2

TRANSMISSION SYSTEM ASPECTS

"Maybe I can't define stability, but I know it when I see it !"[1]
Carson W. Taylor

In this chapter we analyze the rôle played by the transmission system in voltage stability.

We first deal with two basic notions: the maximum power that can be delivered to loads and the relationship between load power and network voltage. Then we briefly and qualitatively explain how these two basic properties may result in voltage instability. Next, we discuss the effect of components that affect the transmission capability, series and shunt compensation on one hand, transformers with adjustable tap ratio on the other hand. We also introduce the notion of VQ curves that express the relationship between voltage and reactive power at a given bus.

Most of the material of this chapter is based on the analysis of a simple single-load infinite-bus system, which allows easy analytical derivations and provides insight into the problem. Basic concepts introduced in this chapter will be generalized in later chapters to large system of arbitrary complexity.

2.1 SINGLE-LOAD INFINITE-BUS SYSTEM

We consider the simple system of Fig. 2.1, which consists of one load fed by an infinite bus through a transmission line. By definition, the voltage magnitude and frequency

[1] Panel Session presentation at the 1997 IEEE/PES Winter Power Meeting

Figure 2.1 Single-load infinite-bus system

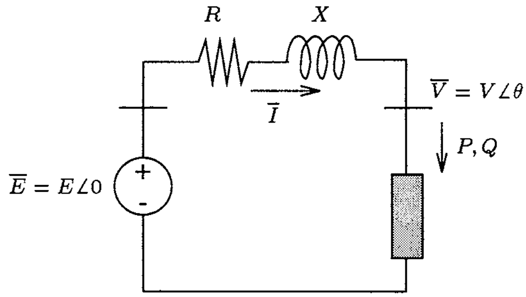


Figure 2.2 Circuit representation

are constant at the infinite bus. We assume balanced 3-phase operating conditions, so that the per phase representation is sufficient. We also consider steady-state sinusoidal operating conditions, characterized by phasors and complex numbers. The phase reference is arbitrary and need not be specified at this stage.

This leads to the circuit representation of Fig. 2.2. The infinite bus is represented by an ideal voltage source E. The transmission line is represented by its series resistance R and reactance X, as given by the classical pi-equivalent. The line shunt capacitance is neglected for simplicity (the effects of shunt capacitors are considered later in Section 2.6.2). The transmission impedance is:

$$Z = R + jX$$

Alternatively, we may think of E and Z as the Thévenin equivalent of a power system as seen from one bus. Note that, because power generators are not pure voltage sources, the Thévenin emf somewhat varies as more and more power is drawn from the system; we will however neglect this variation in a first approximation and consider a constant emf E as mentioned previously.

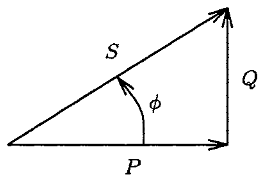


Figure 2.3 Definition of angle ϕ

Finally, let us recall that the *load power factor* is given by:

$$\mathrm{PF} = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}} = \cos\phi$$

where P, Q and S are the active, reactive and apparent powers and ϕ is the angle defined in Fig. 2.3.

2.2 MAXIMUM DELIVERABLE POWER

As pointed out in the Introduction, voltage instability results from the attempt of loads to draw more power than can be delivered by the transmission and generation system. In this section we focus on determining the maximum power that can be obtained at the receiving end of the simple system of Fig. 2.2, under various constraints.

2.2.1 Unconstrained maximum power

For the sake of simplicity we start by assuming that the load behaves as an impedance. In fact we will show later on that this choice does not affect the results. We denote the load impedance by:

$$Z_\ell = R_\ell + jX_\ell$$

where R_ℓ and X_ℓ are the load resistance and reactance, respectively.

We first revisit a classical derivation of circuit theory known as the load adaptation problem [CDK87] or maximum power transfer theorem: assuming that both R_ℓ and X_ℓ are free to vary, find the values which maximize the *active* power consumed by the load.

The current $\bar{I}$ in Fig. 2.2 is given by:

$$\bar{I} = \frac{\bar{E}}{(R + R_\ell) + j(X + X_\ell)}$$

and the active power consumed by the load:

$$P = R_\ell I^2 = \frac{R_\ell E^2}{(R + R_\ell)^2 + (X + X_\ell)^2} \tag{2.1}$$

Maximizing P over the two variables R_ℓ and X_ℓ, the necessary extremum conditions are:

$$\begin{aligned} \frac{\partial P}{\partial R_\ell} &= 0 \\ \frac{\partial P}{\partial X_\ell} &= 0 \end{aligned}$$

which after some calculations yields:

$$\begin{aligned} (R + R_\ell)^2 + (X + X_\ell)^2 - 2R_\ell(R + R_\ell) &= 0 \\ -R_\ell(X + X_\ell) &= 0 \end{aligned}$$

The solution to these equations, under the constraint $R_\ell > 0$, is unique:

$$R_\ell = R \tag{2.2a}$$

$$X_\ell = -X \tag{2.2b}$$

or in complex form:

$$Z_\ell = Z^*$$

One easily checks that this solution corresponds to a maximum of P. In other words:

> load power is maximized when the load impedance is the complex conjugate of the transmission impedance.

Under the maximum power conditions, the impedance seen by the voltage source is $R + R_\ell + jX + jX_\ell = 2R$, i.e. it is purely resistive and the source does not produce any reactive power. The corresponding load power is:

$$P_{max} = \frac{E^2}{4R} \tag{2.3}$$

and the receiving-end voltage:

$$V_{maxP} = \frac{E}{2}$$

where the subscript $maxP$ denotes a value under maximum active power condition.

The unconstrained case is not well suited for power system applications. The first problem is that in a transmission system the resistance R can be negligible compared to the reactance X. Now, making R tend to zero, the optimal load resistance (2.2a) also goes to zero, while the maximum power (2.3) goes to infinity. The two results might seem in contradiction: however, as R and R_ℓ go to zero, the current I goes to infinity (since $X + X_\ell = 0$) and so does the power $R_\ell I^2$! This is obviously unrealistic.

Even when taking into account the nonzero transmission resistance R, the above result is not directly applicable to power systems. Indeed, a highly capacitive load would be required to match the dominantly inductive nature of the system impedance. A modified derivation, closer to power system applications is made by assuming that the power factor of the load is specified. This case is dealt with in the next subsection.

