Preface

Since Max Born's "statistical interpretation of wave functions" proposed in 1926, there has been the long-standing problem of "interpretation of Schrödinger equations". However, the emphasis should not be on "interpretation": The right problem to set is

"What is the Schrödinger equation ?"

This monograph is devoted to an attempt to give an answer to the problem in terms of diffusion theory, namely, in terms of diffusion processes, and it will be shown that the Schrödinger equation and diffusion equations in duality are equivalent. As a result, Schrödinger's conjecture of 1931 will be solved.

The equivalence of Schrödinger and diffusion equations implies that, roughly speaking, if we add (Brownian) noise and specific additional drift to a "classical particle" then we get the movement of a "quantum particle". It must be emphasized that the quantum particle (= diffusion particle) has its well-defined position but no velocity, and hence it is not at all a "classical particle" any more. Therefore, we must look at it carefully in the context of diffusion theory. Moreover, the theory of diffusion processes for the Schrödinger equation will tell us that we must go further into the theory of systems of (infinitely) many interacting quantum (diffusion) particles.

The contents of the monograph are based on my lectures on diffusion theory (mathematics, not physics) which have been given at various places including Tokyo Institute of Technology, Aarhus University, University of Erlangen, Keio University, University of California at San Diego, and the main part of them at the University of Zürich in the last decade.

This monograph may be regarded as "*An Introduction to the Theory of Diffusion Processes with Applications*". There will be no difficulty in reading Chapters 2, 3 and 4 for those who have an elementary knowledge of PDE and diffusion processes, and some fundamentals of functional analysis including measure theory. Those who would like to learn in a hurry the equivalence of Schrödinger and diffusion equations and its implications can read Chapter 4 directly, possibly referring to the necessary pages of Chapters 2 and 3. For Chapters 5 and 6, readers are assumed to have slightly more advanced experience in diffusion processes and perhaps some patience, since they contain delicate analysis in connection with the singularity of coefficients of diffusion equations. For Chapters 7 and 8, I have to assume that readers have some elementary knowledge of and intuition for statistical mechanics and more maturity in mathematical experience. Some applications in biology and physics will be given in Chapter 9. Chapters 10 and 11 offer a self-contained exposition on relative entropy and large deviations, needed in Chapters 5 and 8. Non-linearity induced by the branching property will be briefly explained in Chapter 12.

The text is practically self-contained and proofs are given for all theorems except for some proofs in Chapter 7 which are left in the original articles so that the monograph is kept to a reasonable size.

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Zürich, January 1993 Masao Nagasawa[1]

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