Page 21, line 9 *Replace the second paragraph of 1.4 by the following*

We denote by $\Lambda^s M$ the vector bundle of exterior s-forms on M. Hence $\Lambda^s_x M = \Lambda^s\left(T^*_x M\right)$, this vector space being as usual a quotient space of $\bigotimes^s\left(T^*_x M\right)$ (sometimes we will consider forms as elements of $\bigotimes^s\left(T^*_x M\right)$ through the $Gl(T^*_x M)$-invariant splitting of the canonical projection). More generally, $\wedge$ will denote any alternating procedure and similarly S or $\odot$ will denote symmetrisation.

Page 21, line -5 *Replace the first line of Formula (1.5a) by*

$$\alpha\left(X_0, X_1, \ldots, X_p\right) = \sum_{i=0}^{p} (-1)^i\, X_i \cdot \alpha\left(X_0, \ldots, \hat{X}_i, \ldots, X_p\right)$$

Page 24, line -7 *Replace the second line of the first formula of 1.12 by*

$$+\sum_{i<j} (-1)^{i+j}\, \alpha\left(\left[\tilde{X}_i, \tilde{X}_j\right], X_0, \ldots, \hat{X}_i, \ldots, \hat{X}_j, \ldots, X_p\right).$$

Page 24, line -6 *Replace line -6 (“Then”) by the following*

Notice that, if $p = 0$ and s is a section of E, then $d^{\nabla} s$ is nothing but what we called ∇s in 1.8 Remark a). And then the curvature R of the connection ∇ is given through the formula …

Page 25, line 10 *Replace the proof of Theorem 1.14 by the following*

Sketch of proof : This follows from Formula (1.12) and the fact that

$$\left(d^{\nabla} \circ d^{\nabla}\right) \circ d^{\nabla} = d^{\nabla} \circ \left(d^{\nabla} \circ d^{\nabla}\right),$$

provided we put on the vector bundle $End\,(E)$ the “natural” (see 1.15) connection, still denoted by ∇, induced by the connection ∇ on E, and consider the corresponding d^{∇}.

Page 25, line 12 *Replace paragraph 1.15 by the following*

1.15. Some more definitions. (a) Let E be a vector bundle with a linear connection ∇. Then, on any vector bundle “naturally associated” with E, like E^* or $End\,(E)$, there exists a “natural” connection associated with ∇, which will be also denoted by ∇. We will not give the general definition here and refer to [Ko-No 1] for details. We give only one example here. If s is a section of E and α a section of E^*, then, for any tangent vector X, we have

$$\left(\nabla_X \alpha\right)(s) = X\left(\alpha(s)\right) - \alpha\left(\nabla_X s\right).$$

(b) A section s of a vector bundle E, equipped with a connection ∇ is called parallel if $\nabla s = 0$. Note that, given a point x in M, there always exists a neighborhood U and a finite family $\left(s_i\right)_{i\in I}$ of sections of E over U such that $\left(s_i(y)\right)_{i\in I}$ is a basis of the fiber E_y for each y in U and $\nabla s_i(x) = 0$. (Consider

a local trivialization of E centered at x. Then to find such s_i near x amounts to finding functions with appropriate first jet at x).
(c) If the vector bundle E is equipped with some additional structure (such as a Euclidean fiber metric h, a complex structure J, a symplectic form ω or a Hermitian triple (g, J, ω) , then a linear connection ∇ is called respectively Euclidean (or metric), complex, symplectic or Hermitian, if respectively h, J, ω or (g, J, ω) are parallel (when the vector bundle in which they live respectively is endowed with the "natural" connection induced by ∇). In such a case, the curvature R^{∇} of ∇ satisfies additional properties, namely, for each X, Y in T_xM, the linear map $R^{\nabla}_{X,Y}$ on E_x is respectively skewsymmetric, complex linear, skewsymplectic or skew-Hermitian.
In the particular case where E is the tangent bundle of the base manifold M, further considerations can be developed.

Page 25, line -9 *Replace "(see Remark(1.8.a))" by*
(see Remark (1.8.a) and 1.12)

