Dynamic analysis of the Tripteor X7: model and experiments

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Abstract: This paper deals with the dynamic analysis of a Parallel Kinematic Machine tool (PKM), the Tripteor X7. Parallel manipulators present, most of the time, important variations of their mechanical characteristics (stiffness, natural frequency, etc.) in function of the position of their end-effector. This particularity leads to new issues in the preparation of a machining operation. The part positioning for instance is becoming more and more important in order to guaranty the quality of the final product. A compliant multibody dynamic model is then proposed in order to simulate the dynamic behaviour of the Tripteor and validated through experiments: a contourning operation and experimental modal analysis.

Key words: PKM, Tripteor, modal analysis, dynamic, machine tool

1- Introduction

Parallel Kinematic Machine tools (PKM) are based on a new type of architecture which presents several independent kinematic chains between the base and the end-effector. The Tripteor X7 is a hybrid machine tool as it is composed of a parallel unit and a serial wrist (see Fig. 1). The parallel unit has 3 motorized legs and the serial wrist has 2 additional rotations, so the machine is a 5 axis machine tool [N1].

These machines are the object of many research works as their dynamic potential is higher than Serial Kinematic Machines (SKM) [TD1]. Nevertheless, they present several drawbacks. Most of the time, their command and the identification of their geometrical parameters are complex [P1]. Furthermore, the design choices made for lowering mobile masses and for building the joints lead to a loss of stiffness when compared to serial kinematic machine tools [A1, BC1].

These drawbacks lead to accuracy problems so to defects on the machined parts. The aim of this paper is to study the dynamic behaviour of a given machine tool: the Tripteor X7. Predicting its behaviour before machining a part could lead to a great improvement of its performances and of the quality of the machined parts.

Thus, the focus will be on the variations of the dynamic behaviour in function of the tool position in the workspace. First of all, the formulation of the dynamic problem is presented. The impact of the structure deflections is studied by introducing compliant coordinates in the proposed multibody dynamic model.

A simulation of a contouring operation is then performed and compared to the real machining of the part. Finally, the modal analysis of the machine is realized with the proposed dynamic model and the results are compared with experimental measurements of the modes of vibration.

2- Dynamic model with compliances

In order to determine the modes of vibration of the machine tool, the compliances of the machine have to be introduced. The choice of the parameters has been made after studying the stiffness of the mechanism performed in [BC2]. Most of the deflections come from the behaviours of the joints and of the legs. So these compliances are represented by lumped stiffnesses representing the behaviour of the legs (with the joints). Thus, the radial displacements in the joints between the legs and the mobile platform are considered, see Fig.2. 6 degrees of freedom are introduced and noted:

\[ \delta = [\delta_1 \ \delta_2 \ \delta_{21} \ \delta_{22} \ \delta_3 \ \delta_{32}]^T \] (1)
The stiffness associated to these degrees of freedom depends on the radial stiffness of the bearing and on the stiffness of the roller screw used to modify the leg lengths. \( \delta_i^2 \) is in the direction of leg \( I \) and \( \delta_i^1 \) is radial orthogonal to this direction. These stiffnesses are regrouped in the array \( K_\delta \):

\[
K_\delta = \begin{bmatrix}
K_{rj} & 0 & 0 & 0 & 0 & 0 \\
0 & \left(K_{rj}^{-1} + K_{screw}^{-1}\right)^{-1} & 0 & 0 & 0 & 0 \\
0 & 0 & K_{rj} & 0 & 0 & 0 \\
0 & 0 & 0 & K_{rj} & 0 & 0 \\
0 & 0 & 0 & 0 & K_{rj} & 0 \\
0 & 0 & 0 & 0 & 0 & \left(K_{rj}^{-1} + K_{screw}^{-1}\right)^{-1}
\end{bmatrix}
\]  

Where \( K_{rj} \) is the radial stiffness of the revolute joint with ball bearings between the legs and the mobile platform. \( K_{rj} \) is the stiffness of the roller screw system.

