Abstract

Simulating fluid flow through heterogeneous reservoirs requires the use of upscaling techniques that account for subscale effects, but which nevertheless accurately represent the fluid flow behaviors at the reservoir scale. To simulate single-phase fluid flow through a system, upscaling often involves estimating the unknown permeability at the coarse scale, using information available at the fine scale. Many upscaling methods share a common feature: they define localized fine-scale problems or cell problems and solve them to estimate the permeability at the coarse scale. Cell problems are often calculated with predefined boundary conditions (BCs). If the BCs do not account for actual local spatial variations of flow fields, these methods might result in inaccurate estimates of the coarse-scale permeability. Some recent operator-based multiscale methods allow multiscale variations to be linked simultaneously, and therefore remove the dependency on predefined BCs. This paper reports a preliminary investigation into a finite element technique based on one of such methods, the Heterogeneous Multiscale Method (HMM).

Introduction

Reservoirs are often composed of an assortment of rocks giving rise to permeability heterogeneities at a variety of length-scales usually in a hierarchical fashion. It has been appreciated widely that heterogeneities at a scale of centimeters or even millimeters could influence fluid flow in a reservoir that is often at scales of kilometers [5]. A fine geological model might be able to model heterogeneities at many scales, but a numerical model derived from it directly is likely to contain too many cells to make simulation tractable. Upscaling techniques are often called on to account for subscale effects in coarse-scale simulations.

Without losing the generality of the discussion, this paper is concerned only with upscaling single-phase steady-state fluid flow at two hierarchical scales. The mathematical model of the single-phase steady-state fluid flow is given by

\[ \nabla \cdot v^s = q, \quad v^s = -K^s \nabla p^s \quad \text{on } \Omega \]

subject to proper boundary conditions. \( v^s \) is velocity; \( p^s \) is fluid pressure; \( K^s \) is absolute permeability tensor. \( q \) is source and/or sink term; The superscript \( s \) indicates the scale, taking \( c \) for the coarse scale and \( f \) for the fine scale. Viscosity is set to the unit and the gravity term is omitted for simplicity.

There is a large body of literature on upscaling schemes. They include averaging methods, pressure-solving methods, homogenization methods, renormalization methods, operator-based
methods, multigrid-based methods, and wavelet methods. Durlofsky [6] presents a review of some of these methods.

Pressure-solving methods are reported to be capable of producing reliable permeability estimates in many cases. They use predefined BCs to define cell problems and solve them to estimate $K^c$ for the coarse grid cells. Commonly used BCs include no-flow, periodic and linear pressure gradient BCs. However, if predefined BCs do not reflect the flow fields, the pressure-solving methods might result in inaccurate estimates [4]. This is likely to be the case for extremely heterogeneous media such as faulted/fractured porous media. Both high contrast in permeability between faults/fractures and matrix, and complicated structures could give rise to large local spatial variations in flow fields that predefined local BCs cannot reflect adequately. Recently, alternative pressure-solving methods have been suggested to deal with this problem and their performances for extreme heterogeneous media are to be demonstrated.

Unlike the pressure-solving methods, the operator-based methods do not require the prescription of predefined BCs to define cell problems. Operator-based methods usually do not produce upscaled permeability $K^c$. Instead, for each cell problem they estimate a functional operator associated with $K^c$ to derive coarse-scale flow fields. As a result of this, the fine-scale flow fields need to be recovered from the calculated coarse-scale flow fields. So operator-based methods involve both upscaling and downscaling.

Some operator-based methods require grids at the two scales being nested whereas others do not. Arbogast [1] introduced a multiscale mixed finite element method for subgrid upscaling on nested grids. This method has been demonstrated to be able to capture fine-scale effects on the fluid flow in heterogeneous systems even containing fractures [3], and was applied to upscale a quarter of the 10th SPE competitive model for upscaling [2]. E and Engquist [7] introduced a heterogeneous multiscale method (HMM), which does not require nested-grids. Instead, it requires fine-scale sub-grids only to be defined at chosen sampling points for estimating the operator. HMM is a general numerical multiscale framework rather than a specific method. Techniques under this framework have been shown to have potential to simulate a wide range of physical problems. With respect to the problem defined in (1), specific operator-based methods have been developed utilizing the standard finite element technique [8], called FE HMM, as well as the finite difference technique. In the context of homogenization, E et al [9] established error estimates between the homogenized solution and the FE HMM solution, and proved they converge for periodic and random media. However, HMM has not yet been exploited for reservoir simulation.

One advantage of the non-nested-grid methods over the nested-grid methods is that they are more likely than the others to work with unstructured meshes. Unstructured meshes are required for modeling geologically and geometrically complicated systems such as fault/fracture systems. Because of this, it is authors’ interest to examine FE HMM for both upscaling and downscaling.

