Abstract

In this work we present the mathematical grounds and the numerical developments necessary for introduction of the capillary forces and the wettability into a streamline-based simulator. The corresponding simulation tool (the CapSL simulator) has been developed on the basis of the 3DSL0.25 streamline simulator by R.Batycky, SUPRI-C group, Department of Petroleum Engineering, Stanford. Our simulator is capable of predicting the two phase displacement, with full account for capillary effects varying between different zones of a heterogeneous reservoir.

Introduction

In this paper we briefly describe the mathematical grounds for the streamline simulator with capillary effects (CapSL) [1]. Detailed overviews of the streamline simulation method may be found in King and Datta-Gupta [2], Batycky [3] and Thiele [4]. The theory of the capillary effects in porous media of petroleum reservoirs is given, for example, in Shapiro and Stenby [5], Bedrikovetsky [6], and Dullien [7].

The streamline simulation method, as any other reservoir simulation technique, is, essentially, solution of the mass and the momentum conservation laws in porous media. The momentum conservation law is the Darcy law [7, 8]. Its combination with the incompressibility condition results in the so-called pressure equation. For the flow of two immiscible incompressible fluids the divergence of the total velocity far from the wells must be equal to zero. On the basis of the Darcy momentum law and accounting for the gravity forces, the pressure equation is formulated as:

\[ \nabla \left[ k \lambda_w \left( \nabla P_w + \rho_w g D \right) + k \lambda_o \left( \nabla P_o + \rho_o g D \right) \right] = 0 \]  

(1)

Here k is the permeability; \( \lambda \) is the phase mobility introduced as a ratio of the phase relative permeability to the phase viscosity; P is pressure; \( \rho \) is the phase density; g is the gravity acceleration constant, and D is depth. Subscripts w and o represent the water and the oil phases.

In the immiscible case, the mass conservation law is formulated for each phase presented in the reservoir. In particular, for a two phase immiscible incompressible flow, it is possible to formulate the mass conservation law in terms of the saturation of one of the phases, typically water:

\[ \phi \frac{\partial S_w}{\partial t} + \nabla u_w = q_w \] .  

(2)
The outline of the streamline simulator is as follows: 1) a reservoir is divided into a number of grid blocks in \(x, y\) and \(z\) directions. Each grid block is assigned the porosity, the permeability and the initial water saturation values; 2) the pressure equation is solved implicitly for the pressure values in all the grid blocks. The phase mobilities are treated explicitly during the pressure solution step; 3) the water velocity is calculated in all the grid blocks, using the Darcy velocity equation; 4) the streamlines are traced using the Pollock's semi-analytical method \([9]\); 5) the saturation equation is solved along the streamlines. The saturation values are mapped back to the finite difference grid; 6) the solution procedure returns to step 2 for the next time step.

**Capillary pressure**

The capillary pressure is introduced as a pressure difference between the oil and the water phases, regardless of the rock wettability. The standard representation of the capillary pressure in the petroleum engineering is \([7, 10, 11]\):

\[
P_c = P_o - P_w = \frac{\sigma \cos \theta}{\sqrt{k/\phi}} J(s).
\]  

(3)

Here \(\sigma\) is the interface tension, \(\phi\) is porosity, \(\theta\) is the wettability angle, \(J\) is the dimensionless Leveret function and \(s\) is the water saturation.

**The pressure equation with capillary effects**

By substituting the oil pressure with the water and the capillary pressures from equation (3) and regrouping the variables, the pressure equation (1) is transformed to the form of:

\[
\nabla \left[ k \left( \lambda_o \nabla P_w - \lambda_s \nabla P_c - \lambda_o \nabla D \right) \right] = 0.
\]  

(4)

Here \(\lambda_o = \lambda_s + \lambda_w\) is the total mobility, and \(\lambda_o = (\lambda_w \rho_w + \lambda_o \rho_o) g\) is the gravity mobility.

The pressure equation (4) may readily be solved on a finite-difference grid. This solution method is referenced as a straightforward (SFD) solution method. However, further modifications may be applied to equation (4) in order to improve stability of the solution scheme and handling of sharp variations of the reservoir properties. The capillary pressure may be represented as a product of the saturation-dependent part \(J(s)\) and the porous media dependent term \(\Phi(x, y, z)\):

\[
P_c = \frac{\sigma \cos \theta}{\sqrt{k/\phi}} J(s) = \Phi(x, y, z) J(s).
\]  

(5)

The capillary-viscous potential (CVP) is introduced as \([12]\):

\[
\gamma = P_w + \Phi(x, y, z) \left( J(s) - \gamma_o(s) \right),
\]  

(6)

where

\[
\gamma_o(s) = \int_{s_o}^{s} f dJ(s),
\]  

(7)

and \(f = \lambda_w / (\lambda_w + \lambda_o)\) is the water fractional flow function.

