the stresses around the mixed-mode crack tip at several load angles. The numerical results were then compared with the values obtained using the Tables of Shih [6]. Aoki et al. used finite element to compute the J-integral values \((J_I, J_{II})\) around the crack tip of a compact tension shear specimen made of A5083-O aluminum alloy. The elastic-plastic analysis for the strains at a specific point near the crack tip under the mixed-mode loading was also conducted. The calculated results were in good agreement with the experimental data obtained from the strain gage.

Studies on the elastic-plastic fracture behavior for a ductile material under mixed-mode loading are very few, especially for the determination of the whole crack-tip deformation field. In the paper a stereo vision [9, 10] was used to measure the out-of-plane and in-plane displacement fields in the vicinity of a crack tip subjected to a mixed-mode loading. The in-plane displacement field was then used to study whether the strain field can be characterized by the modified HRR solution. In addition, the crack tip parameters, such as J integral, plastic mixity, and elastic mixity, of a mixed mode fracture specimen were also investigated in the paper.

Theoretical Background
For a mixed-mode crack in elastic-plastic material, the asymptotic crack-tip strain field around the crack tip can be written as [4, 6]

\[
\varepsilon_\psi = \frac{J}{\alpha \sigma_r \varepsilon_r \ln(n, M')} \tilde{\varepsilon}_\psi (\theta, n, M')
\]  (1)
where $\sigma_0$ is the yield stress, $\varepsilon_0 = (\sigma_0/E)$ the yield strain, $\alpha$ the strain hardening coefficient, $n$ the strain hardening exponent, $r$, $\theta$ the polar coordinates at the crack tip, $M^p$ the plastic mixity defined as

$$M^p = \frac{2}{\pi} \tan^{-1} \left( \lim_{\rho \to 0^+} \frac{\sigma_{\rho\rho}(r,\theta)}{\sigma_{\rho\rho}(r,0)} \right), \quad (2)$$

and $J$ is the line integral surrounding the crack tip defined by

$$J = \int_{\Gamma} W \, dy - \sigma_{\nu} \cdot n_j \, \frac{\partial u_i}{\partial x} \, ds,$$  

where $n_j$ is a normal vector perpendicular to the contour $\Gamma$ enclosing the crack tip, $u_i$ is the displacement vector on the contour, $ds$ is the differential arc length along $\Gamma$, and $W$ is the strain energy density. It is noted that $M^p = 1$ for mode I and $M^p = 0$ for mode II. $I_n$ is a dimensionless constant and $\varepsilon_i$ is a dimensionless function of angular position. A detailed tabulation of $I_n$ and $\varepsilon_i$ is given by Shih [6].

In a region called far field whose size is larger than the plastic zone but small compared to the crack length, the asymptotic elastic solution dominates. Then the $J$ integral can be related to the stress intensity factors $K_I$ and $K_{II}$ by

$$J = \frac{K_I^2 + K_{II}^2}{\pi}.$$  

In the far field, Shin [4] defined the elastic mixity $M^e$ to characterize the relative strengths of $K_I$ and $K_{II}$, which is given by

$$M^e = \frac{2}{\pi} \tan^{-1} \left( \lim_{\rho \to 0^+} \frac{\sigma_{\rho\rho}(r,\theta)}{\sigma_{\rho\rho}(r,0)} \right) = \frac{2}{\pi} \tan^{-1} \left( \frac{K_I}{K_{II}} \right), \quad (4)$$

For a central crack in a large plate subjected to a remote uniaxial stress oriented at an angle $\beta$ with respect to the axis of the crack, the ratio of $K_I$ and $K_{II}$ is related to $\beta$ by $K_I/K_{II} = \tan \beta$. Hence equation (4) can be rewritten as

$$M^e = \frac{(2/\pi)\beta}{\pi}. \quad (5)$$

### Material Properties and Specimen Geometries

The fracture specimen used in the experiment was made of 2024-O aluminum. A uniaxial stress-strain tension test was performed for AL 2024-O, and then a Ramberg-Osgood relation was used to fit the stress-strain curve, i.e.,

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left( \frac{\sigma}{\sigma_0} \right)^n.$$

The material properties obtained from the test are shown as follows:

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield stress, $\sigma_0$</td>
<td>120 MPa</td>
</tr>
<tr>
<td>Elastic Modulus, $E$</td>
<td>72 GPa</td>
</tr>
<tr>
<td>Strain hardening exponent, $n$</td>
<td>4.325</td>
</tr>
<tr>
<td>Strain hardening coefficient, $\alpha$</td>
<td>1.234</td>
</tr>
<tr>
<td>Poisson's ratio, $\nu$</td>
<td>0.333</td>
</tr>
</tbody>
</table>

Figure 1 shows the schematic diagram of the CTS specimen mounted on a special loading device. The CTS specimen with $W = 90$ mm was fabricated from an 8 mm thick plate of AL 2024-O. A chevron notch was machined in the notch tip. A fatigue precrack was produced by cyclically loading the notched specimen with a sinusoidal wave form using standard ASTM procedures. A random pattern was produced on the surface using black and white spray paint. The precrack of the CTS specimen is about 6 mm and the $a/W$ ratio is 0.5. In the experiment, a loading device designed by Rechard [11] was used so
that both the normal and shear loads can be applied to the CTS specimen. Hence, the investigation on the mixed-mode crack tip deformation fields can be conducted at some specific loading angle.