The remainder of this paper is organized as follows. The next section describes the FE HMM to be investigated. Section 3 describes some numerical experiments and discusses their results. Section 4 concludes the paper.

**FE HMM**

This section describes FE HMM informally. For formal discussion of this technique, the reader is referred to [8].

Let $T^h_\mu$ be the triangulation of a domain $\Omega'$ and $X^h_\mu$ be the finite element space spanned from test functions $\{N_i : i = 1, \ldots, n_\mu\}$. Following the standard finite element procedure, the global
matrix of the system equations is \( M^i = \{ m^i_{lj} \} \) where \( m^i_{lj} = \sum_k \int_{T^i(k) \in T^i_h} \nabla N^i_{jk} \nabla N^j_{kj} \, d\Omega \). The integrals can be approximated by numerical integration:

\[
\int_{T^i(k) \in T^i_h} \nabla N^i_{jk} \nabla N^j_{kj} \, d\Omega \approx \sum_{l} w^i_l (\nabla N^i_{jk} \nabla N^j_{kj})(x^i_l) \text{ where } \{(w^i_l, x^i_l), l = 1, \ldots, n^i_i\} \text{ is a set of weights and integration points defining a quadrature scheme of a given order.}
\]

Since \( K^c \) is unknown and so is the integrand, FE HMM chooses to reconstruct bi-linear operators as follows:

\[
\nabla \cdot (N^i_{jk} K^c \nabla N^j_{kj})(x^i_l) \equiv \frac{1}{|\Omega(x^i_l)|} \sum_{l} \int_{T^i(k) \in T^i_h} \nabla p(N^i_{jk}) \nabla p(N^j_{kj}) \, d\Omega \text{ where } \Omega(x^i_l) \text{ is a sub-domain around integration point } x^i_l, |x| \text{ stands for the volume of the sub-domain } x. \]

Because \( \{ N^i_{jk}, i=1, \ldots, n_c, \} \) are known, cell problems can be solved independently to approximate the unknown integrands. Therefore global equations can be constructed to calculate coarse-grid pressures. In this way, FE HMM allows sub-grid information to be incorporated into coarse-scale calculations with no need to predefine local BCs.

After coarse-grid pressures are calculated, cell problems can be defined with pressure BCs interpolated from coarse-grid pressures to recover fine-grid pressures at any locations.

**Numerical experiments**

The purpose of numerical experiments is to examine whether the FE HMM technique described above is capable of capturing pressure variations at the coarse scale and recovering flow fields at the fine scale without using any predefined local BCs. Figure 1 shows a two-scale model on a unit-square. This model represents, rather unrealistically, 45 degree cross-bedding structures inclining towards the right and the left. The model contains 96x48 triangles. The red strips have a high permeability of 1000 mD whereas the blue strips have a low permeability of 1 mD. In terms of area, the ratio between the red and blue strips is 1:6. An 8x4 yellow triangular grid was superimposed on the model to define the coarse grid. Each yellow square contains 3x3 unit periodic sub-models in one of two types shown on the right side in Figure 1. Upscaled permeability tensors for the sub-models can be determined analytically as \( \{ k_{ij} \mid k_{11}=126.041, k_{12}=k_{21}=+/-124.709 \} \) where + and - correspond the right and left inclining patterns respectively.

Two cases, called BC1 and BC2 corresponding to the two boundary conditions (BCs), respectively, were examined. BC1 was prescribed with

![Figure 1. A two-scale model](image-url)
no-flow BCs at the top and bottom, and a unit pressure and a zero pressure at the left and right sides respectively highlighted in pink. BC2 was prescribed with no-flow BCs except for two segments highlighted in black. Pressure and velocity results are given in Figure 2 where BC1 results are shown on the left and BC2 results on the right. Because the pressures ranged from 0 to 1, no scale legend is shown. Note that velocity vectors (arrows) are associated with elements but are interpolated onto nodes for display purposes. Figure 2 (a) show the fine-grid pressure and velocity fields.

To examine how well the technique can derive coarse-grid nodal pressure, (1) was solved on the coarse grid in two ways. Firstly, we applied the analytical permeability tensors to coarse-grid elements to define a coarse-grid model. We are aware that the use of the tensors here may not be strictly justified because the structures are spatially limited violating the scale-separation requirement. The calculated pressure fields, to be discussed below, suggested that this model might indeed be appropriate for comparing results. Secondly, we solved the coarse-grid problem using the FE HMM technique described in section 2 to calculate coarse-grid nodal pressures. We choose a single integration point for each coarse-grid triangle cell (i.e. the centroid of the triangle) and all fine-scale triangles falling inside the large one to define a cell problem. However, we only use results for a sub-window containing 32 triangles in each cell-problem domain to estimate the operator. We found it can result in inappropriate estimates if cell problems are defined on a region that is too small.

