

THE MATHEMATICS OF SURVIVAL IN CHINA

The earliest sign of mathematical activity in China is the legend of Yü the Great and the Lo River tortoise. According to this legend, Emperor Yü received a divine gift in the form of a magic square – the Lo Shu diagram – etched into the tortoise’s shell. The earliest archeological indication of Chinese mathematical activity is the tally and code symbols carved on oracle bones in the Shang period, about thirty-four hundred years ago. By the time of Han (2nd century BCE to the 4th century), the Chinese had developed a codified notation and were computing with a counting board and rods. The ideographic script used by the Chinese may have encouraged their advances in notation and computing. Needham (1959: 13) conjectures (following Derek Price) that civilizations which had alphabets available for constructing numerals were prompted to break out of the “ten fingers” constraint and develop systems of more than 9 numerals. The Shang Chinese appear to be the first people able to express any number using no more than nine numerals.

The oldest extant Chinese manuscript which is of mathematical interest is the *Chou Pei Suan Ching* (“Arithmetic Classic of the Gnomon and the Circular Paths of Heaven”). It pre-dates the 3rd century BCE, and includes what we know as the “Pythagorean” theorem, empirical geometry, and basic operations with fractions using the concept of a common denominator. Numbers are expressed in words.

The practical significance of the “Pythagorean” theorem is explained in this manuscript by Shang Kao (Needham, 1959: 23). A plane triangle (right-angled) laid on the ground is used to construct things “straight and square” (using cords). Heights are observed using a recumbent triangle. A reversed triangle is used to measure depth. And a flat triangle serves to determine distance. A circle can be drawn by revolving a triangle; and squares and oblongs can be formed by uniting triangles; Shang Kao concludes:

He who understands the earth is a wise man, and he who understands the heavens is a sage. Knowledge is derived from the straight line (the shadow). The straight line is

derived from the right angle. And the combination of the right angle with numbers is what guides and rules the ten thousand things.

The *Chou Pei* begins as a discussion involving the Duke of Chou, Chou Kung, and Shang Kao. Later, the dialogue is between Chhen Izu and Jung Fang. These speakers gradually fade, and are replaced by the phrases “According to the method” (Fa yeh), and “According to the art” (Shu yeh). These are, according to Needham (1959: 21), apparently intercalated texts. This sort of routinizing or objectifying process tends to occur whenever a certain degree of generational continuity occurs in a field. The substitution of general expressions for persons or attributed statements is a fundamental aspect of the abstraction process and the source of many of the difficulties we have understanding the grounds for pure or formal statements. I discuss this issue in later chapters.

One of the most celebrated texts in the Chinese mathematical literature is the *Chiu Chang Suan Shu*, variously referred to in English as “Arithmetic in Nine Sections”, “Nine Chapters on Mathematical Art”, and “Nine Chapters on Mathematical Techniques”. It is also called the *Chiu-chang suan ching*, “Mathematical Manual in Nine Chapters”. This text is said by some authorities to be a compilation produced in the third or second century BCE. Chang Ts’ang (fl. 165–152BCE) may have been the person who initially compiled this manual. But in fact the compilation may have been written as late as the period 50BCE to 100CE. Different sections, of course, may have been written at different times. The version we have now was probably extant by the first century CE at the latest. There were many commentaries on the manual, but the two that stand out are by Liu Hui (3rd century), and Li ch’un-feng (seventh century) (Needham, 1959: 24ff; Libbrecht, 1973: 267ff.; Yan and Du Shiran, 1987: 33ff.).

The *Chiu Chang Suan Shu* had the same sort of influence on Chinese mathematics that Euclid’s *Elements* had on Greek mathematics, even to the extent of inhibiting mathematical developments. This was a result of the degree to which it systematized mathematics and became a “classic”. It was studied by a wide range of scholars (Needham, 1959: 27). Liu Hui’s commentary in particular influenced the course of Chinese mathematics for more than a millenium. He also wrote another important but much shorter work, the *Hai-tao suan-ching*, “Sea Island Mathematical Manual” in 263. Liu Hui’s works were included in the *Suan-ching shih-shu*, “Ten mathematical Manuals”, of 656. Li Shun-feng (602–670), the

T'ang mathematical worker and astronomer, annotated and added commentaries to these works, and they eventually became standard texts. According to official regulations, mathematics students had to spend three years studying Liu Hui's works. His works were also prescribed texts in the Japanese schools opened in 702. Very little is known about Liu Hui's life besides the facts that he lived in the kingdom of Wei toward the end of the Three Kingdoms period (221–265), and that he was well-known for his writings on mathematics.