The gradient of the CVP has the form of
\[ \nabla Y = \nabla P_w + (J(s) - Y_0(s))\nabla \Phi(x,y,z) + \Phi(x,y,z)(1-f)\nabla J(s). \] (8)

Substituting the water pressure into equation (4) by the CVP from equation (8) and using equation (5) for the capillary pressure, one obtains:

\[ \nabla \cdot \left( k \lambda_w \nabla Y + k \lambda\Omega(s) \nabla \Phi(x,y,z) \right) + \nabla \left( k \lambda_w \nabla D \right) = 0, \] (9)

where \( \Omega(s) = Y_0(s) - J(s) \).

It was shown in [12] that the CVP method allows for larger pressure time steps and results in a smaller mass balance error, compared to the SFD method.

**Modification of the saturation equation**

The saturation equation (2) is reformulated in terms of the total velocity and the capillary pressure. Full derivation of the saturation equation with capillary and gravity effects is given in [1]. The water velocity may be obtained from the overall Darcy equation and the capillary pressure equation (3):

\[ u_w = \frac{\lambda_n}{\lambda_i} u_i + k \lambda_w (1-f) \nabla P_w + k \left( \frac{\lambda_n}{\lambda_i} - \lambda_w \rho_w g \right) \nabla D. \] (10)

Introducing the water velocity into the saturation equation (2) and applying the total incompressibility assumption, one may transform the saturation equation to the form of:

\[ \phi \frac{\partial s_w}{\partial t} + u \nabla f + \nabla \left( k \frac{\lambda_n}{\lambda_i} \nabla P_w \right) + \nabla \left( k \frac{\lambda_n}{\lambda_i} \left( \rho_n - \rho_w \right) g \nabla D \right) = 0. \] (11)

The final system of equation for saturation \( s \) and pressure \( P_w \) (or potential \( Y \)) consists of equations (4) and (11) in the SFD formulation and equations (9) and (11) in the CVP formulation.

**Numerical solution**

The pressure or the CVP equations are solved on the Cartesian 7-point finite-difference grid. Introduction of the capillary pressure leads to a single additional term on the right-hand side of the pressure equation, both for the SFD and the CVP methods, compared to the pressure equation without capillary effects. During the single pressure solution step, the water pressure or the CVP are treated implicitly; however, the phase mobilities and the capillary pressure are treated explicitly as functions of the saturation on the previous step. Full description of the numerical solution of the pressure equation may be found in Aziz and Settari [13], Batycky [3], and Berenblyum et.al. [1].

The saturation equation is solved using the operator splitting method. The viscous forces are accounted for along the streamlines, as shown by Batycky [3], Thiele et.al. [14], and Blunt et.al. [15]. The saturation equation is transformed on the basis of the concept of the time-of-flight \( \tau \) [16]:

\[ \frac{\partial s_w}{\partial t} + \frac{\partial f}{\partial \tau} = 0. \] (12)
The capillary and the gravity forces are accounted for on the finite difference grid. The step on the finite difference grid is referenced to as a corrector step. The corrector step is capable of relocating the fluids inside the reservoir, but not changing their total amounts. The equation for the corrector step has the form of:

$$\frac{\partial s_w}{\partial t} + \frac{1}{\phi} \nabla \left( k \frac{\lambda_w}{\lambda_i} \nabla P_c \right) + \frac{1}{\phi} \nabla \left( k \frac{\lambda_w}{\lambda_i} \left( \rho_n - \rho_w \right) g \nabla D \right) = 0.$$  (13)

Full description of the saturation equation solution procedure is presented in Berenblyum et.al. [1].

The streamline simulators are less restricted by the stability limitations, compared to the finite difference methods [3, 15, 17]. However, introduction of capillary effects and the operator splitting solution may require more frequent time step updates, compared to the conventional streamline simulation techniques. Several saturation steps including consequent solution of equations (12) and (13) may be performed during a single time step. The time step may be rejected if the material balance error exceeds the limits specified by user. For the simulation grids composed of large homogeneous zones, like synthetic test cases or lab scale simulations, it may be recommended to use fully automatic time step selection routine. The currently implemented automatic routine is based on the method presented by Ichiro et. al. [18].

**Capillary effects in porous media**

Capillary effects in porous media may be separated into the three groups: 1) Smearing of the saturation front by the capillary effects; 2) Capillary effects on the borders of the media with different permeabilities (and generally, in the porous media possessing sharp heterogeneity); 3) Capillary effects on the borders of the media with different wettabilities.

Smearing of the saturation front is due to the gradient of the Leverett function and takes place in all the types of the porous media. Smearing the front in one-dimensional homogenous displacement is well covered in the literature (f.ex. [19]) and is not described here.

The wettability effects are illustrated by the two examples. The first example is a simple 1D simulation comprised of 1500 grid blocks of 1 m³ volume each. The first and the last five hundred grid blocks are strongly water wet with $\cos\theta=1$, while the middle grid blocks are strongly oil wet with $\cos\theta=-1$. For simplicity of comparison, we assume that behavior of the water phase in the oil wet rock is similar to behavior of the oil phase in the water wet rock. Hence, the relative permeability curves and the Leverett function are exactly the same in the oil wet and the water wet media, if considered as functions of the saturation of the wetting phase. The relative permeability curves and the Leveret function are presented in figure 1.