2.2.2 Maximum power under a given load power factor

Specifying the load power factor $\cos\phi$ is equivalent to having a load impedance of the form:

$$Z_\ell = R_\ell + jX_\ell = R_\ell + jR_\ell \tan\phi$$

which now leaves R_ℓ as the single degree of freedom for maximizing the load power.

The current $\bar{I}$ is now given by:

$$\bar{I} = \frac{\bar{E}}{(R + R_\ell) + j(X + R_\ell \tan\phi)}$$

and the load active power by:

$$P = R_\ell I^2 = \frac{R_\ell E^2}{(R + R_\ell)^2 + (X + R_\ell \tan\phi)^2} \tag{2.4}$$

The extremum condition is:

$$\frac{\partial P}{\partial R_\ell} = 0$$

or, after some calculations:

$$(R^2 + X^2) - R_\ell^2(1 + \tan^2\phi) = 0 \tag{2.5}$$

which is equivalent to:

$$|Z_\ell| = |Z|$$

The second derivative is given by:

$$\frac{\partial^2 P}{\partial R_\ell^2} = -2R_\ell(1 + \tan^2\phi)$$

which is always negative, thereby indicating that the solution is a maximum. In other words:

> under constant power factor, load power is maximized when the load impedance becomes equal in magnitude to the transmission impedance.

The optimal load resistance and reactance are thus given by:

$$\begin{aligned} R_{\ell maxP} &= |Z|\cos\phi \\ X_{\ell maxP} &= |Z|\sin\phi = R_{\ell maxP}\tan\phi \end{aligned}$$

As an illustration, Fig. 2.4 shows the load power P, the voltage V and the current magnitude I as a function of R_ℓ. An infinite R_ℓ corresponds to open-circuit conditions. As R_ℓ decreases, V drops while I increases. As long as R_ℓ remains larger than $R_{\ell maxP}$, the increase in I^2 gains over the decrease in R_ℓ and hence P increases. When R_ℓ becomes smaller than $R_{\ell maxP}$ the reverse holds true. Finally, $R_\ell = 0$ corresponds to short-circuit conditions.

Lossless transmission

Let us come back to the case where $R = 0$. The optimal load resistance under constant power factor is, according to (2.5):

$$R_{\ell maxP} = X\cos\phi$$

Substituting in (2.4) yields the maximum active power:

$$P_{max} = \frac{\cos\phi}{1+\sin\phi}\frac{E^2}{2X} \tag{2.6}$$

with the corresponding reactive power:

$$Q_{maxP} = \frac{\sin\phi}{1+\sin\phi}\frac{E^2}{2X} \tag{2.7}$$

and receiving-end voltage:

$$V_{maxP} = \frac{E}{\sqrt{2}\sqrt{1+\sin\phi}} \tag{2.8}$$

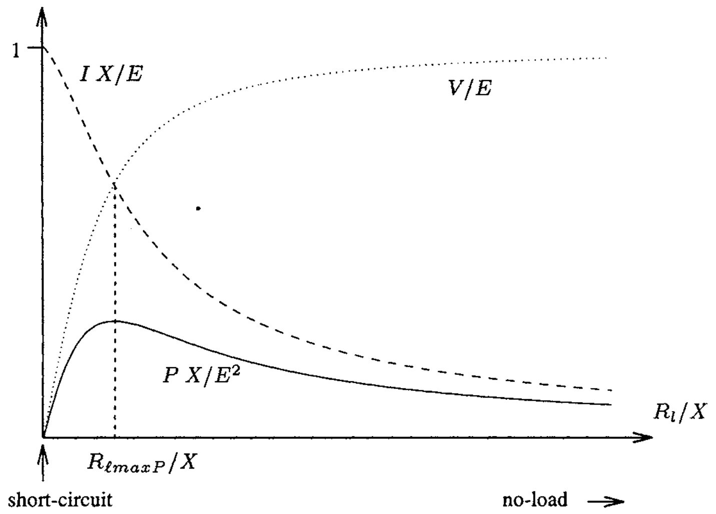


Figure 2.4 P, V and I as a function of R_ℓ, for a lossless system ($R = 0$) and under constant power factor ($\tan\phi = 0.2$)

Lossless transmission and unity power factor

If we assume furthermore that the load is perfectly compensated, so that $\cos\phi = 1$, the optimal resistance, maximum power and receiving-end voltage become respectively:

$$\begin{aligned} R_{\ell maxP} &= X \\ P_{max} &= \frac{E^2}{2X} \\ V_{maxP} &= \frac{E}{\sqrt{2}} \simeq 0.707E \end{aligned}$$

Extensions to multiport systems

Some generalizations of the above results to multiport systems are given in [Cal83]. Let a multiport circuit be characterized by

$$\bar{\mathbf{V}} = \bar{\mathbf{E}} + \mathbf{Z}\bar{\mathbf{I}}$$

where $\bar{\mathbf{V}}$ is the vector of terminal voltages, $\bar{\mathbf{E}}$ the vector of open-circuit voltages, $\bar{\mathbf{I}}$ the vector of injected currents and $\mathbf{Z}$ the (short-circuit) impedance matrix.

If the circuit is purely reactive, and characterized by $\mathbf{Z} = j\mathbf{X}$, it can be shown that the total active power delivered is maximized when a purely *resistive* network with impedance matrix $\mathbf{Z}_\ell = \mathbf{X}$ is connected to the multiport. The corresponding maximum power is easily obtained.

Furthermore if all the elements of the multiport matrix $\mathbf{Z}$ have the same argument ζ and all the elements of the loading matrix $\mathbf{Z}_\ell$ the same argument ϕ, i.e.

$$\mathbf{Z} = \mathbf{N}e^{j\,\zeta} \quad \text{and} \quad \mathbf{Z}_\ell = \mathbf{L}e^{j\,\phi}$$

the total active power is maximum when $\mathbf{N} = \mathbf{L}$.

Note that the individual load powers are not constrained with respect to each other in this derivation. If a pattern of load increase is specified, the maximum power delivered will be smaller. These aspects will be further discussed in Chapters 7 and 9.

Remark on load characteristics

Note that the maximum deliverable power given by either (2.3) or (2.6) depends only on the network parameters (R, X) and is independent of the load characteristic which was assumed to be that of an impedance for simplicity. This will be verified in the sequel, where no assumption will be made as to the nature of the load. For this purpose we now adopt a formulation in terms of powers.

2.2.3 Maximum power derived from load flow equations

For the sake of simplicity, we neglect the transmission resistance R (see Fig. 2.2). We also take the ideal voltage source as the phase reference by setting $\bar{E} = E\angle 0$. We denote the load voltage magnitude and phase angle by V and θ respectively.

