Robustness Evaluation And Tolerance Prediction For A Stamping Process With Springback Calculation By The FEM

M. Strano

Università di Cassino, Dip. Ingegneria Industriale, Italy
m.strano@unicas.it

Abstract. The FEM simulation of a sheet forming process can be seen as a method for calculating a deterministic vector set of output responses, starting from a vector set of input variables. In the real world, though, most process parameters should be considered as random variables (material properties, lubrication conditions, etc.), with a given statistical distribution. As a consequence, any FEM simulation which is run with nominal values of the input variables does not provide the “true” solution, but only a nominal response, related to a distribution around an expected value. The robustness of a given solution can be evaluated by several methods: sensitivity analysis, Montecarlo method, Response Surface Method (RSM), etc.

The sensitivity analysis is usually performed by running one or two simulations for each random variable, setting it at an extreme value. The method does provide a rough measure of the process variability, but it does not usually guarantee a quantitative estimation of the process’s confidence interval. Statistical techniques, such as Montecarlo or RSM are very effective in predicting the true process variation. However, both methods are computationally very expensive, if the problem is complex (e.g., when forming, trimming and springback operations must be simulated for a large part).

In the present paper the solution’s robustness of a Numisheet ’05 benchmark case is faced and solved with an approximate and computationally inexpensive method. The purpose of the study is to determine the tolerance interval of the part surface, induced by the random variation of 11 process parameters. The variability is modeled with a joint multinormal distribution. The analyzed response is the geometrical error of the process with respect to a reference “deterministic” geometry. The proposed method provides an upper bound estimation of the tolerance interval, by exploring the boundaries of the ellipsoidal probability density function of the multinormal input. Finally, the effectiveness of the method is tested by running a conventional Montecarlo analysis of the problem.

INTRODUCTION

Since when the available Finite Element Methods for running sheet forming simulations have become reliable and powerful, the use of statistical techniques has been increasingly registered in the scientific literature. Recent and convincing examples can be found in ref. 2, 3 and 4, where the Response Surface Method and Montecarlo simulations are used in order to evaluate the process robustness or to perform stochastic optimization. Following this approach, the FEM simulation of a sheet forming process can be seen as a method for calculating a deterministic vector set of output responses \( y \), starting from a vector set of input variables \( x \) of size \( N \). In the real world, though, most process parameters should be considered as random variables (material properties, lubrication conditions, etc.), with a given statistical distribution. As a consequence, any FEM simulation which is run with nominal values of the input variables \( x = x_0 \) does not provide the “true” solution, but only a nominal response \( y_0 \), related to a distribution around an expected value \( \mu_y \). The robustness of a given solution \( y_0 \) can be evaluated by several methods: sensitivity analysis, Montecarlo method, Response Surface Method (RSM), etc.

The sensitivity analysis is usually performed by running one or two simulations for each random variable \( x_i \) from the vector \( x \), setting it at an extreme value \( x_i^{\text{min}} \) or \( x_i^{\text{max}} \). The method does provide a rough measure of the process variability, but it does not
guarantee a quantitative estimation of the response’s confidence interval, unless \( N_x = 1 \).

Statistical techniques, such as Montecarlo or RSM are very effective in predicting the true process variation. However, both methods are computationally very expensive, if the problem is complex (e.g., when forming, trimming and springback operations must be simulated for a large part) and \( N_x \) is large (greater than 5).

In the present paper the issue of robustness of the Numisheet ’05 benchmark #2 (an underbody cross member, see Fig. 1 and the instructions in ref. 1) solution is faced and solved with an approximate and computationally inexpensive method. The selected material has been the DP600 (US Steel) dual phase steel.

The purpose of the study is to determine the tolerance interval of the part surface, induced by the random variation of the following parameters \((N_x = 11)\):

The variability of \( x \) is modeled with a joint multinormal distribution \( N(x_0, \Sigma) \), where \( \Sigma \) is the square variance/covariance matrix. The analyzed response \( y \) is the width of the \( 6\sigma \) tolerance interval and is modeled with a normal distribution \( N(\mu, \sigma^2) \). The proposed method provides an upper bound estimation of the \( 6\sigma \) interval, by exploring the boundaries of the ellipsoidal probability density function of the multinormal variable \( x \). The effectiveness of the method is tested by running a conventional Montecarlo analysis of the problem.

\[ \frac{\sigma}{\mu} = K \left( \varepsilon_0 + \varepsilon \right) \]

\[\text{FIGURE 1.} \text{ Forming tool setup.}\]

\[\text{FIGURE 2.} \text{ binder force vs. binder travel.}\]

\[\text{FIGURE 3.} \text{ initial blank mesh with characteristic length.}\]

**SETUP OF THE SIMULATION MODEL**

All the simulations have been run with the commercial software package Pam-Stamp 2G. The main parameters have been set according to the instructions given together with the benchmark (ref. 1), with the exception of the binder force, which has been given as a curve (reported in Fig. 2).
FIGURE 4. refinement levels ate the end of the stamping stage and at the beginning of the springback stage.

