Lower Phase Velocity of Focused Laser Beam and Vacuum Laser Acceleration

N. Cao, Y.K. Ho a), Q. Kong, J. Pang, L. Shao, Y.J. Xie

Institute of Modern Physics, Fudan University, Shanghai 200433, China
a) Author to whom correspondence should be addressed.
FAX: +86-21-65643815. Electronic address: hoyk@fudan.ac.cn

Abstract. An acceleration channel has been found in the field of a focused laser beam propagating in vacuum, which shows similar characteristics to that of a wave guide tube of conventional accelerators: a subluminous wave phase velocity in conjunction with a strong longitudinal electric field component. Relativistic electrons injected into this channel can remain synchronous with the accelerating phase for sufficiently long time and receive considerable energy from the field. We call this acceleration scheme CAS (capture and acceleration scenario). The basic conditions for CAS to occur are examined and the output properties of electrons accelerated by this scheme are also presented in this paper.

Advances in laser technology have resulted in a new class of compact, ultrashort pulsed lasers with extremely high intensity [1-3], and high power lasers continue to spur new concepts for advanced laser-driven accelerators [4-9]. Among them, the far-field laser acceleration of free electrons in vacuum has received wide attention [10] because it avoids a number of potential problems in other accelerating scheme. Nevertheless, vacuum laser acceleration has a foundational and long-standing question that has not been answered totally till now. Can a free electron have net energy exchange with a laser beam in far-field where the interaction length is unlimited [11]?

According to the Lawson-Woodward theorem [12], a light beam in the far-field is the superposition of propagating plane waves, thus its effect on a particle is the sum of plane wave effects. Since a particle moves at velocity less than c and a plane wave moves at c in a vacuum, the particle must slip in the phase of the wave, and because of phase slippage, the effect of light on a particle averages to zero if the interaction is unbounded.

More recently it was shown [10], via experiments and simulations that a focused laser pulse interacting with low energy electrons (at rest or near at rest) could be ponderomotively scattered and receive net energy gain by interacting with the nonlinear ponderomotive laser force.

In our studies, we found that, for a focused laser beam, there exists a region characterized by lower phase velocity ($V_p < c$). If the amplitude of the longitudinal electric field in the region is strong enough, and if one can inject fast electrons into it, then these electrons can remain synchronous with the accelerating phase for sufficiently long time such that they receive considerable energy from the field. We named this acceleration scheme CAS (the capture and acceleration scenario)[13]. Simulations indicated that, at the regime of the CAS, the electron’s trajectory is perturbed significantly by the laser field as it enters the high intensity
region, which in effect limits the interaction region. Hence, large energy gains are obtained without limiting the interaction distance by the use of additional optics. For presently available lasers (>100TW), substantial energy gains (>100MeV) can be obtained.

This paper is organized as follows. The subluminous phase velocity region in the laser field is derived firstly. Secondly, the acceleration channel is described. Then, the dynamics behaviors of CAS electrons are discussed and the output properties of electrons interacting with the laser field are given lastly.

Numerical simulation methods used here are similar to those we used previously [13]. A four-dimensional energy-momentum configuration \((\gamma, p_x, p_y, p_z)\) is used to specify an electron state, where \(\gamma\) is the Lorentz factor and the momentum \(p\) is normalized in the units of \(m_e c\), the angle \(\theta = \tan^{-1}(p_x / p_z)\) is the electron injection angle with respect to the \(z\) axis, the laser propagation direction. The electron dynamics is governed by the relativistic Newton-Lorentz equation. For simplicity, throughout this paper, time and length are normalized by \(1/\omega\) and \(1/k\), respectively. Without losing generality, we assume that, initially electrons move freely without the influence of laser field, the center of the laser pulse and that of the injected electron bunch reach the point \(x = y = z = 0\) simultaneously at \(t = 0\), \(\Delta t_i\) is used to specify the relative delay between the laser pulse and an electron.