One easily obtains from Fig. 2.2:

$$\bar{V} = \bar{E} - jX\bar{I}$$

The complex power *absorbed* by the load is:

$$\begin{aligned} S &= P + jQ = \bar{V}\,\bar{I}^\star = \bar{V}\,\frac{\bar{E}^\star - \bar{V}^\star}{-jX} \\ &= \frac{j}{X}(EV\cos\theta + jEV\sin\theta - V^2) \end{aligned} \tag{2.9}$$

which decomposes into:

$$P = -\frac{EV}{X}\sin\theta \tag{2.10a}$$

$$Q = -\frac{V^2}{X} + \frac{EV}{X}\cos\theta \tag{2.10b}$$

Equations (2.10a,b) are the *power flow* or *load flow* equations of the lossless system. For a given load (P, Q), they have to be solved with respect to V and θ, from which all other variables can be computed. Let us determine for which values of (P, Q) there is one solution.

Eliminating θ from (2.10a,b) gives:

$$(V^2)^2 + (2QX - E^2)V^2 + X^2(P^2 + Q^2) = 0 \tag{2.11}$$

This is a second-order equation with respect to V^2. The condition to have at least one solution is:

$$(2QX - E^2)^2 - 4X^2(P^2 + Q^2) \geq 0$$

which can be simplified into:

$$-P^2 - \frac{E^2}{X}Q + (\frac{E^2}{2X})^2 \geq 0 \tag{2.12}$$

The equality in (2.12) corresponds to a parabola in the (P, Q) plane, as shown in Fig. 2.5. All points "inside" this parabola satisfy (2.12) and thus lead to two load flow solutions. Outside there is no solution while on the parabola there is a single solution.

This parabola is the locus of all maximum power points. Points with negative P correspond to a maximum generation while each point with positive P corresponds to the maximum load under a given power factor, as derived in the previous section.

The locus is symmetric with respect to the Q-axis (i.e. with respect to changing P into $-P$). In other words, the maximum power that can be injected at the load end is exactly equal to the maximum power that can be absorbed. However, this symmetry disappears if one takes into account the line resistance.

Setting $P = 0$ in (2.12) one obtains:

$$Q \leq \frac{E^2}{4X}$$

Noting that E^2/X is the short-circuit power at the load bus, i.e. the product of the no-load voltage E by the short-circuit current E/X, the maximum of purely reactive load is one fourth of the short-circuit power.

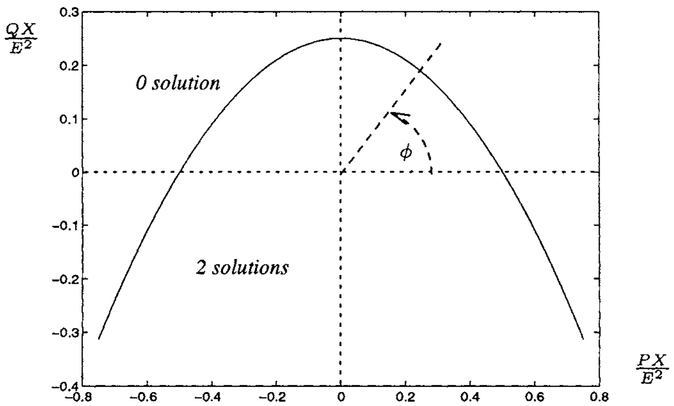


Figure 2.5 Domain of existence of a load flow solution

Similarly, by setting $Q = 0$ in (2.12) one gets:

$$P \leq \frac{E^2}{2X}$$

which is the same power limit we derived for a lossless line with unity power factor, and corresponds to half the short-circuit power.

As can be seen, there is a fundamental difference between the active and reactive powers: any active power can be consumed provided that enough reactive power is injected at the load bus ($Q < 0$), while the reactive load power can never exceed $E^2/4X$. This difference comes from the inductive nature of the transmission system and further illustrates the difficulty of transporting large amounts of reactive power. Note that in practice the large reactive support that is required for large active power will finally result in unacceptably high load bus voltage.

2.3 POWER-VOLTAGE RELATIONSHIPS

Assuming that condition (2.12) holds, the two solutions of (2.11) are given by:

$$V = \sqrt{\frac{E^2}{2} - QX \pm \sqrt{\frac{E^4}{4} - X^2P^2 - XE^2Q}} \tag{2.13}$$

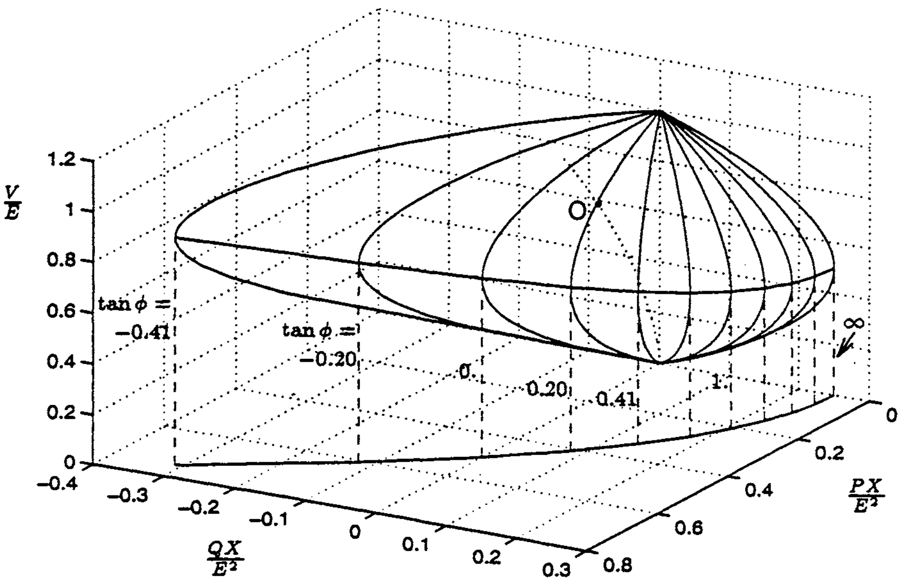


Figure 2.6 Voltage as a function of load active and reactive powers

In the (P, Q, V) space, equation (2.11) defines a two dimensional surface shown in Fig. 2.6. The upper part of this surface corresponds to the solution with the plus sign in (2.13), or the higher voltage solution, while the lower part corresponds to the solution with the minus sign, which is the low voltage one. The "equator" of this surface, along which the two solutions are equal corresponds to the maximum power points as given by (2.6, 2.7, 2.8). The projection of this limit curve onto the (P, Q) plane coincides with the parabola of Fig. 2.5.

