

INDIAN MATHEMATICS: A HISTORY OF EPISODES

A sociologically inspired history of Indian mathematics has yet to be written. And the conventional histories of Indian mathematics are often little more than lists of achievements, sometimes designed to illustrate and not infrequently to exaggerate India's contributions to "world mathematics". Nonetheless, a few sociologically important points can be made by reviewing the basic features of Indian mathematics.

In India as elsewhere, the connection between early mathematical activity and religion is evident. In ancient Vedic society, the male head of a household was required to maintain three fires, each one sheltered in a specially designed, precisely measured altar. He also had to transfer fires from one altar to another. Altars might be the same or different in shape, but their areas had to be related by a simple ratio. In ancient India as elsewhere in the ancient world, the construction of altars was an important source of mathematical problems. There are references to such problems in the *Rig Veda Samhita*, and specific details about "the science of the altar" in the *Taittiriya Samhita* and *Taittiriya Brahmana*.

The rules for constructing altars and transferring fires were codified in the *Sulvasutras*. These sutras are thought by some scholars to have been written as early as 800 to 500 BCE; others date them in the early centuries of the common era. In any case, they are an important document for historians of mathematics; they include such items as the "Pythagorean" theorem (for special cases), and the proposition that the diagonal of a rectangle divides it into equal parts.

"Sulva" seems to have originally meant "to measure", and later "rope" or "cord" (from "rope-measurers", probably more familiar in their Egyptian incarnation). For a long period, geometry was known as "sulva" or "rajji" (rope). The *Sulvasutras* are not strictly speaking "mathematical" in the sense that the *Arya Bhateeya* (discussed below) is. But it is the earliest important codification of mathematical problems and findings in India. As in the case of many other achievements in Indian mathematics, there is little continuity between the results and findings recorded in these sutras and later developments in Indian mathematics. Other religious sutras such as the *Anuyoga Dwara Sutra* and the

Sthananga Sutra (300–500) also include important mathematical items. The former includes an enumeration of powers and roots, and rules for finding roots and powers. The latter lists the ten basic mathematical topics, including the four basic operations, geometry, applied arithmetic, and fractions.

Mahavira, the ninth century founder of Jainism, was a well-known mathematical worker in the court of Amoghavarsha Nripatunga, the Rashtrakoota King of what is today the state of Mysore. *Ganitanuyoga*, or “exposition of mathematical principles”, and *Samkbyana*, “the science of arithmetic and astronomy”, were important branches of Jainist literature. Mahavira’s *Ganita Sara Sangraha* (written about 850) is the first “textbook” that presents arithmetic in its modern form. Mahavira’s text includes problems such as the following (Srinivasiengar, 1967: 72):

Three merchants found a purse in the way. One of them said “If I secure this purse, I shall become twice as rich as both of you with your moneys on hand”. Then the second man said, “I shall become thrice as rich”. The third man said, “I shall become 5 times as rich”. What is the value of money in the purse, as also the money in hand with each of them?

Although he did not systematically study conics, Mahavira does refer briefly to the ellipse; he appears to be the only Indian mathematical worker to do so. His formula for the area of the ellipse is not correct, and was probably based on the formula for the area of the circle.

Indian mathematics, especially in the period before Greek astronomy was introduced (c.400), placed a relatively strong emphasis on large numbers. Geometry, arithmetic, number theory, and algebra were virtually ignored in favor of the use of numbers in social schemes. The *Upanishads* (c.700–500 BCE) contain numerous numerical descriptions: the 72,000 arteries, the 36,360 or 36,000 syllables, the 33,303 or 3306 gods, the 5, 6, 7, or 12 basic elements out of which the world is composed. The wisdom of the Buddha is illustrated by the gigantic numbers he can count out (on the order of eight times twenty-three series of 10^7), and his magnificence is illustrated by the huge number of bodhisattvas and other celestial beings who gather to set the scenes for his various sutras. The Hindu cosmology includes a cyclical view of time that enumerates great blocks of years called *yugas*. There are four *yugas* ranging from 432,000 to 1,728,000 years, all of which together make up one thousandth of a *kalpa* or 4,320,000,000 years.

This emphasis upon immense, cosmological numbers gives a distinctively Hindu view of the near-infinite stretches of being that surround the empirical world. It seems almost inevitable that the Hindus should have invented the “zero” as we know it and use it in modern mathematics, or *sunya* (emptiness) in Sanskrit. The concept *sunya*, developed about 100, was the central concept in Madhyamika Buddhist mysticism, and preceded the introduction of the mathematical zero about 600. Classical Indian world views are permeated with mathematics but of a special kind. It is a mathematics for transcending experience, but not in the direction of rationalistic abstraction. Instead, numbers are used for purposes of *mystification* or to convey the notion that some thing or being is *impressive*; they are symbols in a mathematical rhetoric designed to awe the listener into a religious posture. In general, numbers were used for *numerological* rather than mathematical purposes. The social roots of this distinctive mathematical system lie in the particularly exalted status of Indian religious specialists. The concrete as opposed to abstract nature of Hindu large numbers may also have been suggested by a social reality: the great variety of ethnic groups making up Indian society, institutionalized in the ramifications of the caste system (see Restivo, 1983: 60–63, for a discussion of the sociology of large number systems in India).