For a laser beam of Hermite-Gaussian(0,0) mode polarized in \(x\)-direction and propagating along \(z\)-axis, the phase of the Gaussian beam field is given by

\[
\varphi(x, y, z) = k z - \omega x - \varphi(z) - \varphi_0 + \frac{k(x^2 + y^2)}{2R(z)}
\]  

(1)

where \(\varphi(z) = \tan^{-1}(z/Z_R)\) is the Grouy phase shift, \(Z_R=kw_0^2/2\) is the Rayleigh length and \(R(z) = z(1 + Z_R^2/z^2)\) is the radius of the curvature. The phase velocity \((V_\varphi)\) of the wave along a particle trajectory can be calculated using the equation

\[
\frac{\partial \varphi}{\partial t} + (V_\varphi)_t (\nabla \varphi)_t = 0
\]  

(2)

where \((\nabla \varphi)_t\) is the gradient of the phase field along the trajectory. From it, we can get the phase velocity along the direction normal to the equal-phase face (The minimum phase velocity) \(V_{\varphi_m}\) as follows

\[
V_{\varphi_m} = \frac{ck}{|\nabla \varphi|} = \frac{ck}{\sqrt{\left(\frac{2\rho l}{w_0(1+l^2)}\right)^2 + \left(k - \frac{1 - f_\varphi}{Z_R(1+l^2)}\right)^2}}
\]  

(3)

with

\[
f_\varphi = \frac{r^2(1-z^2/Z_R^2)}{w_0^2(1+z^2/Z_R^2)}
\]  

(4)

and \(p=\sqrt{x^2 + y^2}/w_0\), \(l=zt/Z_R\), \(r=\sqrt{x^2 + y^2}\). From Eq.(3), it is straightforward to find that the subluminous phase velocity region: \(V_{\varphi_m} < c\) occurs approximately in
the region \( r > w(z) \). In this region, the magnitude of the minimum phase velocity is of the order \( V_{\text{eqm}} \sim c[1 - 1/(k w_0)^2] \), with an angle relative to z-axis \( \sim 1/(k w_0) \). The distribution of \( V_{\text{eqm}} \) on the plane \( z=0 \) is shown in Fig 1. Another feature of Fig.1 is that at \( z=0 \), the subluminous phase velocity region occurs only for \( r > w_0 \).

**FIGURE 1.** The distribution of the minimum phase velocity \( v_{\text{eqm}}/c \) versus \( \beta = x/w_0 \) and \( \eta = y/w_0 \) in the \( z=0 \) plane of the focused laser beam with \( k w_0 = 60 \).

For accelerating particles, in addition to the subluminous phase velocity, the field strength, i.e., the amplitude of the longitudinal electric field, is also an important factor. We thereby introduce a quantity \( Q \) that combines these two factors together to represent the ability of the laser field for accelerating charged particles. We call it acceleration quality factor, which is defined by

\[
Q = Q_0 \left( 1 - V_{\text{eqm}}/c \right) \left[ x/w(z) \right] \exp \left[ -\left( x^2 + y^2 \right) / w(z)^2 \right]
\]

for \( V_{\text{eqm}} \leq c \), and \( Q=0 \) for \( V_{\text{eqm}} > c \) (5)

Where \( Q_0 \) is a normalization constant chosen to make \( Q \) in the order of unit. In Eq.(5), \( 1 - V_{\text{eqm}}/c \) represent the contribution from the phase velocity, and the remaining term is proportional to the amplitude of the longitudinal electric field, it describes the effects of laser intensity on electron’s behavior.

The distribution of the acceleration quality factor \( Q \) on the plane \( z=0 \) for a focused laser beam with \( k w_0 = 60 \) is given in Fig.2.
FIGURE 2. The acceleration quality factor Q versus $\rho = x/w_0$ and $\eta = y/w_0$ in the $z=0$ plane of a focused laser beam with $kw_0=60$.

It can be seen from Fig.2 that significant values of Q emerge just beyond the beam width and concentrated in the areas near the polarization plane. This is because the lower phase velocity region locates outside the beam width and the large amplitudes of the axial electric field distribute near the polarization plane. This is the favorable region for accelerating electrons and we call it the acceleration channel of the laser beam propagation in vacuum. It is also of interest to note that in the region near the beam axis, the factor Q=0 because $V^n>c$ there. It means the region near beam axis is not good for accelerating charged particles. This is because the phase velocity here is the highest, and the amplitude of the axial electric field is small. Along the diffraction angle ($\sim 1/kw_0$), exceeding several Rayleigh lengths, the intensity of the laser field become much weaker and therefore, the effect of acceleration become inconspicuous. From the above discussions, we may say that there exists an acceleration channel in the field of the focused laser beam propagating in vacuum, which shows similar characteristics to that of a wave guide tube of conventional accelerators: subluminous phase velocity in conjunction with a strong longitudinal electric field component. Consequently, if one can inject fast electrons into this channel, then some of them may remain synchronous with the accelerating phase for sufficiently long time such that they receive considerable energy from the field.