The "meridians" drawn with solid lines in Fig. 2.6 correspond to intersections with vertical planes $Q = P \tan \phi$, for ϕ varying from $-\pi/8$ to $\pi/2$ by steps of $\pi/16$. Projecting these meridians onto the (P, V) plane provides the curves of load voltage as a function of active power, for the various $\tan \phi$. These famous curves, shown in Fig. 2.7, are generally referred to as the *PV curves* or *nose curves*. They play a major role in understanding and explaining voltage instability.

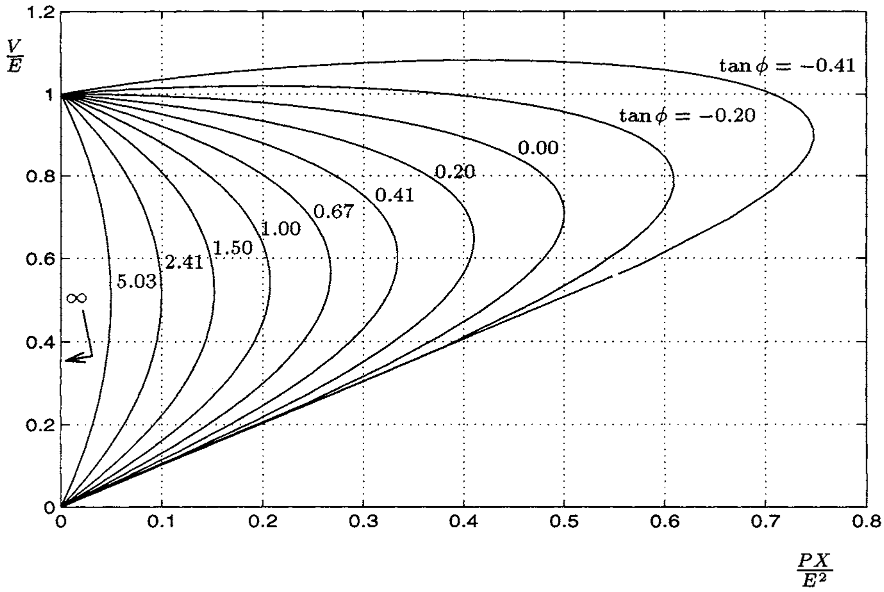


Figure 2.7 The famous PV curves

Although they are probably the most popular, the PV curves are not the only possible projection of the surface of Fig. 2.6 onto a plane. We could similarly:

- project the meridians onto the (Q, V) plane, thereby producing QV curves
- take the apparent power $S = \sqrt{P^2 + Q^2}$ as the abscissa, and consider SV curves
- consider QV curves corresponding to constant active power P
- or PV curves under constant reactive power Q.

All these curves have basically the shape shown in Fig. 2.7, the only difference being that curves drawn under constant P or constant Q do not go through zero voltage (except, of course, when the power held constant is equal to zero).

The following observations can be made regarding the curves of Fig. 2.7:

1. For a given load power below the maximum, there are two solutions: one with higher voltage and lower current, the other with lower voltage and higher current. The former corresponds to "normal" operating conditions, with voltage V closer to the generator voltage E. Permanent operation at the lower solutions is unacceptable, as will be discussed in the next section.

2. As the load is more and more compensated (which corresponds to smaller $\tan\phi$), the maximum power increases. However, the voltage at which this maximum occurs also increases. This situation is dangerous in the sense that maximum transfer capability may be reached at voltages close to normal operation values. Also, for a high degree of compensation and a load power close to the maximum, the two voltage solutions are close to each other and without further analysis it may be difficult to decide if a given solution is the "normal" one.

3. For over-compensated loads ($\tan\phi < 0$), there is a portion of the upper PV curve along which the voltage increases with the load power. The explanation is that under negative $\tan\phi$, when more active power is consumed, more reactive power is produced by the load. At low load, the voltage drop due to the former is offset by the voltage increase due to the latter. The more negative $\tan\phi$ is, the larger is the portion of the PV curve where this takes place.

2.4 GENERATOR REACTIVE POWER REQUIREMENT

In this chapter, generators are treated as voltage sources of constant magnitude. As will be discussed in the next chapter the main defect of this assumption lies in the limited reactive power capability of generators. It is therefore of interest to determine how the reactive generation increases with load.

Pursuing the example of Fig. 2.2, in the lossless case $R = 0$, we express the generator reactive production as the sum of the load and the network losses:

$$Q_g = Q + XI^2 \tag{2.14}$$

where the line current I relates to the generator apparent power S_g through:

$$I = \frac{S_g}{E} = \frac{\sqrt{P_g^2 + Q_g^2}}{E}$$

Substituting I in (2.14) and noting that $P_g = P$ in the absence of real power losses, we get:

$$Q_g = Q + \frac{X}{E^2}(P^2 + Q_g^2)$$

which can be reordered into:

$$Q_g^2 - \frac{E^2}{X}Q_g + \frac{E^2}{X}Q + P^2 = 0 \tag{2.15}$$

Solving this equation with respect to Q_g yields:

$$Q_g = \frac{E^2}{2X} \pm \sqrt{\left(\frac{E^2}{2X}\right)^2 - \frac{QE^2}{X} - P^2} \tag{2.16}$$

Note that (2.15) has a solution only when the condition (2.12) is satisfied. Equation (2.15) defines a surface in the (P, Q, Q_g) space. Cutting this surface with constant power factor planes - as we did in Fig. 2.6 - one obtains the $P\ Q_g$ curves shown in Fig. 2.8. These curves are similar to the PV curves, except that normal operating points now lie on the *lower* part of the curves. Starting from open-circuit conditions $(P = 0, Q_g = 0)$ and increasing the load, the reactive generation increases nonlinearly with P up to the maximum power. Beyond this point, P decreases while reactive losses continue to increase, up to the point $(P = 0, Q_g = \frac{E^2}{X})$ which corresponds to a short-circuit at the load bus. Note finally that all the maximum power points are characterized by:

$$Q_{g\,maxP} = \frac{E^2}{2X}$$

whatever the load power factor be.

2.5 A FIRST GLANCE AT INSTABILITY MECHANISMS

The purpose of this section is to emphasize why the existence of a maximum deliverable power may result in system instability and voltage collapse. We propose here some intuitive views, keeping a more rigorous analysis for later chapters.

