

Preface

Cuts and metrics are well-known and central objects in graph theory, combinatorial optimization and, more generally, discrete mathematics. They also occur in other areas of mathematics and its applications such as distance geometry, the geometry of numbers, combinatorial matrix theory, the theory of designs, quantum mechanics, statistical physics, analysis and probability theory. Indeed, cuts are many faceted objects, and they can be interpreted as graph theoretic objects, as metrics, or as probabilistic pairwise correlations. This accounts for their fecund versatility.

Due to the wealth of results, in writing this book we had to make a selection. We focus on polyhedral and other geometric aspects of cuts and metrics. Our aim is to collect different results, established within diverse mathematical fields, and to present them in a unified framework. We try to show how these various results are tied together via the notions of cuts and metrics and, more specifically, the cut cone and the cut polytope. One of our guidelines for selecting topics was to concentrate on those aspects that are less well-known and are not yet covered in a unified way elsewhere. The book has, moreover, been written with a special attention to interdisciplinarity. For this reason, while some topics are treated in full detail with complete proofs, some other topics are only touched, by mentioning results and pointers to further information and references.

The book is intended as a source and reference work for researchers and graduate students interested in discrete mathematics and its interactions with other areas of mathematics and its applications. In particular, it is of interest for those specializing in algebraic and geometric combinatorics and in combinatorial optimization.

The book is subdivided into five parts, in which we consider the following topics: ℓ_1 -metrics and hypercube embeddable metrics, hypermetrics and Delaunay polytopes in lattices, the metric structure of graphs, designs in connection with hypercube embeddings, and further geometric questions linked with cut polyhedra.

We do not cover extensively the topic of optimization. Indeed, research on cuts and metrics in this direction is already well-documented. Some survey papers are available; for instance, by Frank [1990] on multicommodity flows and by Poljak and Tuza [1995] on the max-cut problem. Nevertheless, we do present some of the recent breakthroughs. For instance, we discuss the new semidefinite programming approximative algorithm of Goemans and Williamson, as well as

the result of Bourgain on Lipschitz ℓ_1 -embeddings with small distortion, and its application to approximating multicommodity flows by Linial, London and Rabinovich. Moreover, we mention en route a number of further results relevant to the max-cut problem and binary matroids.

We made each of the five parts of the book as self-contained as possible; in principle, each of them can be read independently of the other. Moreover, we have chosen a common labeling system for all items such as theorems, examples, figures, etc., in order to simplify the search throughout the text. Some portions of text that contain side information or lengthy proofs are in small print and can be avoided at first reading.

The part of the book treating links with the geometry of numbers, is based on a survey paper coauthored by the authors with V.P. Grishukhin of the Academy of Sciences of Russia in Moscow (cf. Deza, Grishukhin and Laurent [1995]). We are grateful to Slava Grishukhin for kindly permitting us to include the material in this book.

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