Now we turn to study the conditions under which CAS phenomenon can occur. Simulation results show that there exists a threshold value of laser intensity ($a_0)_{th}$ which is sensitive to the beam width $w_0$. One can observe the CAS phenomenon only as $a_0\geq(a_0)_{th}$. Fig.3 shows the electron maximum final energy $\gamma_{fm}$ vs. the laser intensities $a_0$ at different beam width, $\gamma_{fm}$ is the maximum value of $\gamma$ as $\phi_0$ is varied over the range $0, 2\pi$.
FIGURE 3. The maximum output energy $\gamma_m$ as a function of the laser intensity $a_0$ at different laser beam width. Solid line for $w_0=60$, $P_x=0.9$, $P_z=9$; dashed line for $w_0=80$, $P_x=1.1$, $P_z=11$; dotted line for $w_0=100$, $P_x=1.2$, $P_z=12$; and dash dot dot line for $w_0=120$, $P_x=1.4$, $P_z=14$; momentums in y-direction in all cases equal zero, we terminate the calculation at $\omega t=400000$.

Fig.3 illustrates that, (i) $(a_0)_{th}$ is strongly dependent on the laser beam width $w_0$, it increases quickly with $w_0$. (ii) The scaling law for net energy gain of electrons from the laser field is approximately as $\gamma_m \propto a_0^n$ with $n \sim 1$ as $w_0 \ll 150$. This feature is consistent with the mechanism underlying CAS, namely the acceleration occurs primarily in the acceleration channel by the longitudinal electric field. (iii) With the increasing of beam width $w_0$, the maximum output energy $\gamma_m$ increase also. This is due to the fact that the work $\Delta W$ done by the laser field on the electron in the longitudinal direction is proportional to the beam width and can be approximately estimated by $\Delta W \approx E_z \cdot Z_R \approx \frac{E_0}{k w_0^2} \cdot \frac{k w_0^2}{2} \approx E_0 w_0$, where $E_0$ is the transverse electric field amplitude at focus, and $Z_R$ the Rayleigh length.

In addition to the laser intensity, the electron initial incident momentum and incident angle are also important factors for CAS to emerge.

Roughly speaking, for CAS to occur, the laser intensity should be large enough as $a_0 \geq 5$, and electrons should be injected into the laser field in a small angle (typically $\tan \theta = 0.1$) with injection energy ranging 5-15MeV.

The output properties of the electrons interacting with the laser pulse in vacuum are shown in Fig.4.
Our study shows that the output property of the interaction is a high-energy electron macro-pulse consisted of many micro-pulses whose duration corresponds to the periodicity of the laser field (Fig. 4(a)). Due to the fact that the incident electrons encounter all phases of each laser period, energies of CAS electrons spread widely. For the specific example of Fig. 4, the output energy range from 245 MeV to 491 MeV, with average energy about 356 MeV and the number of CAS electrons is about 34% of the total incident electrons.

The output electrons can be roughly divided into two groups: CAS electrons concentrated in the small scattering angle region and inelastic scattering (IS) electrons correspond to the broad peaks in the larger angle region (Fig. 4(c)). The simulation results of the energy-angular correlation are roughly consistent with the equation derived from the classical Hamilton-Jacobi theory [14] (Fig. 4(d)).

\[
\theta_f = \tan^{-1}\left(\sqrt{\frac{p_{\perp}^2 + 2(\gamma_f - p_{\perp})(\gamma_f - \gamma_i)}{(p_{\perp} + \gamma_f - \gamma_i)}}\right)
\]

The deviations are due to that we use Gaussian beam to describe the laser field, and in CAS case, the longitudinal component plays a very important role [13]. Another reason is that we set the emittance of the initial states of the electrons 0.1π mm·mrad in the calculations.
In conclusion, we find that for a focused laser beam propagating in free-space, there exists, surrounding the laser beam axis, a subluminous wave phase velocity region. Relativistic electrons injected into this region can be trapped in acceleration phase and remain in phase with the laser field for sufficiently long time, thereby receiving considerable energy from the field. The basic conditions for CAS to occur are the laser intensity should be large enough as $ao>5$, and electrons should be injected into the laser field in a small angle (typically $\tan \theta = 0.1$) with injection energy range be in 5-15MeV. The output electrons can be divided into two groups: the CAS electrons with higher energy concentrated in small scattering angle region and the IS electrons with lower energy gains scattered widely in space. The output electrons can be run through a magnetic spectrometer to select a near-monoenergetic electron micro-pulse train. Studies show that CAS is an attractive laser acceleration scheme.

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