2.5.1 Network vs. load PV characteristics

The power consumed by loads varies with voltage and frequency. In this book we will concentrate mainly on variations with voltage. We call *load characteristic* the expression of the load active and reactive power as a function of voltage V and an independent variable z, which corresponds to the amount of connected equipment. We

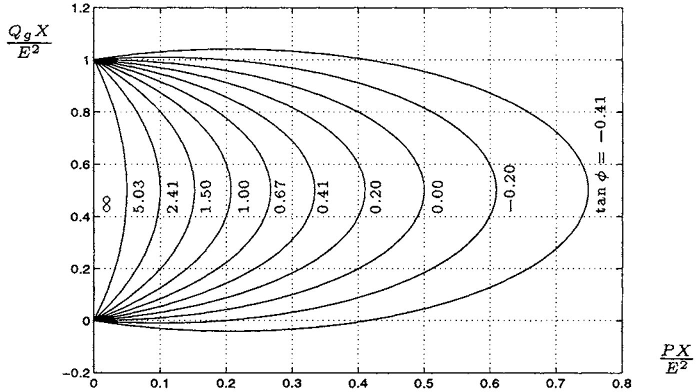


Figure 2.8 Generator reactive production as a function of load power

call z the *load demand.* Thus the load characteristic takes on the general form:

$$P = P(V, z) \tag{2.17a}$$

$$Q = Q(V, z) \tag{2.17b}$$

For a specified demand z, equations (2.17a,b) define a curve in the (P, Q, V) space. This curve intersects the $V(P, Q)$ surface at one or more points. These are possible operating points for the specified demand. When the latter changes, the intersection points move on the surface. If we project the set of intersection points for all values of the demand onto the (P, V) plane, we obtain what we call the *network PV characteristic* as opposed to the *load PV characteristic* given by (2.17a). Alternatively, we may project the set on the (Q, V) plane and consider the *load QV characteristic.* Note that the network characteristic cannot be defined without considering how the load power varies with voltage.

Consider for instance the widely used load characteristic known as the *exponential load model*:

$$P = zP_o\left(\frac{V}{V_o}\right)^{\alpha} \tag{2.18a}$$

$$Q = zQ_o\left(\frac{V}{V_o}\right)^{\beta} \tag{2.18b}$$

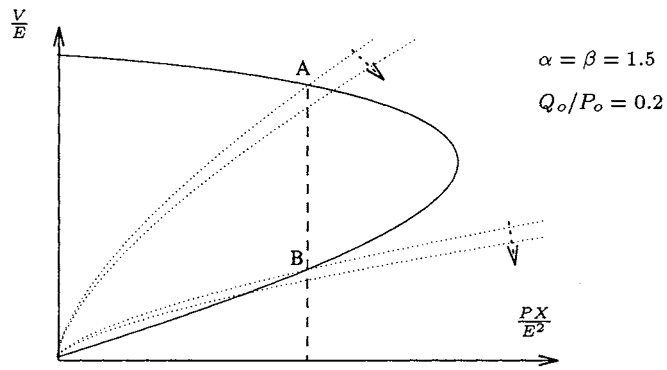


Figure 2.9 Network and load PV curves

In this model P_o (resp. Q_o) is the active (resp. reactive) power consumed for $z = 1$ and a voltage V equal to the reference voltage V_o. As an example, the dotted curve shown in Fig. 2.6 corresponds to (2.18a,b) with $\alpha = \beta = 1.5$ and $Q_o/P_o = 0.2$. It intersects the $V(P, Q)$ surface at point O and at the origin. As the demand z changes, so does the intersection point O. The set of points O for all possible demands, projected on the (P, V) plane is the solid line in Fig. 2.9. This is the network characteristic *corresponding to the assumed change in the load active and reactive power components*. In the above specific example:

$$\frac{Q}{P} = \frac{Q_o}{P_o}\left(\frac{V}{V_o}\right)^{\beta-\alpha}$$

and since $\alpha = \beta$ in this example, the load power factor is constant whatever the voltage. Hence, the network PV curve is merely the curve of Fig. 2.7 corresponding to $\tan\phi = 0.2$. This shortcut is no longer possible when $\alpha \neq \beta$.

2.5.2 Instability scenarios

Each dotted line in Fig. 2.9 is the load PV curve for some value of P_o. A and B are two operating points characterized by the same power P but different demands z.

Consider the effect of a small increase in demand z, as depicted in Fig. 2.9. At point A, the higher demand causes some voltage drop but results in a higher load power. This is the expected mode of operation of a power system. At point B however, the larger demand is accompanied by a decrease in *both* the voltage *and* the load power. If the load is purely static, operation at point B is possible, although perhaps non-viable due to low voltage and high current; this is however a matter of viability, not of stability. On the other hand, by assuming a load controller, or some inherent mechanism built in the load, that tends to increase the demand in order to achieve a specified power consumption, the operating point B becomes *unstable*. It will be shown in Chapter 4 that induction motors, load tap changers and heating thermostats are typical components which exhibit, directly or indirectly, the above behaviour.

Consider now a load which, following some disturbance, behaves instantaneously according to the dotted PV characteristic of Fig. 2.9 but tends dynamically to a constant power characteristic as given by the dashed line in the same figure. Anticipating a little about the dynamic notions of Chapter 5, we will say that this dashed vertical line is the *load equilibrium characteristic*, or *load steady-state characteristic*. Similarly, the network PV curve, if properly determined, corresponds to the equilibrium condition of the generation and transmission systems.

An obvious prerequisite to stable system operation is the existence of an equilibrium, given by the intersection of both characteristics. It happens precisely that an important class of voltage instability scenarios corresponds to changes in system parameters that lead to the disappearance of an equilibrium.

A first mechanism is illustrated in Fig. 2.10.a: an increase in demand causes the load equilibrium characteristic to change until finally it does not intersect the network characteristic.

A second, practically even more important scenario corresponds to a large disturbance. Disturbances of concern are the loss of transmission and/or generation equipments. In our two-bus example this corresponds to an increase in X and/or a decrease in E. The instability mechanism is depicted in Fig. 2.10.b: the large disturbance causes the network characteristic to shrink drastically so that the post-disturbance network PV curve does no longer intersect the (unchanged) load characteristic. Voltage collapse results from the loss of an equilibrium in the post-disturbance network.