![Figure 1. The relative permeabilities and the Leveret functions](image-url)
For this example, both the Eclipse and the CapSL show good agreement of the saturation profiles, see figure 2:

![Comparison of the Eclipse and the CapSL saturation profiles](image)

**Figure 2. Comparison of the Eclipse and the CapSL saturation profiles**

Main difference in the predicted saturation profiles is around the border of wettability. The saturation profiles predicted by Eclipse show a single saturation peak in the upwind zone. The saturation profiles predicted by CapSL show two saturation peaks: the maximum in the water wet zone, and the minimum in the oil wet zone. The picture with the two peaks seems to be more physically correct, since in this case the capillary pressure between the oil and the water zones is closer to being continuous. Disappearance of one peak in the Eclipse simulation may probably be explained by the numerical dispersion.

Geometry of the reservoir, the relative permeabilities and the Leverett function are shown in figure 3.

![Geometry of the reservoir and the constituting dependences of the displacement](image)

**Figure 3. Geometry of the reservoir and the constituting dependences of the displacement**

The grid is comprised of the two types of the rock. Both of them have the porosity of 0.2 and the permeability of 100 mD. The gray-colored part of the reservoir is slightly water-wet, with \( \cos\theta = 0.1 \), while the white-colored part is strongly water wet with \( \cos\theta = 1 \). The production pressure is fixed at 20MPa, the volumetric flow rate is 0.1m\(^3\)/day. The system pore volume is 1500m\(^3\).

The CapSL and the Eclipse show a perfect match of the displacement pictures in the water wet zone. The simulators predict the same breakthrough time and the oil production curves show good match as well. Two possible reasons for small differences in the saturation profiles may be: 1) Numerical dispersion in Eclipse; 2) The operator splitting solution error of the saturation equation in CapSL.
Reservoir-scale simulations

The CapSL and the Eclipse 100 (IMPES mode) [20] are applied to simulate the real-scale reservoir scale displacement problem, based on the SPE10 comparison project [21]. In this simulation the phase relative permeabilities are the second order dependencies of the water saturation:

\[
k_{rw} = 0.8 \left( \frac{s - s_{wi}}{1 - s_{wi} - s_{or}} \right)^2, \tag{14}
\]

\[
k_{ro} = 0.9 \left( 1 - \frac{s - s_{wi}}{1 - s_{wi} - s_{or}} \right)^2. \tag{15}
\]

Here \(s_{wi}\) is the irreducible water saturation equal to 0.1, and \(s_{or}\) is the residual oil saturation with the value of 0.25.

The Leverett function is modeled by the van Genuchten dependence [22]:

\[
J(s) = 0.6 \left[ \left( \frac{s - s_{min}}{1 - s_{max} - s_{min}} \right)^{-1/0.9} - 1 \right]^{1/10} - 0.017. \tag{16}
\]

Here \(s_{min}\) takes the value of 0.0999 and \(s_{max}\) of 0.1999999. It should be pointed out that the end-point values of the Leveret function are very sensitive to the values of \(s_{min}\) and \(s_{max}\) due to sharp variation of the van Genuchten dependence close to the end points. The porosity field and the permeability field are presented in figure 5. The injection wells are shown by red dashed lines, the production wells by solid blue lines.

The production pressure is fixed at 30 MPa, the injection rate at 1085 m³/day. The water viscosity is fixed at 0.25 cP, the oil viscosity at 0.4 cP. The interfacial tension is 30 mN/m. The reservoir is simulated using the Cartesian grid with 60*110*8 equally sized grid blocks (6*6*6.5m each). 2000 days or around 2PV of water injected are simulated.

The saturation profiles at 0.1 and 0.5PVI, as well as the oil production curves obtained using the CapSL and Eclipse 100, are presented in figure 6.

Both simulators are in a good agreement. The streamline simulator captures fine features of the displacement front and is less affected by the numerical diffusion. In the course of the
simulation, the Eclipse performed 6548 time steps and the simulation took 3 hours 49 minutes. The CapSL simulation finished within 8 minutes 20 seconds, using 20 pressure time steps.

Conclusions

- The streamline simulator fully accounting for capillary effects and wettability has been developed.
- Applicability of the streamline simulator to complex reservoir simulation problems is demonstrated on an example of the full-scale reservoir simulation.
- The displacement fronts and the oil production curves predicted by the Eclipse and the CapSL are in excellent agreement, especially, with regard to the global characteristics of displacement. The streamline simulator has a speed-up factor of around 27.
- The fine pictures of displacement may differ. The CapSL produces a more detailed picture less affected by the numerical dispersion. This is important, in particular, when the wettability effects on the boundaries of the zones with mixed wettability are considered.
Acknowledgements

Authors would like to acknowledge the Danish Energy Authority and the member companies of the IVC-SEP consortium for financial support. Hallur Stakksund is acknowledged for the contribution to the wettability modeling.

References


