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I. Linear Partial Differential Equations. Elements of the Modern Theory

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Translated from the Russian
by P.C. Sinha

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