Figure 2.11 illustrates the same two scenarios for a load characterized by $\alpha = \beta = 0.7$ (instead of $\alpha = \beta = 0$) at equilibrium.

Assuming a smooth load increase as in Fig. 2.10.a and 2.11.a, the point where the load characteristic becomes tangent to the network characteristic defines the *loadability limit* of the system. As mentioned above, a load increase beyond the loadability limit results in loss of equilibrium, and the system can no longer operate. In Fig. 2.10.a the point where the load and network PV curves are tangent coincides with the maximum

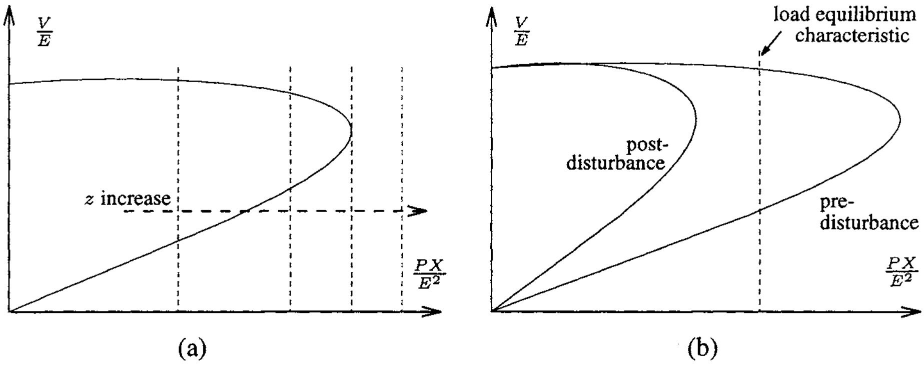


Figure 2.10 Instability mechanisms illustrated with PV curves; load equilibrium characteristic with $\alpha = \beta = 0$

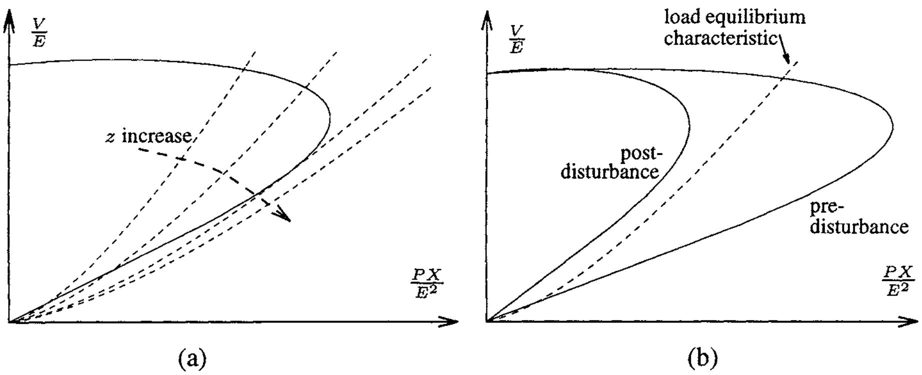


Figure 2.11 Instability mechanisms illustrated with PV curves; load equilibrium characteristic with $\alpha = \beta = 0.7$

deliverable power, because the load is assumed to restore to constant power, an important case in practice. However, a loadability limit does not necessarily coincide with the maximum deliverable power, since it depends on the load characteristic. This can be seen from Fig. 2.11.a. Note also that for certain load characteristics (e.g. the one in Fig. 2.9) there is no loadability limit, i.e. there is an operating point for all demands. Of course, some of these operating points may be infeasible for other reasons, such as unacceptably low voltage.

The load characteristics will be analyzed further in Chapter 4, while a more thorough discussion on loadability limits is left for Chapter 7.

The above scenarios do not tell us the course of events that occur as a result of the loss of equilibrium. They only tell us that, as far as network and load PV curves are the equilibrium characteristics of the system dynamics, system operation will experience a disruption. An in-depth investigation of the instability mechanism requires that we consider the dynamic behaviour of each component. Moreover, there are instability mechanisms that cannot be foreseen from purely static characteristics.

2.6 EFFECT OF COMPENSATION

Generally speaking, compensation consists of injecting reactive power to improve power system operation, more specifically keep voltages close to nominal values, reduce line currents and hence network losses, and contribute to stability enhancement [Mil82].

Most often compensation is provided by capacitors, counterbalancing the predominantly inductive nature of either the transmission system, or the loads. It may also consist of reactors where reactive power absorption is of concern.

Regarding voltage stability, the effects of *load compensation* have been discussed in Section 2.3. In this section we focus on *network compensation*, which may consist of either capacitors installed in series with transmission lines or shunt elements connected to system buses.

2.6.1 Line series compensation

Series compensation is used basically to decrease the impedance of transmission lines carrying power over long distances, as shown by the simple equivalent of Fig. 2.12 (the

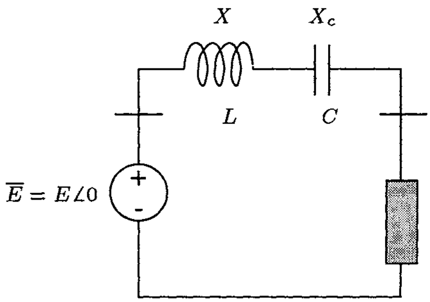


Figure 2.12 Series compensation

latter does not take into account the series capacitors location, e.g. at the mid-point or $1/3$ or $1/4$ points of the line).

The line net reactance is given by:

$$X_{net} = X - X_c = \omega L - \frac{1}{\omega C}$$

with the degree of compensation

$$\frac{X - X_{net}}{X} = \frac{X_c}{X}$$

being usually in the range 0.3 – 0.8.

Replacing X by X_{net} in (2.6, 2.8) it is clearly seen that the maximum deliverable power is increased, while the voltage under maximum power is left unchanged.

Series compensation addresses a fundamental aspect of voltage instability, namely the electrical distance between generation and load centers. In this respect it is a very efficient countermeasure to instability .

2.6.2 Shunt compensation

The connection of shunt capacitors (or reactors) is probably the simplest and most widely used form of compensation. To investigate its effect in some detail, we consider the simple system of Fig. 2.13, which combines the effect of line charging (susceptance B_l) with that of an adjustable shunt compensation (susceptance B_c).