Figure 2 (b1) shows the pressure field at the coarse grid derived in BC1 using the analytical permeability tensors. Clearly, it very well resembles the fine-scale pressure field. The two approaches produced similar pressure fields. Instead of showing all of them, their differences are shown in (b2). The red line shows the differences between pressures calculated using analytical permeability tensors in BC1 and BC2, and indicates the degree of differences due to BCs. The small differences suggest that in BC2 the model using the analytical permeability tensors may not be the most appropriate one since the pressure fields of the two cases at the fine scale are quite different. The pink and blue lines show the differences of the pressures calculated using analytical permeability tensors and FE HMM in BC1 and BC2 respectively. The small differences suggest that FE HMM can derive comparably accurate coarse-grid nodal pressures with reference to the model with analytical permeability tensors. We have also used 3 integration points for each coarse cell to estimate the operator. The corresponding results are very similar to those shown above.

After calculating coarse-grid pressures, we examined how well the technique can perform to recover the flow fields at the fine scale. This time, the same cell problems were solved with pressure BCs derived from the coarse-grid nodal pressures. Two approaches were employed. The first one uses coarse-grid nodal pressure values and test functions to interpolate coarse-grid nodal pressures to boundary nodes for each cell problem, solving it to recover fine-grid nodal pressures and velocities. This approach is referred to as the ‘direct’ approach.

Figure 2 (c) shows the pressure and velocity fields calculated using the ‘direct’ approach. In BC1 the pressure field is similar to the fine-scale pressure field, whereas in BC2 the pressure field has a quite different pattern to the fine-scale pressure field. The respective pressure errors compared with fine-scale pressures are shown in Figure 2 (d). In BC1 the errors are small. In BC2 the errors are larger and the recovered pressure over-estimated the fine-scale pressure at the upper left-hand corner and under-estimated it at the lower right-hand corner.

For both cases, the velocity fields look to be dominated by those cells with large magnitudes. They appear along the straight edges of the coarse-grid triangles where pressures were interpolated onto boundary nodes of the cell problems. By examining error and velocity plots, it becomes clear that those large velocity values are likely to be the result of the combination of
pressure errors due to interpolation and high permeability cells along the straight edges. This did not happen along the diagonal edges because they pass through low permeability strips. After filtering out extreme velocity values as shown in Figure 2 (d) they exhibit similar patterns as the fine-scale velocity fields.

It has been suggested in [1] that the use of higher-order finite elements at the coarse scale may help reduce interpolation errors. We did not investigate this approach here. Instead we explore a different approach. The basic idea of this second approach is to design the coarse grid so that they contain smaller cells overlapping with those boundary nodes in cell problems to allow a smooth pressure interpolation. In this work, we did not redesign such a coarse grid but use the fine-scale grid as the coarse grid. For each coarse triangular cell, we applied the estimated operator to every small triangle in it. This approach is referred to as the ‘fineup’ approach.

Figure 2 (e) and (f) show the pressure, velocity and pressure errors for both cases obtained using the ‘fineup’ approach. Note that the velocity fields are shown as they were without being filtered. Unlike the pressure field in BC1, the pressure field in BC2 exhibits a different pattern to the pressure field derived with ‘direct’ approach but similar to the fine-scale pressure field. The pressure errors seem to be more localized with the ‘fineup’ approach than the ‘direct’ approach. This indicates that for the same coarse-grid pressures the different approaches to determining boundary nodal pressures for cell problems could have a big impact on recovered fine-scale flow fields.

Conclusions

We describe a finite element technique based on the heterogonous multiscale method for upscaling. We investigate the technique in calculating coarse-grid pressure fields and recovering pressure and velocity fields at the fine scale. Two cases were used in this work and the results suggested the technique could derive accurate coarse-scale pressures to some extent. With respect to recovering fine-scale flow fields from the coarse-scale flow fields, there is an issue of how to determine boundary nodal pressures for each cell problem. Interpolation errors can influence the recovered flow fields significantly. Further work is required to investigate FE HMM in more detail.

References


### (a) Fine-scale pressure and velocity fields

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<th>BC1</th>
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- (b1) Pressure field for BC1 using analytical permeability tensors
- (b2) Coarse-grid pressure differences
- (c) Recovered fine-scale pressure and velocity fields by the 'direct' approach
(d) Errors in the recovered fine-scale pressures by the ‘direct’ approach and the filtered velocity field

(e) Recovered fine-scale pressure and velocity fields by the ‘fineup’ approach

(f) Errors in the recovered fine-scale pressures by the ‘fineup’ approach.

Figure 2. Fine-scale, coarse-scale and recovered fine-scale pressure and velocity fields.