Solutions, answers, and hints for selected problems

Complete solutions of some problems are given. Answers only are given for some other problems. For still others, only hints or partial solutions are given. Asterisks in "A Modern Approach to Probability Theory" by Fristedt and Gray identify the problems that are treated in this supplement.

For Chapter 31

31-2. For the last assertion one may for each ω, view Q as a probability measure on $(\mathbf{D}([0,\infty),\Psi),\mathcal{H})$. Then Q_t is the distribution of the Ψ-valued random variable $\varphi \rightsquigarrow \varphi_t$ defined on the probability space $(\mathbf{D}([0,\infty),\Psi),\mathcal{H},Q)$. Since $\varphi_u \to \varphi_t$ as $u \searrow t$ and almost sure (in this case sure) convergence implies convergence in distribution, $Q_u \to Q_t$ as $u \searrow t$ (for each ω, not just the requested 'a.s.').

31-3. TO BE DONE

31-9. Let R_t denote the distribution of the Lévy process at time t. Then

$$T_t f(x) = \int f(x+y)\, R_t(dy)\,.$$

Let R denote the distribution of the Lévy process. Then the corresponding Markov family $(Q^x \colon x \in \mathrm{R})$ is defined by

$$Q^x(B) = R\big(\{\varphi \colon [t \rightsquigarrow (x+\varphi_t)] \in B\}\big)\,.$$

31-15. TO BE DONE

31-21. Suppose that Q_0 is an equilibrium distribution for $\widetilde{T}$. Then

$$Q_0 T_t = e^{-ct} \sum_{k=0}^{\infty} \frac{(ct)^k}{k!} Q_0 \widetilde{T}^k = e^{-ct} \sum_{k=0}^{\infty} \frac{(ct)^k}{k!} Q_0 = Q_0\,.$$

Hint: for converse: Use Problem 16.

31-23. *Hint:* Let f be the indicator function of the one-point set $\{y\}$ and use Theorem 14.

31-25.

$$
\begin{aligned}
p_{00}(t) &= (q_{01}+q_{10})^{-1}\big[q_{10}+q_{01}\exp[-(q_{01}+q_{10})t]\big]\\
p_{01}(t) &= (q_{01}+q_{10})^{-1}\,q_{01}\big(1-\exp[-(q_{01}+q_{10})t]\big)\\
p_{10}(t) &= (q_{01}+q_{10})^{-1}\,q_{10}\big(1-\exp[-(q_{01}+q_{10})t]\big)\\
p_{12}(t) &= (q_{01}+q_{10})^{-1}\big(q_{01}+q_{10}\exp[-(q_{01}+q_{10})t]\big)
\end{aligned}
$$

The limits at ∞ of both p_{00} and p_{10} are the same: $(q_{01}+q_{10})^{-1}q_{10}$, the value the equilibrium distribution assigns to $\{0\}$. The limits at ∞ of both p_{01} and p_{11} are the same: $(q_{01}+q_{10})^{-1}q_{01}$, the value the equilibrium distribution assigns to $\{1\}$.

31-28. TO BE DONE

31-29. TO BE DONE

31-36. $\frac{d\nu}{d\lambda}(y) = b(1-b)ce^{-(1-b)cy}$, $\nu\{\infty\} = 0$; equilibrium distribution assigns value $(1-b)b^x$ to x; jump-rate function is

$$
x \rightsquigarrow \begin{cases} cb^{-1} & \text{if } x = 0 \\ c & \text{if } x > 0\,; \end{cases}
$$

transition probabilities from x to $x-1$ equal 1 for $x > 0$ and from 0 to $x > 0$ equal $(1-b)b^{x-1}$; transition rates from x to $x-1$ equal c for $x > 0$ and from 0 to $x > 0$ equal $c(1-b)b^x$ and all others equal 0