Experiments

The stereo vision was used to measure the three-dimensional displacement components surrounding the crack tip of a CTS specimen. The experimental setup for the fracture experiment at the loading angle of 75° is shown in Figure 2. The stereo vision is formed by two Sony XC-77 CCD cameras, one Matrox frame grabber and one Pentium III PC. The region of interest is about 25 × 22 mm. For the loading angle of 45°, the JAI CCD cameras were used and the region of interest is about 34 × 25 mm.

After calibrating the stereo system, an approximate error of 2 µm for each displacement component was obtained. A displacement loading was applied to the CTS specimen with an Instron material test machine. The images around the crack tip were taken at several loading stages and stored for later analysis. Combined with digital image correlation [12-14], both the in-plane and out-of-plane displacement fields of the interested area were determined using the stereo vision [9, 10].

Figure 1 CTS specimen with a special loading device

Figure 2 Experimental setup (Loading angle = 75°)
**Determination of J integral**

Based on the plane stress assumption, equation (3) can be used to evaluate the J integral along contours surrounding the crack tip. First, the experimental in-plane displacement field was smoothed [15] to obtain the strain field. Using deformation theory, the strains were separated into elastic strain and plastic strain and the stresses were obtained. Then the J integral was computed along several rectangular paths around the crack tip. Figure 3 shows the rectangular integration contour, along which the values of J integral were obtained.

![Figure 3 Integration contour for the J integral](image)

**Results and Discussions**

By using the smoothing technique [15], the in-plane displacement components \( u \) and \( v \) were smoothed to obtain the strains. Figure 4 shows the experimentally determined \( \varepsilon_{yy} \) strain distribution around the crack tip. Figure 5 presents the J integral against distance measured from the crack tip to the path of the line integral. It is well known that the three-dimensional effects [16-18] usually prevent the data lying inside a circle of radius \( r = 0.5 \, h \) from being analyzable using two-dimensional analyses. Hence, it can be seen in Fig. 5 that the J integral value only shows path independence for \( r/h > 0.5 \).

![Figure 4 Contour plot of \( \varepsilon_{yy} \) strain field for the CTS specimen at 23,400 N applied load (Loading angle of 45°)](image)
The in-plane strains determined experimentally at several points near the crack tip are compared with the values calculated using Shih’s plane strain solution. Figures 6 and 7 show the comparison of the experiment data and the calculated values at the loading angles of 75° and 45°, respectively. It is found that the measured values follow the trends of the Shih’s plane strain solution. It is also noted that the location of the points indicated in Figs. 6 and 7 is beyond the region ($r/h > 0.5$) where the three dimensional effects exist.
From the Tables of Shih, $M^p = 0.835$ for the loading angle of 75° and $M^p = 0.5$ for the loading angle of 45°. By equation (5), $M^e = 0.833$ for the loading angle of 75° and $M^e = 0.5$ for the loading angle of 45°. The evaluation of $(2/\pi)\tan^{-1}\left[\sigma_{yy}/\sigma_{xy}\right]$ along the radial line of $\theta = 0$ is presented in Figures 8 and 9. The values of elastic mixity evaluated using the measured crack tip stress field are close to those obtained from analytical solution. However the evaluated plastic mixity deviates from the analytical solution due to the three-dimensional effects.

![Figure 8](image1.png)

**Figure 8** Values of $(2/\pi)\tan^{-1}\left[\sigma_{yy}/\sigma_{xy}\right]$ along the radial line of $\theta = 0$ at 23,000 N applied load (loading angle of 75°)

![Figure 9](image2.png)

**Figure 9** Values of $(2/\pi)\tan^{-1}\left[\sigma_{yy}/\sigma_{xy}\right]$ along the radial line of $\theta = 0$ at 23,400 N applied load (loading angle of 45°)

**Conclusions**

The three-dimensional deformation field near the crack tip of a CTS specimen at the loading angles of 75° and 45° was determined using the stereo vision. The in-plane displacement field was smoothed to obtain the in-plane strain field. Using the deformation theory of plasticity and the plane stress assumption, the in-plane stress field was determined. The J integral values were then evaluated and show path independence for $rh > 0.5$. The in-plane strains determined experimentally at several points, ahead of crack tip and located beyond the region ($rh > 0.5$) where the three dimensional effects exist, are compared with the values calculated using Shih’s plane strain solution. Results indicate that the measured values follow the trends of the Shih’s plane strain solution. The values of elastic mixity evaluated using the measured far field stress data are close to those obtained from analytical solution. The plastic mixity determined using the near field stress data deviates from the analytical solution due to the three-dimensional effects.
Acknowledgements

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References