Electromagnetic localized structures in a relativistic laser-plasma interaction

Lj. Hadžievski*, M. M. Škorić*, M. S. Jovanović† and Lj. Nikolić**

*Vinča Institute of Nuclear Sciences, POB 522, 11001 Belgrade, Yugoslavia
†Faculty of Natural Sciences, University of Niš, 18000 Niš, Yugoslavia
**Graduate University for Advanced Studies, National Institute for Fusion Science, Toki, Japan

Abstract. Localized electromagnetic structures with trapped laser light inside are often found in simulations of relativistic laser-plasmas. First, we examine existence and stability of one-dimensional electromagnetic solitons formed in a relativistic laser interaction with an underdense cold plasma. In a weakly relativistic model, the original equation of the nonlinear Schrödinger type, with local and non-local cubic nonlinearities is derived. Standing electromagnetic solitons are analytically shown to be stable in agreement with the model simulation. Next, we discuss a novel example of a stimulated backscattering of a laser light from the slow trapped electron-acoustic wave, involving a standing (localized) Stokes light sideband. Conditions are examined when above underdense instability dominates over standard stimulated Raman backscattering.

INTRODUCTION

Slow intense electromagnetic structures with trapped laser light inside are frequently observed in simulations of relativistic laser-plasmas. A complex interplay between relativistic electronic instabilities which produces depletion and frequency down-shift of a laser pulse is often followed by a phenomenon of light localization. Such, a laser pulse gets partially transformed into close to zero velocity light structures, such is e.g. a train of ultra-short relativistic solitons. We first discuss formation and stability of electromagnetic solitons in a relativistic laser propagation through a long underdense plasma. In conditions dominated by stimulated Raman scattering (SRS) and relativistic modulational instability standing light solitons are analytically found. Regions of stability for weak and strong relativistic case are discussed and compared with simulations. Next we turn to a new phenomenon of stimulated electron-acoustic wave scattering (SEAS) proposed recently to explain anomalous scattering data. In first particle simulations of SEAS, in moderately underdense plasma (> n_cr/4), not accessible to SRS, a strong backscatter at the electron plasma frequency is observed. A mechanism of resonant coupling between laser, standing backscattered light and trapped low-frequency electron-acoustic mode is proposed to explain an onset of SEAS in hot relativistic plasmas. Conditions for SEAS and implication on relativistic laser-plasma scattering and electron heating are discussed.
I ONE-DIMENSIONAL ELECTROMAGNETIC SOLITONS

Relativistic electromagnetic (EM) solitons in laser driven plasmas were analytically predicted and found by particle simulations [1-7]. Relativistic solitons are localized structures with EM energy self-trapped by locally modified plasma refractive index via two effects: the relativistic electron mass increase and the electron density drop by the ponderomotive action of intense laser light [1,3]. A large effort was put into studies of one-dimensional (1D) relativistic EM solitons in an ultraintense laser interaction with underdense and overdense plasmas [8-13]. For laser pulses longer than the electron plasma wave wavelength, spatially modulated depleted pulse, due to stimulated Raman scattering, readily in nonlinear stage breaks up into a slow train of ultra-short 1D relativistic EM solitons [5,8-9]. It was estimated, for ultra-short laser pulses [9], that 30 to 40% of the laser energy can be trapped inside these low-frequency electromagnetic solitons creating a significant channel for laser beam energy conversion. In this section, the problem of existence, stability and dynamics of linearly polarized electromagnetic solitons is studied by 1D analytical model for a weak relativistic nonlinearity.

Nonlinear Schrödinger model

The fully nonlinear relativistic one-dimensional wave equation, the continuity equation and the cold electron momentum equation, in the Coulomb gauge for the plasma with fixed ions, read

\[
\left( \frac{\partial^2}{\partial t^2} - c_0 \frac{\partial^2}{\partial x^2} \right) a = -\frac{\omega_p^2}{n_0} n, \quad (1)
\]

\[
\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} \left( np \frac{1}{m\gamma} \right) = 0, \quad (2)
\]

\[
\frac{\partial p}{\partial t} = -eE_{||} - mc^2 \frac{\partial \gamma}{\partial x}, \quad (3)
\]

where \( a = eA/mc^2 \) is the normalized vector potential in y direction, \( n \) is the electron density, \( p \) is the electron momentum in x direction, \( \gamma \equiv \left( 1 + a^2 + p^2/m^2c^2 \right)^{1/2} \), \( E_{||} \) is the longitudinal electric field, \( n_0 \) is the unperturbed electron density and \( \omega_p = \left( 4\pi e^2 n_0/m \right)^{1/2} \) is the background electron plasma frequency. In distinction to circular polarization [9,13], linearly polarized waves have odd harmonics of the vector potential \( a \) and even harmonics of the electron density \( \delta n \) [1,12].