Modeling The Variability Of The Input Process Parameters

As stated in the introduction, the variability of the input vector $x$ has been modeled with a joint multinormal distribution $N(x_0, \Sigma)$, where $\Sigma$ is the square variance/covariance matrix. Obviously, the matrix $\Sigma$ is unknown and must be estimated, starting from the available data. The considered random input variables of the $x$ vector are:

- the parameters $x_1=K$, $x_2=n$, $x_3=\varepsilon_0$ of the Krupkowsky hardening law $\overline{\sigma} = K(\overline{\varepsilon} + \varepsilon)^n$;
- the sheet thickness $x_4=t$
- the anisotropy parameters $x_5=r_0$, $x_6=r_{45}$, $x_7=r_{90}$;
- the young modulus $x_8=E$;
- the friction coefficients between the blank and the binder $x_9=f_b$, the upper die $x_{10}=f_d$ and the lower punch $x_{11}=f_p$.

Clearly, some of these 11 parameters can be considered independent from each other. As an example, there is no reason for the friction coefficients to be correlated to the hardening parameters. On the contrary, all the material parameters (from $x_1$ to $x_3$) might be the correlated and the covariance terms might have a non-zero value. Therefore, a correlation analysis is needed. Before that, an ANOVA (Analysis of Variance, ref. 3) has been conducted on the available material data, using the tensile tests’ testing direction as a factor with three levels (0, 45 and 90°) and $x$ as the vector of response variables. The ANOVA proved that $x_5$, $x_6$, $x_7$ and $x_8$ are dependent on the testing direction. Then, the potential correlation among the input variables has been studied for $x_i$ with $i=1..8$. The 8 listed material variables. No experimental data is available for the three friction coefficients ($x_i$ with $i=9..11$). Therefore, they have been assumed as normally, equally and independently distributed with mean 0.12 and standard deviation 0.036.

Thanks to these preliminary statistical observations, the parameters of the multinormal distribution $N(x_0, \Sigma)$ have been estimated. The results is summarized by the (estimated) variance-covariance matrix given in Table 1, showing that only $x_2$, $x_3$ and $x_4$ are correlated. In Table 2, the (estimated) mean $x_0$ vector is reported.

Modeling The Variability Of The Simulation Geometrical Response

Once Tables 1 and 2 have been estimated, thanks to the available material data, several simulations have been run according the method described in the following section. This method is aimed at estimating, with a minimum number of simulations, the amount of geometrical deviation induced on the process by the statistical variation of the mentioned random variables. Therefore, a geometrical indicator of the simulated
TABLE 1. Estimated variance-covariance matrix \( \Sigma \)

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( x_5 )</th>
<th>( x_6 )</th>
<th>( x_7 )</th>
<th>( x_8 )</th>
<th>( x_9 )</th>
<th>( x_{10} )</th>
<th>( x_{11} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.832E+02</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.220E-05</td>
<td>6.30E-07</td>
<td>-4.71E-06</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6.30E-07</td>
<td>2.665E-08</td>
<td>-1.80E-07</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-4.71E-06</td>
<td>1.90E-06</td>
<td>1.233E-05</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.233E-05</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.8970</td>
<td>203125</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE 2. Estimated mean vector \( \bar{x}_0 \)

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( x_5 )</th>
<th>( x_6 )</th>
<th>( x_7 )</th>
<th>( x_8 )</th>
<th>( x_9 )</th>
<th>( x_{10} )</th>
<th>( x_{11} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>units</td>
<td>MPa</td>
<td>-</td>
<td>-</td>
<td>mm</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>MPa</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \bar{x}_0 )</td>
<td>1032</td>
<td>0.1595</td>
<td>0.000344</td>
<td>1.622</td>
<td>0.7377</td>
<td>0.9230</td>
<td>0.8970</td>
<td>203125</td>
<td>0.12</td>
<td>0.12</td>
</tr>
</tbody>
</table>

FIGURE 5. Deviations from the reference geometry.