The connection between early number work and religion is universal. The reason for this is that religion is a somewhat loosely applied term for the everyday and ritual activities in which people create and recreate feelings of social solidarity and sustain “moral codes”. In a context where the differentiation of social activities into role, organizational, or institutional sectors was minimal, number work inevitably was associated with both mundane counting and ritualistic ordering. The connection between religion, political power, and the calendar was another factor that linked number work; especially in astronomical matters, and religion.

A “GOLDEN AGE”

A “golden age” in Indian mathematics begins with the rise to power of King Gupta in 290, and ends in the midst of the political turmoil of the twelfth century. While the publication of Arya Bhata’s *Arya Bhateeya* in 499 is the first major indicator of this golden age in mathematics, the “age of the Siddhantas” (Boyer, 1968: 231) is an important prelude. These works seem to have been a part of the renaissance in Sanskrit culture

inaugurated under the Gupta dynasty. The *Siddhantas* all seem to have been works in which rules related to astronomy were listed in cryptic Sanskrit verse. There is little or no explanation of these rules, and no proofs are offered. While there is general agreement that the *Siddhantas* belong to the late fourth or early fifth century, there is some controversy about the origin of the facts they list. Some authorities claim that they reflect the influences of Paul of Alexandria and Ptolemy.

The *Surya Siddhantas*, the only one of the manuscripts that seems to be complete, has been dated ca. 400. This “system of the Sun” is written in epic stanzas, and identified as the work of Surya the Sun God. It includes a decimal system, positional notation, and ciphered forms for ten numerals. Not one of these is due to the Indians; but they are probably the first people to combine these three mathematical inventions. As in other manuscripts, references to predecessors are infrequent.

The *Arya Bhateeya* of 499 is the first treatise on mathematics as a more or less distinct activity published in India. It was written by Arya Bhata of Kusuma Pura (near modern Patina). There is some evidence that there was a “school” of mathematical workers at Kusuma Pura.

The *Arya Bhateeya* is a relatively brief text that reports what was known at the time about such topics as progressions, determining square and cube roots, and solving quadratic equations. Arya Bhata uses the sort of alphabet numerals used by the Greeks which are well-suited for the poetic style used in ancient writings on mathematics and other topics. He is the earliest mathematical worker-astronomer mentioned in the extant historical documents. Among his contributions was the proposal that the diurnal motion of the heavens is due to the rotation of the earth on its axis.

The most important mathematical workers of this “golden age” are, like Arya Bhata, astronomers. Varaha-Mihara is the author of an astronomical treatise, *Pancha Siddhantika* (505), and several works on astrology. And Brahmagupta (born in 598 and a member of the Ujjain School) wrote the *Brahma Sphuta Siddhanta*, the major source of Arabic-Islamic learning about Indian astronomy. A number of chapters in this manuscript are devoted to topics in arithmetic (*Ganita*) and algebra (*Kuttaka*, literally “pulverizer”). (The term *Bija Ganita* eventually came to mean “algebra”: it appears for the first time in a mid-ninth century work by Prithudaka Swami, a commentator on Brahmagupta).

Brahmagupta is known for his solution of the indeterminate equation $Nx^2 + 1 = y^2$. He deals with interest problems, provides geometrical

solutions for constructing right triangles that anticipate Fibonacci (1202) and Vieta (1580), and gives other results that anticipate the achievements of seventeenth century European mathematical workers.

At the age of 67, Brahmagupta wrote the *Khanda Khadyaka* (665), an expository astronomical treatise. Here he gives a rule ("Brahmagupta's rule") equivalent to what is generally known as the Newton-Sterling formula. He also has a claim to being the "inventor" of interpolation theory; but neither he nor his successors gave much attention to this topic.

The most celebrated mathematical worker in ancient India is Bhaskara (b.1104) "the gem of the circle of mathematicians". Bhaskara was a Brahmin, and (like Mahavira) from the area now known as Mysore. Bhaskara was a "pure mathematician" in the sense that he focused on simplifying, improving, and adapting the contributions of his predecessors. Unlike most earlier Indian mathematical workers, Bhaskara is aware of and refers to predecessors, including Brahmagupta and Mahavira, as well as Sridhara and Padmanabha (whose works are lost), Sripathi, Prithedaka Swarmi, and Varaha-Mihara.

Bhaskara's *Siddhanta Siromani* (1150) is a textbook that deals with arithmetic, algebra, the celestial globe, and the planets. The core text is written in a poetic style, with accompanying prose commentaries. It includes sections on the mechanical application of methods ("leelavati") and the theory underlying those methods ("bijaganita"). The text begins with a prayer to Lord Ganesa, and includes tables, the eight operations (+, -, $\times$, $\div$, x^2 , x^3 , $^2\sqrt{}$, $^3\sqrt{}$), fractions, zero, the rule of 3, interest, mensuration, and permutations. The Leelavati chapters are especially noted for their interesting "recreational" problems. A typical problem is formulated as follows (Srinivasiengar, 1967: 85):

O Girl! out of a group of swans $7/2$ times the square root of the number are playing on the shore of a tank. The two remaining ones are playing with amorous fight in the water. What is the total number of swans?

Comparable "interesting" problems are, of course, not uncommon in other mathematical traditions.

Bhaskara is the first writer to give $a/0 = \infty$. He did not use infinitesimals, but did work out some differential calculus results. Using summation methods used by mathematical workers from Archimedes to Kepler, Bhaskara determined the area and volume of the sphere.

Bhaskara is the last representative of the "golden age" of Indian



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