In a weakly relativistic limit for \( |a| << 1 \) and \( |\delta n| << 1 \) we expand the right hand side of (1) and introduce the normalized perturbed electron density \( \delta n = (n - n_0)/n_0 \) and dimensionless variables \( x \to (e\omega_p^{-1})x \) and \( t \to (\omega_p^{-1})t \) to obtain the wave equation for the vector potential envelope \( A \)

\[
i \frac{\partial A}{\partial t} + \frac{1}{2} A_{xx} + \frac{3}{16} |A|^2 A - \frac{1}{8} (|A|^2)_{xx} A + \frac{1}{24} (A^2)_{xx} A^* = 0. \quad (4)
\]
The eq. (4) has a form of the generalized nonlinear Schrödinger (NLS) equation [16] with two extra nonlocal (derivative) nonlinear terms. We calculate the conserved quantities: photon number \( P \)

\[ P = \int |A|^2 dx, \quad (5) \]

and Hamiltonian \( H \)

\[ H = \frac{1}{2} \int \left\{ |A_{xx}|^2 - \frac{3}{16} |A|^4 - \frac{1}{8} (|A|^2_x)^2 - \frac{1}{6} |A|^2 |A_x|^2 \right\} dx. \quad (6) \]

We look for a stationary, localized solution of (4), found in an implicit form

\[ \pm \lambda x = \frac{1}{2 \sqrt{2}} \ln \left( \frac{\sqrt{1 - \frac{\lambda^2}{32 \lambda^2}} + \sqrt{1 - \frac{\lambda^2}{3}}}{\sqrt{1 - \frac{\lambda^2}{32 \lambda^2}} - \sqrt{1 - \frac{\lambda^2}{3}}} \right) - \frac{4}{3} \lambda \ln \frac{\frac{1}{3} \sqrt{32 \lambda^2 - 3 \lambda^4} + \sqrt{1 - \frac{\lambda^2}{3}}}{\sqrt{1 - \frac{\lambda^2}{32 \lambda^2}}}, \quad (7) \]

with a soliton amplitude \( a_0 = \frac{4 \sqrt{2}}{\sqrt{3}} \lambda \) of a vector potential \( a \) oscillating at \( \omega = 1 - \lambda^2 \), slightly below the plasma frequency. For the soliton strength \( \lambda \) above the critical value \( \lambda \geq \lambda_c = \frac{3}{4 \sqrt{2}} (a_0 \geq \sqrt{3}) \) the solution has a form of a "cusp" soliton [16]; the centrally highly pointed waveform. For small amplitudes \( \lambda \ll \lambda_c \) one neglects the non-local terms and the solution becomes the well-known secant hyperbolic (NLS) soliton.

**Soliton Stability and Model Simulations**

To check the soliton (7) stability we use the Vakhitov-Kolokolov criterion [16,17]

\[ \frac{dP_0}{d\lambda^2} > 0, \quad (8) \]

where \( P_0 \) is the soliton photon number defined by (5). The function \( P_0(\lambda) \) for the soliton solution (7) is calculated analytically as

\[ P_0(\lambda) = \frac{16 \sqrt{2}}{3} \lambda + 2 \left( 1 - \frac{32}{9} \lambda^2 \right) \ln \left| \frac{1 + \frac{4 \sqrt{2}}{3} \lambda}{1 - \frac{4 \sqrt{2}}{3} \lambda} \right|. \quad (9) \]

