

Contents

Chapter I. Set Systems and Set Functions	1
1. Set Systems.....	1
Basic Notions and Notations.....	1
Inverse Images of Pavings.....	4
The Transporter	6
Complements for Ovals and σ Ovals	9
2. Set Functions	10
Basic Properties of Set Functions	10
Contents and Measures	13
New versus Conventional Contents and Measures	15
The Main Example: The Volume in \mathbb{R}^n	19
3. Some Classical Extension Theorems for Set Functions	22
The Classical Uniqueness Theorem	22
The Smiley-Horn-Tarski Theorem	23
Extensions of Set Functions to Lattices	27
Chapter II. The Extension Theories Based on Regularity	33
4. The Outer Extension Theory: Concepts and Instruments	33
The Basic Definition	33
The Outer Envelopes	34
Complements for the Nonsequential Situation	38
The Extended Carathéodory Construction	40
The Carathéodory Class in the Spirit of the Outer Theory	42
5. The Outer Extension Theory: The Main Theorem	45
The Outer Main Theorem	45
Comparison of the three Outer Theories	49
The Conventional Outer Situation	50
6. The Inner Extension Theory	53
The Basic Definition	54
The Inner Envelopes	54
The Carathéodory Class in the Spirit of the Inner Theory	56
The Inner Main Theorem	57
Comparison of the three Inner Theories	58
Further Results on Nonsequential Continuity	59
The Conventional Inner Situation	60

7. Complements to the Extension Theories	64
Comparison of the Outer and Inner Extension Theories	65
Lattices of Ringlike Types	68
Bibliographical Annex	72
Chapter III. Applications of the Extension Theories	79
8. Baire Measures	79
Basic Properties of Baire Measures	79
Inner Regularity in Separable Metric Spaces	83
Extension of Baire Measures to Borel Measures	84
The Hewitt-Yosida Theorem	85
9. Radon Measures	87
Radon Contents and Radon Measures	87
The Classical Example of a Non-Radon Borel Measure	91
The Notion of Support and the Decomposition Theorem	94
10. The Choquet Capacitability Theorem	98
Suslin and Co-Suslin Sets	98
The Extended Choquet Theorem	101
Combination with Basic Properties of the σ Envelopes	104
The Measurability of Suslin and Co-Suslin Sets	105
Chapter IV. The Integral	109
11. The Horizontal Integral	109
Upper and Lower Measurable Functions	109
The Horizontal Integral	112
Regularity and Continuity of the Horizontal Integral	117
12. The Vertical Integral	121
Definition and Main Properties	121
Regularity and Continuity of the Vertical Integral	125
Comparison of the two Integrals	126
13. The Conventional Integral	128
Measurable Functions	128
Integrable Functions and the Integral	133
Integration over Subsets	137
Comparison with the Riemann Integral	139
Chapter V. The Daniell-Stone and Riesz Representation Theorems	143
14. Elementary Integrals on Lattice Cones	143
Introduction	143
Lattice Cones	146
Elementary Integrals	148
Representations of Elementary Integrals	151
15. The Continuous Daniell-Stone Theorem	154
Preparations on Lattice Cones	154
Preparations on Elementary Integrals	156
The New Envelopes for Elementary Integrals	156

The Main Theorem	159
An Extended Situation	163
16. The Riesz Theorem	165
Preliminaries	165
The Main Theorem	167
An Extended Situation	169
17. The Non-Continuous Daniell-Stone Theorem	171
Introduction	171
The Maximality Lemma	172
Subtight Sources	173
The Main Theorem	176
Chapter VI. Transplantation of Contents and Measures	179
18. Transplantation of Contents	179
Introduction and Preparations	179
The Existence Theorem	181
Specializations of the Existence Theorem	183
The Theorem of Łoś-Marczewski	186
The Uniqueness Theorem	189
19. Transplantation of Measures	190
Preparations	190
The Existence Theorem	191
Specializations of the Existence Theorem	192
The Uniqueness Theorem	194
Extension of Baire Measures to Borel Measures	195
Chapter VII. Products of Contents and Measures	201
20. The Traditional Product Formations	201
The Basic Product Formation	201
The Traditional Product Situation	205
Product Measures	206
21. The Product Formations Based on Inner Regularity	210
Further Properties of the Basic Product Formation	210
The Main Theorem	213
The Sectional Representation	215
22. The Fubini-Tonelli and Fubini Theorems	222
Monotone Approximation of Functions	223
The Fubini-Tonelli Theorems	225
The Fubini Theorems	228
Chapter VIII. Applications of the New Contents and Measures	231
23. The Jordan and Hahn Decomposition Theorems	231
Introduction	231
The Infimum Formation	234
The Jordan Decomposition Theorem	236

The Existence of Minimal Sets	239
The Hahn Decomposition Theorem	241
24. The Lebesgue Decomposition and Radon-Nikodým Theorems ..	242
The Lebesgue Decomposition Theorem	243
The Radon-Nikodým Theorem	244
Bibliography	249
List of Symbols	255
Index	257
Subsequent Articles of the Author	261

Measure and Integration
An Advanced Course in Basic Procedures and
Applications

König, H.

1997, XXII, 260 p., Hardcover

ISBN: 978-3-540-61858-4