### 2.1 – Problem formulation

The motion equations are regrouped in an index 3 Differential Algebraic Equation (DAE). They can be deduced from Lagrange’s equations:

\[
M(q) \ddot{q} + C(q, \dot{q}) + g(q) + \frac{\partial}{\partial q} \lambda = s + r_{ext}
\]  

(3a)

\[
\phi(t, q) = 0
\]  

(3b)

Where \( q \) contains the generalized coordinates, \( M(q) \) and \( C(q, \dot{q}) \) are respectively the mass and Coriolis matrices, \( g(q) \) contains the efforts due to gravity, \( \phi(t, q) \) contains the constraint equations and \( \phi_i = \frac{\partial \phi}{\partial q} \), \( \lambda \) contains Lagrange’s multipliers, \( s \) and \( r_{ext} \) are respectively the command effort and the generalized force applied to the machine.

The main difficulty with PKMs is the choice of the parameters in the array \( q \). It is often simpler to consider more parameters than only those of the motorized joints [JB1]. Some constraints equations have then to be written for modelling correctly the mechanism kinematics [B1].

For the parallel unit of the Tripteor, 12 parameters are retained (see Fig. 3), and only 3 are independent. If only these 3 parameters (the leg lengths \( q_1, q_2, q_3 \)) are kept the equations of motion would be too large for any further computation.

So the array \( q \) can be written:

\[
q = [\phi_{17}, \phi_{11}, \phi_{24}, \phi_{54}, \phi_{18}, \phi_{58}, \phi_{19}, \phi_{77}, \phi_{74}, \phi_{79}, \phi_{27}]^T
\]

Fig. 3: Parameterization of the parallel unit.

There are finally 9 dependent parameters, so 9 constraints equations have to be written. They are obtained by considering the vectorial closed loop passing by legs 1 and 2 (3 equations) and legs 1 and 3 (2 equations, projections along \( x \) and \( y \)). The sixth equation expresses the orthogonality of vectors \( u_2 \) and \( w_2 \). The last three equations rely on the mobile platform orientation [P2]. These relations are regrouped in the array \( \phi(t, q) \). They are linearly independent and allow to determine all passive joints parameters for every position of the machine in its workspace.
\[
\begin{bmatrix}
M_u & M_a & M_d & M_s \\
M^T_u & M^T_a & M^T_d & M^T_s \\
M^T_u & M^T_a & M^T_d & M^T_s \\
M^T_d & M^T_s & M^T_u & M^T_a
\end{bmatrix}
\begin{bmatrix}
\dot{\delta} \\
\dot{\phi} \\
\dot{\phi} \\
\dot{\phi}
\end{bmatrix}
= \begin{bmatrix}
C_u & C_a & C_d & C_s \\
C_u & C_a & C_d & C_s \\
C_u & C_a & C_d & C_s \\
C_u & C_a & C_d & C_s
\end{bmatrix}
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{v} \\
\dot{v}
\end{bmatrix}
\]

(6a)

\[
\phi(u, v, \delta) = 0 \quad \text{(6b)}
\]

\[
\phi_u \delta + \phi_v v + \phi_{\delta} \delta = 0 \quad \text{(6c)}
\]

\[
\phi_u \delta + \phi_v \delta + \phi_{\delta} \delta = 0 \quad \text{(6d)}
\]

The matrices \(K_\delta\) and \(D_\delta\) are respectively the stiffness and dissipation matrices associated to the compliant coordinates \(\delta\).

The best way to obtain the evolutions of the compliant coordinates is to eliminate the Lagrange's multipliers \(\lambda\) by expressing them in function of the arrays \(u\) and \(\delta\)

\[
\lambda = -\phi^T \left( g_u + M^T_u \delta M_u + M^T_a \delta M_a + M^T_d \delta M_d + C_u \delta u + C_a \delta a + C_d \delta d + r_{conv} \right)
\]

(7)

And arrays \(\delta\) and \(\delta\) can be expressed with equations (6b), (6c) et (6d) in function of \(u\), \(\phi\), \(\delta\) and \(\delta\).

\[
\delta = -\phi^{-1} \left( \phi_u \delta u + \phi_v \delta v + \phi_{\delta} \delta \delta \right)
\]

(8)

\[
\delta = \phi^{-1} \left( \phi_u \delta u + \phi_v \delta v + \phi_{\delta} \delta \delta \right)
\]

(9)

The resolution of the inverse dynamic model consists in determining \(\delta\) knowing \(u\), \(\phi\) and \(\delta\).

\(\delta\) is the solution of a 2nd order Ordinary Differential Equation (ODE) obtained by substituting arrays \(\delta\) and \(\delta\) in Eq. (6a) by their expression (8) and (9).