The curve \( P_0(\lambda) \) represents the stationary solution which corresponds to the minimum of the Hamiltonian \( H \) for the fixed photon number \( P \). According to the condition (8) the soliton (7) turns out to be stable in the region \( \lambda < \lambda_c \approx 0.44 \) \((a_0 < a_c \approx 1.44)\) indicating that cusp solitons are also unstable \((\lambda_c < \lambda_c)\). More generally, we can now conclude that small amplitude linearly polarized solitons \((a_0 < 1)\) within the weakly relativistic model are stable. The shape of the curve \( P_0(\lambda) \) predicts the soliton instability region \( \lambda > \lambda_c \approx 0.44 \). The Vakhitov-Kolokolov criterion just solves a soliton linear
stability problem therefore, giving no prediction about the nonlinear evolution of unstable solitons or about stability of the arbitrary shape localized structures. According to nonlinear analysis [18] for the generalized NLS equation with a similar shape of $P_0(\lambda)$, besides the stationary solution there exist two other regimes: (a) soliton collapse and (b) long-lived relaxation oscillations around the stable soliton amplitude. In our case, due to the local cubic nonlinear (NLS) term in (4), one can plausibly expect such dynamical regimes. However, the presence of two extra nonlocal nonlinear terms in (4) indicates the possibility of other dynamical states.

To check analytical results and predicted soliton regimes we perform direct simulation of the model equation (4) using an algorithm based on the split-step Fourier method [19], originally developed for NLS equation. Results prove that the initially launched solitons (7) with the parameters inside the stability region $\lambda < \lambda_s \approx 0.44$ remain stable. Solitons with parameters outside the stability region evolve toward the corresponding stable soliton with long-lived relaxation oscillations. The evolution of the initially launched soliton with amplitude $a_0 \approx 1.6 (\lambda \approx 0.2)$ outside the stability region is shown on Fig. 1a. A similar behavior exhibit initially perturbed solitons with photon number $P < P_{\text{max}} = P_0(\lambda_s) \approx 4.79$ inside the stability region $\lambda < \lambda_s$ or different localized structures with small deviation from the stable equilibrium. As an example of such dynamics, the time evolution of Gaussian structure with the initial amplitude $A_0 = 0.536$ and photon number $P = 2.875$ is shown in Fig. 1b. The evolution is long-lived relaxation oscillations around the stable soliton amplitude (7) with $\lambda \approx 0.2 (a_0 \approx 0.653)$ which corresponds to the exact value of the photon number $P = 2.875$. These dynamical regimes exist also for NLS equation and they are analytically predicted and numerically confirmed in [18]. However, when the initial perturbation increases, the period grows with oscillations becoming strongly nonlinear to exhibit new types of long-lived localized dynamical structures (Fig. 1c). Further deviation from stable equilibrium leads to aperiodic growth and soliton collapse (Fig. 1d). The understanding of different regimes is important for an insight into the low-frequency process of formation of stable relativistic solitons behind the laser pulse front inside the photon condensate [8]. More detailed determination of these regions in the parameter plane $(P, \lambda)$ and separatrix curves, requires additional analytical and numerical studies [20].

II STIMULATED ELECTRON-ACOUSTIC-WAVE BACKSCATTERING

Intense laser-plasma interaction can be a source of various electronic instabilities [21-25]. Recently, stimulated backscattering from a trapped low-frequency electron-acoustic wave (SEAS) [D. S. Montgomery et al., Phys. Rev. Lett. 87, 155001 (2001)] was proposed to reinterpret reflection spectra earlier attributed to stimulated Raman scattering (SRS) from unrealistically low densities. Namely, in the linear theory, the so-called electron-acoustic wave (EAW) exists, i.e. a strongly damped linearized Vlasov-Maxwell (VM) mode whose phase velocity is between an electron plasma wave and an ion-sound wave; often neglected in studies of wave-plasma instabilities [26-28]. However, analytical studies of 1D non-linear VM solutions have found that strong electron trapping
can occur even for small amplitude electrostatic wave, resulting in undamped nonlinear traveling waves (BGK-like) [27,28] or, with an inclusion of small dissipation, in weakly damped traveling solutions [29]. The first observation has encouraged further investigation of domains and conditions for SEAS. In this section, excitation of SEAS and its interconnection with SRS instability is addressed by EM relativistic 1d3v particle simulation of linearly polarized laser interacting with a plasma layer placed in vacuum.

