# Preface

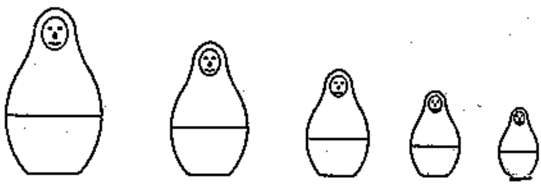
In recent years topology has firmly established itself as an important part of the physicist's mathematical arsenal. Topology has profound relevance to quantum field theory—for example, topological nontrivial solutions of the classical equations of motion (solitons and instantons) allow the physicist to leave the framework of perturbation theory. The significance of topology has increased even further with the development of string theory, which uses very sharp topological methods—both in the study of strings, and in the pursuit of the transition to four-dimensional field theories by means of spontaneous compactification. Important applications of topology also occur in other areas of physics: the study of defects in condensed media, of singularities in the excitation spectrum of crystals, of the quantum Hall effect, and so on. Nowadays, a working knowledge of the basic concepts of topology is essential to quantum field theorists; there is no doubt that tomorrow this will also be true for specialists in many other areas of theoretical physics.

The amount of topological information used in the physics literature is very large. Most common is homotopy theory. But other subjects also play an important role: homology theory, fibration theory (and characteristic classes in particular), and also branches of mathematics that are not directly a part of topology, but which use topological methods in an essential way: for example, the theory of indices of elliptic operators and the theory of complex manifolds.

Faced with the necessity of studying topology, physicists have found out that there are many excellent textbooks (see [1] to [10] in the bibliography), but mostly written in a style unfamiliar to them. (Among the references just cited, perhaps the most accessible to physicists is [3].) At the same time there are also surveys and books that present the basic concepts of topology in a form easily accessible to physicists, generally together with their applications to physics (see [11] to [14], for instance). The original plan for the present work was also to combine the exposition of topology with its applications to physics: much of the material covered here appeared in Russian as part of a monograph on the applications of topology to quantum field theory [15]. Later it became clear that it would be preferable to publish the mathematical exposition separately, especially in view of the interest of physicists working in areas other than quantum field theory. Thus the present book is separate from the author's *Quantum Field Theory and Topology* [16]. For those interested in quantum field theory, it is advisable to read this book and [16] in parallel; and all readers might profit from filling out the discussion of homotopy theory given here with the applications to topologically stable defects in condensed media given in [16].

I have tried to make the study of topology as easy as possible for physicists. This, however, presented certain problems. On the one hand, I wanted the presentation to be brief, so that the reader, having acquired the indispensable topological concepts, would be able to move on at once to the physical applications. On the other hand, it seemed reasonable to include information beyond the barest minimum: not only topological facts that have already been used in the physics literature, but also those that may foreseeably be used in the future. I also wanted the exposition to be as simple as possible for physicists, even at the expense of mathematical rigor; at the same time, I wanted to satisfy those readers who want to know the whole story.

To reconcile these contradictory requirements, I have used the principle of a Russian *matryoshka*:



Therefore you can regard this book as several texts nested inside one another. The main, and relatively brief, text contains the basic concepts. The rest of the text expands on the basic material, adding rigor and filling in details; it can be omitted upon first reading, and is marked like this: ► ◄, ►► ◄◄, and ►►► ◄◄◄, in order of decreasing priority.

Many statements are formulated twice, once in a more intuitive way, the second time rigorously. This has led to the abundant use of paraphrases and parenthetical remarks.

Here is a brief overview of the contents.

Chapter 0 covers the background necessary for the understanding of the book. It is a good idea to glance through this material to make sure you are familiar with these basic concepts, and especially with the idea of gluing, or identification, of spaces, which is frequently used later on.

Chapter 1 defines and illustrates some fundamental topological notions—in particular, homotopies and homotopy equivalences.

Chapters 2 and 3 are devoted to the degree of a map, the fundamental group, and covering spaces. These, too, are fundamental concepts, but the purpose of these chapters is purely pedagogical, because the material covered in them is subsumed in later chapters as particular cases of more general results obtained by less elementary methods. (Degree theory can easily be derived by homological methods, the fundamental group is the first homotopy group, and covering spaces are particular cases of fiber spaces.)

Chapter 4 gives the basic definitions of the theory of smooth manifolds; readers familiar with smooth manifolds, vectors and tensors on manifolds, and orientability will miss nothing by skimming through the first three sections of this chapter.

Next come differential forms and homology theory; Chapter 5 deals with the case of open subsets of $\mathbf{R}^n$, while Chapter 6 discusses the general case. Throughout Chapter 5 and the beginning of Chapter 6, little rigor is used in introducing homology theory; the details come in Sections 6.5 and 6.6.

Chapter 7 studies homotopy groups for simply connected spaces, and Chapter 8 the same groups for arbitrary spaces (this chapter can be left out on a first reading).

Chapter 9 gives the main definitions of the theory of fiber spaces, and Chapter 10 indicates ways of computing homotopy groups using fiber spaces, by listing several relations between the homotopy groups of the base, the fiber and the total space (whose proofs are postponed till Chapter 11). The homotopy groups of certain important examples are also given in Chapter 10.

Chapters 12 and 13 contain a brief summary of the theory of Lie groups and Lie algebras from our point of view. Chapter 14 studies the homotopy and homology groups of Lie algebras and homogeneous spaces (on the whole we limit ourselves to homology and homotopy in dimensions up to three, since these are the dimensions that occur most often in physics).

Chapter 15 is devoted to the geometry and topology of gauge fields.

Finally, there is a set of problems of varying complexity, from simple exercises to important and subtle results not contained in the main text.

I hope that this book will allow physicists to familiarize themselves with many important areas of topology, without stumbling on secondary details. The book may also prove useful to mathematicians specializing in fields other than topology and desiring to apply topological concepts to their own work (in certain fields of applied mathematics, for example).

I take this opportunity to express my deep appreciation to M. A. Baranov and A. A. Rosly for their help, and my heartfelt thanks to my wife, L. M. Kissina, for her patience and support.

In this second printing I have corrected some misprints and inaccuracies found by attentive readers. My special thanks to Keith Conrad.

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