tolerance interval of the stamping and trimming process is needed. The shape resulting by the simulation run with mean values of \( \bar{x} = \bar{x}_0 \) can be assumed as the reference or target geometry. Any \( j \)-th other simulation, run with perturbed process parameters \( \bar{x}_j \), according to the variance-covariance matrix, will generate a different shape. The geometrical error \( \delta_j \) measured on this \( j \)-th shape might be measured as its maximum distance \( d_j^+ \) from the reference geometry on the positive \( Z \) direction, plus the maximum distance \( d_j^- \) on the negative \( Z \) direction. This can be better graphically explained on a two-dimensional section, as in Fig. 5. However, the actual \( \delta_j \) must be calculated on the whole 3D final part.

An important issue must be taken into account, if a proper value of \( \delta_j \) must be evaluated: every shape calculated after spring back is referred in space thanks to two fixed points (A and B in Fig. 6, as indicated in ref. 2), located on the symmetry plane. This particular constraint will inevitably produce increasing values of \( \delta_j \), as the measuring position gets farther from the symmetry plane. In other words, the maximum errors will be concentrated within the zone circled in Fig. 6 and therefore, the error \( \delta_j \) will be largely underestimated. Two of the several viable ways of solving this problem are described as follows:

1) rotating each calculated shape \( j \) around the \( y \) axis (the rotation around the \( z \) and \( x \) axes is prohibited by the symmetry plane) and translating it in the \( x \) and \( z \) directions until the error \( \delta_j \) is minimized;

2) fixing the position of one reference point (e.g. the vertex point C in Fig. 6) and rotating each calculated shape \( j \) around the \( y \) axis until it touches a positioning plane.


The first solution is clearly the most accurate from a numerical point of view. However, it is computationally expensive, since the optimal position of the perturbed shape \( j \) must be determined by a proper searching algorithm.

The second solution still overestimates the actual deviation of \( j \) from the reference shape, but it is very
similar to the real method of measuring the geometrical errors in stamping shops, using measurement tooling and it is computationally inexpensive. For these reasons, this second solution has been adopted in the remainder of the paper.

EVALUATING THE TOLERANCE INTERVAL

Sensitivity analysis

The robustness of the simulated process can be estimated by several methods. The simplest and most commonly used is a conventional sensitivity analysis, that does provide an indication of the potential output variability, but does not provide any statistical indication of this variability. Since \( N_x = 11 \), the minimum number of needed simulations, in addition to the reference (deterministic) case is 11, by perturbing only one of the \( x_i \) at a time and setting it at:

\[
x_i = x_0 \pm 3 \cdot \sigma_i
\]

choosing at random the plus or minus sign in equation 1. This analysis has been run and indicated that the most significant input factor is \( x_11 = f_p \), i.e. the coefficient of friction between the blank and the lower punch. And the less significant factors are the three anisotropy parameters \( x_5, x_6, x_7 \). The average sampled geometrical errors due to a \( 3\sigma \) perturbation of a single factor \( i \) are:

\[
\begin{align*}
\delta_d^- &= 0.72 \text{ mm}, \\
\delta_d^+ &= 0.82, \\
\delta_d &= 1.54 \text{ mm}.
\end{align*}
\]

Montecarlo analysis

One of the most potentially accurate ways of estimating the real values of the geometrical tolerance interval is to run an extensive Montecarlo simulation, by randomly generating \( N_{mc} \) of \( x \) sampled from the multinormal \( \mathcal{N} (\mathbf{x}_0, \Sigma) \). In Fig. 7, the estimation of \( d, d_+ \) and \( \delta \) are printed, together with their \( 3\sigma \) upper confidence limits vs. the simulation number \( N_{mc} \). Clearly, all the estimates tend to stabilize as \( N_{mc} \) increases. For instance, the upper confidence limits (UCL) values of the errors when \( N_{mc} = 36 \) are \( d_{\text{UCL}} = 2.62 \text{ mm}, \quad d_{+\text{UCL}} = 2.22, \quad \delta_{\text{UCL}} = 4.05 \text{ mm} \), significantly greater than the values calculated by the sensitivity analysis. The value of \( \delta_{\text{UCL}} \) is particularly important, since it can be taken as the estimation of the “6\sigma” tolerance interval of the stamping + trimming + springback process.