Page 27, line 10 *At the end of 1.23, add the following*
Notation : We denote by $S_{X,Y,Z}$ the "cyclic sum" with respect to X, Y, Z, i.e., if $A(X, Y, Z)$ is any expression involving X, Y and Z, we write

$$S_{X,Y,Z}A(X,Y,Z) = A(X,Y,Z) + A(Y,Z,X) + A(Z,X,Y)$$

Page 30, line 5 *Replace paragraph 1.38 by the following*
1.38. Let (M, g) be a pseudo-Riemannian manifold. Then at each point x of M, the nondegenerate quadratic form g_x induces a canonical isomorphism $T_xM \to T_x^*M$. This isomorphism is often denoted by $\flat$ ("flat") and its inverse by $\sharp$ ("sharp") since in classical tensor notation, they correspond to lowering (resp. raising) indices, see below 1.42. More generally, g_x induces various isomorphisms between any $T_x^{(p,q)}M$ and $T_x^{(r,s)}M$, as soon as $p+q = r+s$, these isomorphisms depending of which indices are lowered or raised.
If we consider the unique isomorphism $T_x^{(p,q)}M \to T_x^{(q,p)}M$ which lowers the p "covariant" indices and raises the q "contravariant" indices, and compose it with the "evaluation map" (pairing any vector space with its dual, respecting the order of the indices), we get b a nondegenerate quadratic form, still denoted by g_x on any $T_x^{(p,q)}M$. Note that if g_x is positive definite, then it is also positive definite on any $T_x^{(p,q)}M$. Similarly, g_x induces an isomorphism between $\Lambda_x^s M = \Lambda_x^s (T_x^*M)$ and $\Lambda_x^s (T_xM)$, or $S^s (T_x^*M)$ and $S^s (T_xM)$, and (through pairing) g_x induces a nondegenerate quadratique form on $\Lambda_x^s M$ and $S^s (T_xM)$, also denoted by g_x. Notice that if $\left(e^i\right)$ is an orthonormal basis for T_xM, the "dual basis" (e_i^*) for T_x^M, which is defined by $e_i^*\left(e^j\right) = \delta_i^j$ (Kronecker symbol, see 1.42), is also an orthonormal basis, and furthermore $e_{i_1}^* \otimes \cdots \otimes e_{i_p}^* \otimes e^{j_1} \otimes \cdots \otimes e^{j_q}$ is an orthonormal basis for $T_x^{(p,q)}M, e_{i_1}^* \wedge \cdots \wedge e^* i_s$ an orthonormal basis for

$\Lambda_x^s M\,(i_1 < \cdots < i_s)$, and $e_{i_1}^* \odot \cdots \odot e_{i_s}^*, i_1 < \cdots < i_s$, an orthonormal basis for $S^s\left(T_x^* M\right)$.

Page 32, line 14 *Replace the beginning of the first line of 1.45 by*

1.45. Theorem (see [Ru-Wa-Wi] or [Eps]). Local...

Page 32, line 18 *Replace by*

$$\sum_{j=1}^{n} g_{ij}\left(x^1, \ldots, x^n\right) x^j = \sum_{j=1}^{n} g_{ij}\left(0\right) x^j.$$

Page 34, line 5 *Replace lines 5 and 6 by*

a) the *gradient* of f is the vector field *grad* $f = \sharp df$ $\left(\text{or } df^\sharp\right)$, i.e., *grad* f satisfies $g\,(grad\ \text{f,X}) = X\,(f) = df\,(X)$ for any X in TM ; we will sometimes denote *grad* f by Ddf to keep the notations shorter, despite the possible confusion with df.

Page 34, line -8 *Replace the formula by*

$$\delta = \begin{cases} -*_g \circ f \circ *_g, & \text{if } n \text{ is even ;} \\ (-1)^p *_g \circ d \circ *_g & \text{if } n \text{ is odd.} \end{cases}$$

Page 34, line -7 *Replace b) by*

b) δ may be viewed as the restriction of D^* for a suitable embedding of $\Lambda^{p+1} M$ into $\Lambda^1 M \otimes \Lambda^p M$

Page 35, line 4 *Replace paragraphe 1.59 by*

1.59. Instead of forms, we may also consider symmetric tensors. Il we consider the covariant derivative

$$D = S^p M \to \Omega^1 M \otimes S^p M = S^1 M \otimes S^p M,$$

and compose with the "symmetrization" sym

$$sym : S^1 M \otimes S^p M \to S^{p+1} M$$

given by

$$sym\,(\alpha, \omega)\,(X_0, \ldots, X_p) = \frac{1}{p+1} \sum_{i=0}^{p} \alpha\,(X_i)\,\omega\left(X_0, \ldots, \hat{X}_i, \ldots, X_p\right),$$

we obtain a differential operator, *denoted* by δ^*,

$$\delta^* = S^p M \to S^{p+1} M,$$

whose formal adjoint is called the divergence, and denoted by δ,

$$\delta : S^{p+1}M \to S^{p}M.$$

Notice that δ may be viewed as the restriction of D^* for a suitable embedding of $S^{p+1}M$ into $T^{(p+1,0)}M$.