The ODE to solve, isolated from system (6) can be written:

\[
\begin{bmatrix}
M^T_u A & M^T_a & M^T_d & M^T_s \\
M^T_u B - C_u \phi^T \phi & C_a & C_d & C_s \\
-M^T_d C_u \phi & C_a & C_d & C_s \\
-M^T_s C_u \phi & C_a & C_d & C_s
\end{bmatrix}
\begin{bmatrix}
\dot{\phi} \\
\dot{\phi} \\
\phi \\
\phi
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

(10)

\[
s = g_u + M^T_u \delta M_u (A \delta \delta M_a \delta + M^T_a \delta M_a \delta) + M^T_d \delta M_d \delta + M^T_s \delta M_s \delta \]

(14)

The interesting parameters of this flexible dynamic model are not the evolutions of the motors torque but the computed deformations for a followed path. The solving scheme used to obtain is represented Fig. 4.

The influences of the deformations are then studied for a contouring operation next paragraph.

3- Contouring simulation

This method can now be used to perform machining simulation on Tripteor X7. These simulations are able to show the influence of structure deformation on the machined part. The next paragraph concerns the application of the model on a machining operation: the contouring of an aluminium part. The results of the simulation are also compared with a réal machined part realized on the Tripteor.

The chosen machining operation does not represent the main application aimed at for the Tripteor X7 (which are 5 axis pocketing, difficult drilling, etc.). Nevertheless, this example is ideal to realize measurements on the machine and on the machined surface. It is also convenient in order to validate the developed modelling method.

Next paragraph, the experimental protocol and the measured parameters while machining are detailed. The machining operation is finally simulated and the results are compared to the experimental measurements obtained on the machine.

3.1 – Experimental protocol

Cutting conditions for this test are detailed Tab. 1. The machined part and the machining trajectory are represented Fig. 5.
In order to simulate the realized machining operation, machining efforts are directly measured with a Kistler table (9257B type).

Cutting conditions have been chosen in order to stress each axis of the machine. In this test, the tool bending is supposed to be negligible compared to the elastic deformations of the structure (620mm carbide cutter with a depth of cut of 10 mm and a radial depth of cut of 5 mm).

The parameters measured during this test are:
- The evolutions of the leg lengths ($q_1$, $q_2$, and $q_3$)
- The evolutions of the motor torques

The choice of an aluminium alloy is also important in order to make measurements on the machined part. Indeed, this material is easily marked by a milling cutter. Some important vibrations were generated during the machining of the edge 3 of the part. Fig. 6 shows the marks left on the machined surface in rough milling.

The measured profiles of the machined faces 1 and 3 confirm the visual aspect of the part. These profiles, measured longitudinally to the trajectory with a Mitutoyo Roughness Surftest SV 500 are represented Fig. 7.

These results will now be compared with the simulation of this contouring operation next paragraph.

3.2 – Simulation results

The resolution scheme of this dynamic problem is detailed Fig. 3. The input data are the measured trajectories of the tool (in the articulation space) and the measured cutting efforts. The evolutions of the articulation coordinates (leg lengths) are measured with the Numerical Command of the machine. The cutting efforts are measured with a Kistler dynamometer (force plate) linked to the multichannel analyser LMS Pimento.

The output data of the model are the evolutions of the compliant coordinates. With these evolutions the real position of the TCP can be computed and the influence of the compliant coordinates on the overall machine tool behaviour can be studied. The compliant coordinates are computed at the first step of the resolution by solving the ODE (14).

However, the result obtained with the ODE15s solver of Matlab is noisy. Indeed, even if this solver is recommended for stiff problem, a numerical noise appears in the solution and the interpretation of the results is then difficult. The computed time of this ODE on a PC with 2Gb ram and a 3Ghz Dual Core processor lasts 6H.

The computed and measured torques are represented Fig. 8. Larger gaps appear for the machining of face 3 (see Fig. 5). The vibrations of the torques observed experimentally are minimized by the introduction of a filtering needed to minimize the noise in the solution. This filtering acts like an artificial damping of the structure.

Nevertheless, larger torque variations are computed for the machining of face 3 than for the machining of face 1.
This result agrees with the measured torques obtained by the Numerical Command of the Tripteor X7.