From our simulations, SEAS is identified as a resonant 3-wave parametric interaction [30] involving the laser pump \((\omega_0, k_0)\), the backscattered lightwave \((\omega_s, k_s)\) and the trapped electron-acoustic wave (EAW) \((\omega_a, k_a)\). In the linear instability stage, resonant conditions \(\omega_0 = \omega_s + \omega_a\) and \(k_0 = -k_s + k_a\) are well satisfied, while electromagnetic waves (pump and Stokes wave) satisfy standard dispersion equation \(\omega_{0,s}^2 = \omega_p^2 + c^2 k_{0,s}^2\). The backscattered wave is always found to be driven near critical, i.e. \(\omega_s \approx \omega_p\) which implies \(k_s \approx 0\) and \(V_s \approx 0\) \((\omega_p\) is the plasma frequency, and \(V_s = c^2 k_s / \omega_p\) is the light group velocity). Therefore, the Stokes sideband is a slow, almost standing (localized) electromagnetic wave. The above scheme is observed for wide range of laser intensities, plasma densities and temperatures. It is known that the high temperatures can significantly change the growth rates and sometimes suppress parametric instabilities [31]. However, according to [29], efficient excitation of trapped EAW, is expected in the range \(v_{ph}/v_t = 1 - 2\) \((v_{ph}\) and \(v_t = (T/m)^{1/2}\) are the phase and electron thermal velocities). Thus, for SEAS excitation at the threshold, high thermal velocities which closely match the EAW phase velocity are important.

Fig. 2a shows the discrete spectrum in an early phase of SEAS instability. The density...
and the plasma length are \( n = 0.6 n_{cr} \) \( (n_{cr} = n(\omega_p/\omega_0)^{1/2}) \) and \( L = 40 \alpha/\omega_0 \), respectively, the longitudinal thermal velocity is \( v_t/c = 0.28 \) and the laser strength is \( \beta = 0.3 \). The backscattered EM wave grows at the electron plasma frequency \( \omega_p \approx 0.72 \omega_0 \) (the laser wave at \( \omega/\omega_0 = 1 \) is not shown), while corresponding EAW is at \( \omega_0 - \omega_p \approx 0.28 \omega_0 \). Note that apart from ES noise around a natural plasma mode \( (\omega_0 \approx 0.72 \omega_0) \), ponderomotively driven non-resonant modes are also present (not shown in Fig. 2) at 2nd, \( \omega = 2 \omega_0 \) and \( k = 2 k_0 \) \( (v_{ph}/c \approx 1.3) \), as well as at zero-harmonic [32]. From obtained data, it follows that the phase velocity of the EAW is \( v_{ph}/c = (\omega_0 - \omega_p)/k_0 \approx 0.45 \).

We model SEAS as a resonant parametric 3-wave coupling

\[
\frac{\partial a_0}{\partial t} + V_g \frac{\partial a_0}{\partial x} = -M_0 a_s a_s, \\
\frac{\partial a_s}{\partial t} - V_g \frac{\partial a_s}{\partial x} = M_0 a_0 a_a, \\
\frac{\partial a_a}{\partial t} + V_g \frac{\partial a_a}{\partial x} + \Gamma_a a_a = M_0 a_0 a_s,
\]

where \( V_i > 0 \) are the group velocities, \( \Gamma_a \) is damping rate for EAW \( (\Gamma_0 = \Gamma_a = 0 \) for light waves is used), \( M_i > 0 \) are the coupling coefficients and \( a_i \) are the wave amplitudes, where \( i = 0, s, a \), stand for the pump, backscattered wave and EAW, respectively. Our model is a short plasma and for high reflectivity, instability has to be Absolute. With boundary conditions \( a_0(0) = E_0, a_s(L) = a_a(0) = 0 \), the backscatter is absolute for \( L/L_0 > \pi/2 \), [33-34], where \( L_0 = (V_s V_0)^{1/2}/\gamma_0 \) is the interaction length and \( \gamma_0 = E_0 (M_s M_a)^{1/2} \) is the uniform growth rate. Since observed \( V_s \approx 0 \) for the backscatter, the condition (13) is readily satisfied \( (L_0 \approx 0) \). However, for EAW (12) a linear dispersion relation in explicit form does not exists [36-39]. Since damping rate \( \Gamma_a \neq 0 \), the EAW is characterized by the longitudinal absorption length \( L_a = V_a/\Gamma_a \), SEAS-backscatter instability becomes absolute under an extra condition [34],

\[
L_0/L_a < 2.
\]