The *Chiu Chang Suan Shu* was designed as a handbook for architects, engineers, officials, and merchants. Thus, many of the two-hundred and forty six problems in the nine chapters deal with building canals, dikes, and city walls, taxation, barter, public services, and other aspects of everyday life. The nine chapter headings are (1) Land Surveying (rules for finding areas; and for adding, subtracting, multiplying, and dividing fractions), (2) Millet and Rice (Percentages and proportions), (3) Distribution by Progression (distributing properties among partners according to given rates, taxing goods which vary in quality, and other problems involving arithmetical and geometrical progressions – all solved using proportions), (4) Diminishing Breadth (for example, finding the sides of a rectangle given the area and one side; finding the circumference of a circle given the area; extracting square and cube roots), (5) Consultations on Engineering Work (volumes of prisms, pyramids, wedges, cones, and other solids), (6) Impartial Taxation (deals with problems of pursuit and allegation such as how long it takes taxpayers to transport their grain contributions to the capitol; and ratio problems related to allocating tax burdens according to population size), (7) Excess and Deficiency (use of the rule of “false position” to solve problems of the type $ax + b = 0$), (8) Calculation by Tabulation (simultaneous linear equations, using positive and negative numbers), and (9) Right Angles (applications of the “Pythagorean” Theorem). There are several notable problems in this manuscript. Problem 18, Chapter 8, with its five unknowns and four equations, heralds the indeterminate equation. Problem 13, Chapter 9, is similar to a problem found in the writings of Brahmagupta (seventh century): “There is a bamboo 10 feet high. When bent, the upper end touches the ground 3 feet away from the stem. Find the height of the break”. Problem 20, Chapter 9 seems to require solving the quadratic equation

$$x^2 + (20 + 14)x - 2(20)1775 = 0;$$

however, there is no method given for solving the problem.

Procedures for extracting square roots were known in the ancient Eastern and Western civilizations. The methods were derived in basically the same way, from the equation,

$$(a + b)^2 = a^2 + 2ab + b^2.$$

The Greek method was geometrical; it was based entirely on the two-dimensional Euclidean figure. Apparently, no efforts were made to generalize this method by constructing three-dimensional figures and finding cube roots. Very little was known about extracting cube roots in the West until the sixteenth century. The Chinese method for extracting square roots was more advanced than the Greek method in some respects, and more abstract. It was thus possible for mathematical workers to eventually generalize the square root method – by introducing the coefficient of the highest power x^2 as the *hsia fa* (literally, the lowest divisor) – and applying it in the solution of any numerical equation. Liu Hui and Li Shun-feng, commentators on the *Chiu chang suan shu*, give some indication that cubical blocks may have been used to demonstrate the cube root method (Lam Lay Yong, 1977: 56).

The nine problems (chapters) of Liu Hui's *Hai-tao suan ching*, "The Sea Island Mathematical Manual", deal with everyday applications of a classical Chinese method for determining the distance from the sun to a flat earth using double triangulation. The manuscript was originally known as the *Ch'ung ch'a*, "Method of Double Difference" (referring to double or repeated application of proportions to sides of right triangles). It was appended to the *Chiu-chang suan-shu* as Chapter 10, then separated from it in the seventh century when the ten mathematical manuals were selected and renamed, probably, after the first problem which concerns an island. The problems involve finding (1) the elevation of an island and its distance from a pole; (2) the height of a tree on a hill; (3) the size of a distant walled city, (4) the depth of a valley, (5) the height of a tower on a plain as seen from a hill, (6) the width of a gulf seen from a distance on land, (7) the depth of water at the bottom of an abyss, (8) the width of a river seen from a hill, and (9) the size of a city seen from a mountain. All of these problems were worked out *rhetorically*; no algebraic generalizations were arrived at – none were thought necessary by the Chinese calculators.

Many Han mathematical texts are lost, including the *Li Li Suan Fa*, “Mathematical Methods Concerned With Pitchpipes and the Calendar” (author unknown); and the *Suan shu* books of Tu Chung and Hsu Shang (1st century BCE). (How much of this is due to the burning of the books in 213BCE is not clear; the Confucianist writings were the target, and books on practical subjects were not so much affected. One copy of each of the burned books was kept for the State Library, but that library was burnt down during the violence at the end of the Ch’in dynasty; Eberhard, 1971: 66). With the beginning of the Common Era, and the end of the Han dynasty, China entered a period during which very little progress was made in mathematics. There was very little significant mathematical activity between the third and sixth centuries. The mathematical work carried out during this period, including the writing of texts, followed (and repeated the mistakes in) the *Chiu chang suan shu* and did not further the achievements recorded in that manuscript. The *Sun Tsu Suan-ching* (“Arithmetical Classic of Sun Tsu”) was written between 280 and 473.

Toward the end of the second century, the *Shu-shu chi-i*, “Memoir on Some Traditions of Mathematical Art”, written by Hsu Yeh (or Hsu Yo), appeared. Because of references in this text to large numbers and their presentation in the form of arithmetical series, many scholars regard it as having been fabricated after the introduction of Indian ideas about *Kalpas*; the fabrication may even have been carried out by the commentator Chen Luan, who flourished between 560 and 580. Chen Luan was a Buddhist convert and there are references to Buddhism in his commentary. The text is also closer to Taoism and divination than earlier books. Needham dates this text around 190. He points out that the Indian system has very different progressions and nomenclatures. There is some evidence of Chinese opposition to associating mathematics and mysticism, and to Buddhist large numbers in particular. Needham (1950: 88) cites Shen Tso-Chi, a twelfth century scholar on this point:

It is only things which are beyond shape and number (*hsiang shu*) which cannot be investigated. How can there be mathematics beyond the reach of shape and number?

The following passage from this text illustrates the influence of the counting board on how the Chinese thought and wrote about calculating:

In making calculations we must first know the position (and structure) (*wei*) (of



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