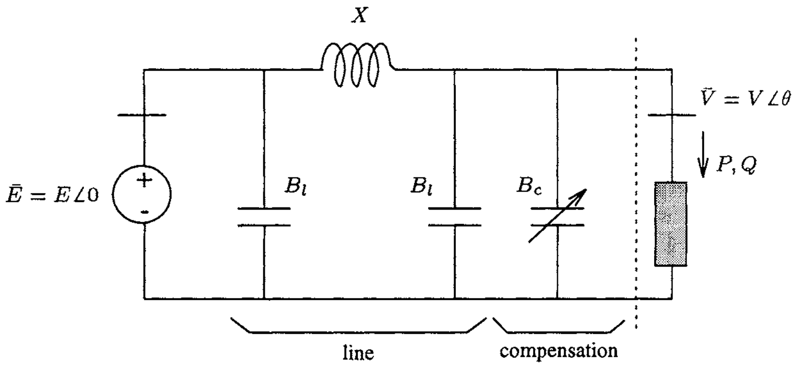


Figure 2.13 Network capacitances and shunt compensation

The Thévenin equivalent as seen by the load (i.e. to the left of the dotted line in Fig. 2.13) has the following emf and reactance:

$$E_{th} = \frac{1}{1-(B_c+B_l)X} E$$
$$X_{th} = \frac{1}{1-(B_c+B_l)X} X$$

Replacing E by E_{th} and X by X_{th} in (2.6, 2.8) gives the maximum deliverable power (under power factor $\cos\phi$):

$$P_{max} = \frac{\cos\phi}{1+\sin\phi}\frac{E_{th}^2}{2X_{th}} = \frac{1}{1-(B_c+B_l)X}\frac{\cos\phi}{1+\sin\phi}\frac{E^2}{2X}$$

and the corresponding load voltage:

$$V_{maxP} = \frac{E_{th}}{\sqrt{2}\sqrt{1+\sin\phi}} = \frac{1}{1-(B_c+B_l)X}\frac{E}{\sqrt{2}\sqrt{1+\sin\phi}}$$

A quick comparison with (2.6, 2.8) shows that both P_{max} and V_{maxP} increase by the same percentage when network capacitances are taken into account and/or capacitive compensation is added.

Figure 2.14 shows a situation where as load power increases, more shunt compensation has to be added in order to keep the voltage within the limits shown by the dotted lines (typically 0.95 and 1.05 pu respectively). The resulting PV curve is shown in heavy line in Fig. 2.14. Note that the addition of shunt compensation may come from an

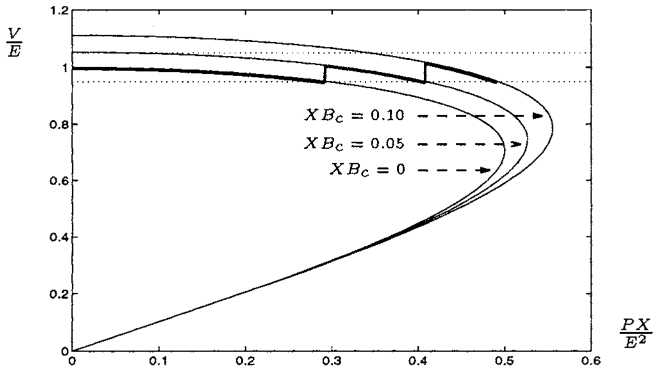


Figure 2.14 PV curves for various compensation levels

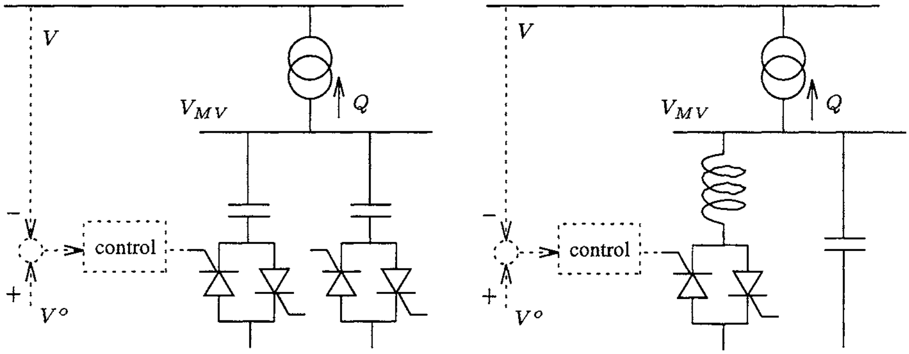
operator action or an automatic device. In the latter case, capacitors may be either mechanically switched or thyristor controlled.

The figure also illustrates a factor of critical importance in voltage instability. As the load grows in areas lacking generation, more and more shunt compensation is used to keep voltages in the normal operating range. By so doing, normal operating points progressively approach maximum deliverable power and in stressed conditions, the scenarios depicted by Figs. 2.10 and 2.11 could become a real threat.

Similarly, in systems with large capacitive effects, shunt reactors ($B_c < 0$) must be connected under light load conditions to avoid overvoltages. This is often the case in Extra High Voltage (EHV) systems where power transfers over long distances, limited by stability considerations, are below surge impedance loading. This requires shunt reactors to absorb the excess reactive power generated.

2.6.3 Static Var Compensators

Simply stated, a *Static Var Compensator* (SVC) is a voltage controlled shunt compensation device. In transmission system applications, the shunt susceptance connected to a Medium Voltage (MV) bus is quickly varied so as to maintain the voltage at a High Voltage (HV) or EHV bus (nearly) constant. SVCs are fast devices, acting typically over several cycles. The significantly higher cost of an SVC is justified when speed of



(a) Thyristor Switched Capacitor (TSC) (b) Thyristor Controlled Reactor (TCR)

Figure 2.15 Schematic representation of SVCs

action is required for stability improvement. This is the case in angle instability and short-term voltage instability problems. Beside voltage control, SVCs can be also used to damp rotor angle oscillations through additional susceptance modulation [Mil82].

The following are the main two techniques used to obtain a variable susceptance:

- in the *Thyristor Switched Capacitor* (TSC) (see Fig. 2.15.a) a variable number of shunt capacitor units are connected to the system by thyristors used as switches;
- in the *Thyristor Controlled Reactor* (TCR) (see Fig. 2.15.b), the firing angle of thyristors connected in series with a reactor is adjusted to vary the fundamental frequency component of the current flowing into this reactor, while the harmonics are filtered out by different techniques. This is equivalent to having a variable shunt reactor in parallel with a fixed capacitor.