The influence of the end-effector position is obvious when we consider the difference between the Tool Center Point (TCP) coordinates computed with the compliant model and the theoretical TCP coordinates computed with the rigid geometrical model. This difference computed along axis $x$ is represented Fig. 9.

The results along $x$ axis are relevant for faces 1 and 3 as this axis is the normal of these faces and the computed difference is then an image of the marks realized on the part.

The computed differences show variations of TCP along $x$ which can be important. In addition, these computed variations are filtered so they are underestimated. Fig. 9 shows position gaps of the TCP maximum for the machining of face 3. They can reach around 10 $\mu$m after filtering. These values agree with the marks observed on the machined part. Even if the defaults are underestimated, the simulation clearly shows that the elastic deformation have a higher influence for the machining of face 3.

![Figure 9: Computed position gap due to compliant coordinates](image)

4- Modal analysis

In order to validate the mass distribution of our dynamic model, the modes of vibration computed with the proposed model are compared to the modes obtained through experimental modal analysis.

4.1 - Experimental measurements

To determine experimentally the modes of vibration of the Tripteor X7, the structure has to be stressed. An electrodynamic shaker is used. It is able to generate efforts within the same range than those encountered while machining a part (up to 700 N).

The displacements of several points of the structure are measured with accelerometers placed on the legs, the mobile platform and on the spindle. Finally, 32 dof are considered: 9 by leg and 4 on the spindle (see Fig. 10 and 11) below.

All the data are stored and treated with LMS software Pimento. The method used to identify the modes relies on an LSCE algorithm. The modes are manually picked on a stabilization diagram computed by Pimento.

The evolution of the modes in the workspace is studied in order to highlight the variations of the dynamic behaviour of the machine. As the experimental setup is heavy, only three configurations are considered in the workspace. They are shown on Fig. 11.

![Figure 10: Experimental setup.](image)

![Figure 11: Machine configurations for the measurement of the modes](image)

![Figure 12: Evolution of the modes in function of the configuration](image)

First of all, we can notice on Fig. 12 that the frequencies of modes 1 and 3 are varying in function of the considered
configuration. A difference of 20% can be observed between configuration 2 and 3 for the first mode.

We also observe that the highest frequencies are measured when the legs are shorter (so stiffer).

These measurements can now be compared to the values obtained with the dynamic model presented earlier.

4.2 – Computed frequencies

The natural frequencies of the Tripteor X7 are computed with the dynamic model presented in §2. The problem considered is a classical eigenvalue problem, which can be written:

\[ M\ddot{\delta} + D\dot{\delta} + K\delta = 0 \]  \hspace{1cm} (15)

The natural frequencies are computed in the same \((x,y)\) plane than for the measurements presented in §4.1. They can be computed for the whole plane as the computing time for one configuration lasts less than 0.5s on a Pentium IV 3.0 GHz computer with 2 Gb of RAM.

![Computed and measured 1\textsuperscript{st} mode of vibration](image)

Figure 13: Computed and measured 1\textsuperscript{st} mode of vibration

After identification of the masses distribution in the model, the vibration frequencies computed and measured are compared. For the three measurement points, the computed and measured values of the first mode are in good agreement (difference under 5%). Moreover, the model shows the evolution of this mode in function of the position of the tool in a plane parallel to the table (200 mm above the table), see Fig. 13. The influence of the leg lengths is noticeable and confirms the importance of part positioning for guarantying the respect of the specified tolerances for a given operation.

Finally, other modes of vibrations can be computed with this method. Nevertheless, as 6 degrees of freedom are considered, they cannot be all determined. The second computed mode is very close to the 3\textsuperscript{rd} measured mode (around 70 Hz for Config. 1) and the 2\textsuperscript{nd} mode cannot be computed with this model.

5- Conclusion

This paper concerns the dynamic behaviour of a PKM, the Tripteor X7. A compliant multi-body dynamic model is proposed with an original resolution method. This model is then used to simulate a contouring operation of an aluminium part. Finally, a vibration analysis of the structure is performed through experimental modal analysis. The results are compared to the vibration frequencies computed with the developed multi-body model.

Currently, the compliances introduced are relatively simple. The influence of all the elements is not represented and only the overall behaviour of the leg is considered. It could be of great interest to introduce distributed compliances in order to have a better representation of the leg and be able to predict more modes of vibration.

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- modelling of the manufacturing process;
- virtual machining;
- emergence of new manufacturing methods.

7- References