Response Surface Methodology

Another possibility for estimating, with a very good accuracy, the response variability is to conduct an RSM. A first order model of \( \delta \) vs. the input vector \( x \) is probably not able to represent the real effect of factors on the response. Therefore, a second order model is needed, and this requires a very large number of runs, if a sufficient model robustness is required. In order to reduce the dimensionality of the problem, the three anisotropy parameters have been merged together, by calculating a new variable (the normal anisotropy \( r = x_{12} \)) as:

\[
r = x_{12} = x_2 + x_7 + 2x_6 \quad \text{(2)}
\]

and a new input vector with 9 variables is now used:

\[
\mathbf{z} = [x_1; x_2; x_3; x_4; x_5; x_6; x_9; x_{10}; x_{11}; x_{12}] \quad \text{(3)}
\]

Then, a full quadratic RSM model for \( \delta \), with interactions, has been built using 67 runs:

\[
\delta = \beta_0 + \sum_{i=1}^{9} \beta_i z_i + \sum_{i=1}^{9} \beta_{ii} z_i^2 + \sum_{i=1}^{9} \sum_{j=i+1}^{9} \beta_{ij} z_i z_j \quad \text{(4)}
\]

The most significant factor, according to the RSM model, is the \( z_9 = x_{11} \), as confirmed by the sensitivity analysis. Also the A sequence of 2500 values of \( \mathbf{z} \) has been randomly generated, knowing that \( \mathbf{z} \sim \mathcal{N} (\mathbf{x}_0, \Sigma) \). For the \( j \)-th value of \( \mathbf{z} \), we can calculate the sum of deviations \( \delta \) according to equation 4. Finally, the

\[1\] 1 The significance of a model term \( z_i \) has been by running t-tests on the regression coefficients \( \beta_i \) and \( \beta_{ii} \) (ref. 3)
mean and UCL values of $\delta$, estimated by the RSM method are $\delta_{\text{UCL}} = 1.32$ and $\delta_{\text{UCL}} = 4.56$ mm.

**An upper bound method**

A new approximate and inexpensive method is proposed for estimating the process geometrical variability, based on the following considerations. The input vector $\hat{x} \sim N(\hat{x}_0, \Sigma)$, which is multinormally distributed can be transformed into a new vector formed by uncorrelated and standardized variables, $\hat{y} \sim N(\hat{y}_0, \hat{y}_\Sigma)$, where $\hat{y}_\Sigma$ is a diagonal matrix, thanks to a so-called Mahalanobis transformation (ref. 7). Geometrically, the probability density function of $x$ can be represented by an 11-D ellipsoid, while the density function of $y$ can be represented by a spheroid. The proposed method consists in randomly selecting a limited number of $y$ values with $y_i = +3$ or $y_i = -3$. In other words, all the casually selected values of $y$ will belong to the outer boundary of the spheroid. The set can be transformed back into a set of $x$ values. One can expect that the geometrical errors resulting from this $x$-values set are very large, since they are sampled from the periphery of the ellipsoid. The calculated average values of $d_-, d_+$ and $\delta$, can be taken as an upper bound estimation of $\delta_{\text{UCL}}$, $d_{\text{UCL}}$ and $d_{\text{UCL}}$. In the present case, 19 function evaluations have been used and the method gives, respectively, the values 2.52, 1.88 and 4.39 mm.

**Comparison of the different methods**

The geometrical process variability estimated by the different methods is shown in the following Fig. 8 and summarized by the points described as follows. The best estimate of the most important parameter, $\delta_{\text{UCL}}$ (it is a measure of the simulated 6$\sigma$ tolerance interval) should be given by the RSM method, because it has been modeled with the highest number of runs (67). 2) The solutions provided by the Montecarlo and RSM methods are quite comparable. 3) The newly proposed method gives a reasonably close upper bound of $\delta_{\text{UCL}}$, at a smaller computational cost than the response surface method.

**CONCLUSIONS**

In the present paper the issue of robustness of the Numisheet ’05 benchmark #2 (stamping + trimming + springback) process is solved with different statistical methods, in order to evaluate the potential geometrical error induced by the input variability. A sensitivity analysis, a Montecarlo analysis, an RSM model and a new proposed upper bound method have been used. The results can be summarized as follows.

1) The sensitivity analysis is computationally not very expensive, but is not able to provide any estimate regarding the 6$\sigma$ tolerance interval $\delta_{\text{UCL}}$.
2) The solutions provided by the Montecarlo and RSM methods are quite comparable.
3) The newly proposed method gives a reasonably close upper bound of $\delta_{\text{UCL}}$, at a smaller computational cost than the response surface method.

**REFERENCES**

1. 2005 Numisheet Benchmark 2 - Springback Prediction of A Cross Member, Detroit, 2004