Page 35, line 16 *Change a sign in the formula.The correct sign is*

$$\delta^* \alpha = \frac{1}{2} L_{\alpha^\sharp} g,$$

Page 35, line 23 *Change a sign in the formula. The correct sign is*

$$= \frac{1}{2} \left(L_{\alpha^\sharp} g \right) (X, Y)$$

Page 43, line 16 *Replace formula (1.92) by*

$$(n-1)\, g\,(\dot{c}, \dot{c})\, \ddot{\phi} + r\,(\dot{c}, \dot{c})\, \phi \leq 0$$

Page 52, line 11 *In the first line of 1.133, the equation is*

$$\check{R} = \frac{1}{4} q\,(R, R)\, q$$

Page 53, lines -4 and -1, page 54, lines 3 and 4 *Replace E by E_p*
Page 54, 1.144 *The correct formula is*

$$\Gamma T = -2 c_\rho^2(R) T$$

Page 55, line 11 *Replace line 11 by*

Thus, since σ is parallel,

Page 55, line 18 *Add*

Although σ is not a representation, c_σ can be given the same meaning as c_ρ in 1.139.

Page 65, line 7 *Change a sign in the formula. The correct sign is*

$$\mu_g' h = \frac{1}{2} \left(\mathrm{tr}_g h \right) \mu_g .$$

Page 65, line 10 *Change a sign in the formula. The correct sign is*

$$Vol\,(M)'_g\,h = \int_M \mu'_g h = \frac{1}{2}\int_M (\mathrm{tr}_g h)\mu_g$$

Page 74, 2.46 *The correct formula is*

$$\omega = \sum_{\alpha=1}^{n} e^{\flat}_{\alpha} \wedge J e^{\flat}_{\alpha}$$

Page 77, 2.64 *The correct formula is*

$$B = \frac{\mathrm{tr}B}{(m^2-1)} Id_{|\Lambda_0^{1,1}M} + B_0$$

i.e. $\Lambda_0^{1,1}$ *instead of* $\Lambda^{1,1}$

Page 92, 2.151 *The correct formula is*

$$\frac{1}{2}\Delta f + (dd^c f, \rho) = 0.$$

Page 166, 6.56 *Replace the first three lines of the statement by*
Let (M, g) be a compact connected Riemannian manifold. If the Ricci curvature of M is non-negative then

$$\dim H^1(M, R) = b_1 \leq \dim M$$

Moreover, the universal cover of M is a Riemannian product of the form $R^{b_1} \times \overline{M}$, where $\overline{M}$ is a manifold of dimension $\dim M - b_1$.
Page 310, line 2 *Replace by*
for $A^2 : f\,(z)\,dx^2 + 2dydz + g\,(y)\,dz^2$, for nonconstant positive functions f and g.

Page 310, line 15 *Replace the expressions for* B^3_θ and B^4 by

$$B^3_\theta = \begin{pmatrix} e^{t\theta} & b \\ 0 & e^{-t\theta} \end{pmatrix}\ t \in \mathbf{R}, b \in \mathbf{C} \text{ where } \theta \in \mathbf{C} \text{ has norm } 1,$$
$$B^4 = \begin{pmatrix} a & b \\ 0 & 1/a \end{pmatrix},\ a \in C^*, b \in C.$$

Page 310, line -7 *Replace the formula by*

$$dx^2 + dy^2 + 2dudv + 2\rho dxdu + \left(\omega - \frac{\partial\rho}{\partial x}\right) du^2.$$

Page 379, line 6 The reference should be [Hit 1]

Page 380, line 7 the almost complex *structure* is integrable...

Page 382, line -6 The reference should be [Hit 2]

Page 385, line -9 The reference should be [Hit 5]

Page 459, line 7 conformal *deformations...*

Page 478, line 10 *Replace* $\mu^{-1}(G)$ by $\mu^{-1}(0)$

Page 489, line [Hit 6] *the complete reference is*

[Hit 6] N. Hitchin, Metrics on moduli spaces, Proceedings of Lefschetz Centennial Conference, Mexico City (1984), Contemporary mathematics **58,** Part I, 157-178, Amer. Math. Soc., Providence (1986).

Page 489, line [H-K-L-R] *the complete reference is*

[H-K-L-R] N.J. Hitchin, A.Karlhede, U. Lindström, M. Rocek, Hyperkähler metrics and supersymmetry, Commun. Math. Phys. **108,** 535-549 (1986).

Page 492, line [Kro] *the complete reference is*

[Kro] P. Kronheimer, Instantons gravitationnels et singularités de Klein, C. R. Acad. Sci. Paris **303,** 53-55 (1986).

Page 496, line [Run] *Between references* [*Run*] *and* [*Rya*]*, insert*

[Ru-Wa-Wi] H.S. Ruse, A.G. Walker and T.J. Willmore, Harmonic spaces, Consiglio Nacionale delle Ricerce, Monografie Matematiche 8, Edizioni Cremonese, Roma (1961).

Page 499, line [Wu3] The last page of that reference is 392 instead of 352.

Page 506 (index), line Futaki No reference to page 375.

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