In linear theory EAW is a highly damped mode, so the absorption length \( L_a \) is taking small values. The key factor for an onset and growth of SAES is nearly critical "standing" backward Stokes wave \( (L_0 \approx 0) \), so that \( V_s \approx 0 \) also minimizes the threshold \( E_0 \) for the instability [34], \( \gamma_0 > 0.5 \Gamma_a (V_s/V_0)^{1/2} \). Initial discrete EM spectrum, later changes to modulated, broadened blue-shifted profiles, consistent with 3-wave complexity induced by a nonlinear phase, as proposed by some of these authors [25, 35].

The temperature effect is seen near the threshold for SEAS (laser strength \( \beta = (eE_0)/(mc\omega_0) \approx 0.3 \), where \( E_0 \) is the amplitude of the electric field). There is an optimum temperature for perfect matching with an EAW which gives a maximum SEAS reflectivity. For \( v_t/c = 0.2 \) observed reflectivity is very high - nearly 140% of the incident
FIGURE 2. a) Spectrum of electromagnetic (top) and electrostatic (bottom) waves in the plasma layer \((n = 0.6n_{cr}, l = 40c/\omega_0)\) for time interval \(t\omega_0 = 322 - 2438\). The initial electron thermal velocity is \(\nu_t/c = 0.28\). b) Reflectivity in time from two connected plasma layers. Initial bursts from ordinary SRS in \(L_1\) are followed at late times by a huge SEAS pulse generated in \(L_2\), after it was heated by SRS-produced hot electrons in \(L_1\).

laser light. One calculates \(v_{ph}/\nu_t = 2.71, 2.58, 1.84\) and 1.72 for \(\nu_t/c = 0.19, 0.20, 0.28\) and 0.30, respectively. For temperatures \(\leq \nu_t/c = 0.18\) and \(\beta = 0.3\) the instability was not observed during time period of \(t\omega_0 = 5000\). However, for laser intensities well above the threshold there appears no need for high electron temperatures to excite SEAS. For example, already at \(T = 500\text{eV}\), with a strong relativistic pump \(\beta = 0.6, n = 0.6n_{cr}\) and \(l = 40c/\omega_0\) instability develops fast and quickly saturates within \(t\omega_0 = 400\).

Finally, we discuss a coexistence and interrelation between SRS and SEAS. The simulated system consists of two connected underdense plasma layers \(L_1\) and \(L_2\), of the length \(l_1 = 20c/\omega_0\) and \(l_2 = 80c/\omega_0\) with corresponding densities \(n_1 = 0.2n_{cr}\) and \(n_2 = 0.6n_{cr}\), respectively. Initial temperature is taken at \(500\text{eV}\). Our choice of densities makes \(L_1\) strongly active for Raman instability, while \(L_2\) (overdense for SRS) is practically in a role of a heat sink. Simulations show common picture, strong SRS with intermittent reflectivity pulsations [25, 35]. The instability eventually gets suppressed by strong electron heating. Hot electrons quickly escape the Raman region to enter and heat the sink \((L_2)\). A striking feature emerges at late times, with an a second intense reflected pulse much larger than the original Raman signal (see Fig.3). This is readily identified as SEAS which comes from the large "sink", once its temperature increased to resonate with EAW and excite SEAS. Therefore, SEAS mediated by SRS becomes a dominant process, as an example of a complex interplay possibly relevant to our understanding of future NIF experiments.
In summary, in first particle simulations in a plasma not accessible to SRS, strong isolated SEAS reflection was observed near the electron plasma frequency. While in reported experiments [26] SEAS to SRS signal ratio was smaller than $10^{-3}$, conditions in which SEAS dominates over SRS were found.

ACKNOWLEDGMENTS

This work was supported by Ministry of Science and Technology, Serbia, Project 1964.

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