In steady-state conditions, the reactive power produced by the SVC is given by:

$$Q = B\, V_{MV}^2 \tag{2.19}$$

where V_{MV} is the MV-bus voltage and B the variable susceptance. The latter obeys:

$$B = K(V_o - V) \tag{2.20}$$

subject to:

$$B^{min} \leq B \leq B^{max} \tag{2.21}$$

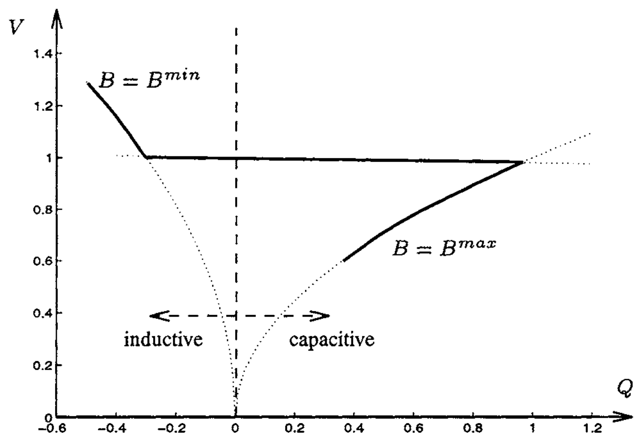


Figure 2.16 Steady-state characteristic of an SVC (B^{min} =- 0.3, B^{max} = 1., K = 50, V_o= 1 , in pu on the compensator rating)

where K is the SVC gain, V_o the voltage reference and B^{min}, B^{max} correspond to extreme thyristor conduction conditions.

The corresponding QV characteristic is shown with solid line in Fig. 2.16. The step-up transformer impedance has been neglected for simplicity (hence making $V = V_{MV}$ in per unit) but is taken into account in detailed simulation. The almost flat portion of the characteristic corresponds to (2.19) and (2.20). It is very close to a straight line with a small droop, due to the high value of K (of the order of 25–100 on the SVC rating). The parabolic parts correspond to (2.19) with B at one of the limits (2.21).

The term *Static Var System* (SVS) is used to designate the combination of an SVC with a mechanically switched capacitor [Mil82, Kun94]. Most often the rôle of the latter is to reset the SVC operating point so that the compensator is left with an adequate reactive reserve to face sudden disturbances.

Coming back to voltage stability considerations, consider the system of Fig. 2.13 with the adjustable capacitor replaced by an automatic SVC. With a TSC the network PV characteristic is close to that shown in Fig. 2.14, with the small steps corresponding to capacitor units successively switched in. With the continuously acting TCR, the characteristic becomes the heavy line in Fig. 2.17.

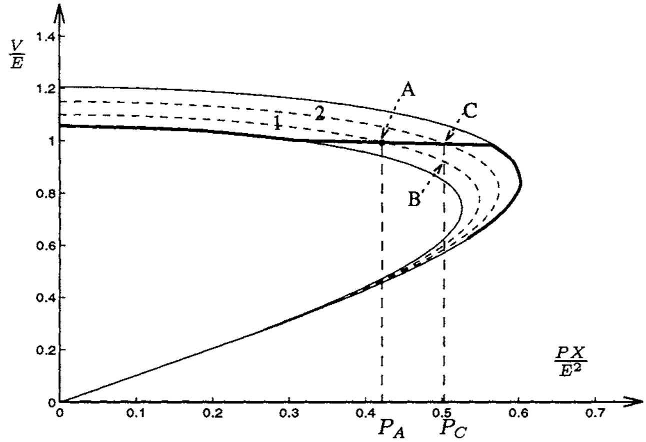


Figure 2.17 PV curves in the presence of an SVC

Assume for instance that the system is operating initially at point A on the dashed PV curve numbered 1 and that the load power is increased from P_A to P_C. In the absence of SVC reaction, the new operating point would be B. However this causes a voltage drop that the SVC will counteract by increasing its susceptance. In accordance with Fig. 2.14, the resulting network characteristic is the dashed PV curve numbered 2 and the new operating point is C. All points like A, C, etc. fall on the slightly sloping line, which corresponds to voltage control by the SVC. The slope of this line is dictated by the gain K. The two PV curves shown with solid line correspond to the susceptance limits (2.21).

As can be seen, the SVC significantly affects the shape of the network characteristic. Similar discontinuities, caused by generator reactive power limits, will be discussed in Section 3.6.

When limited, the SVC behaves as a mere shunt capacitor (or reactor), with the reactive power proportional to the square of the voltage. Comparatively, a better reactive support is offered by a synchronous generator or condenser under limit. Also more favorable, the recently proposed GTO-thyristor based STATic synchronous COMpensator (STATCOM) exhibits a constant current characteristic under limit [Gyu94].

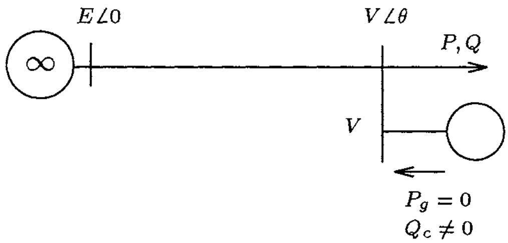


Figure 2.18 Use of a fictitious generator to produce VQ curves

2.7 VQ CURVES

A VQ curve expresses the relationship between the reactive support Q_c at a given bus and the voltage at that bus. It can be determined by connecting a fictitious generator with zero active power and recording the reactive power Q_c produced as the terminal voltage V is being varied [CTF87, MJP88]. Because it does not produce active power, this fictitious generator is often referred to as a synchronous condenser. Since voltage is taken as the independent variable, it is a common practice to use V as the abscissa and produce VQ instead of QV curves, as was done for loads earlier in this chapter. We will conform to this practice.

We illustrate the technique on the 2-bus example sketched in Fig. 2.18. The load flow equations (2.10a,b) become :

$$P = -\frac{EV}{X}\sin\theta \tag{2.22a}$$

$$Q - Q_c = -\frac{V^2}{X} + \frac{EV}{X}\cos\theta \tag{2.22b}$$

It must be noted at this point that the VQ curve is a characteristic of both the network and the load. As the curve aims at characterizing the steady-state operation of the system, the load must be accordingly represented through its steady-state characteristic. In this simple example we assume a constant power load.

For each value of the voltage V, θ is first obtained from (2.22a), then the reactive power Q_c is computed from (2.22b). Three such VQ curves are shown in Fig. 2.19. Curve 1 refers to system operation far below the maximum power. The two intersection points with the V-axis correspond to no compensation. Referring to a previous discussion, the higher voltage solution (marked O in Fig. 2.19) is the normal operating point. As can

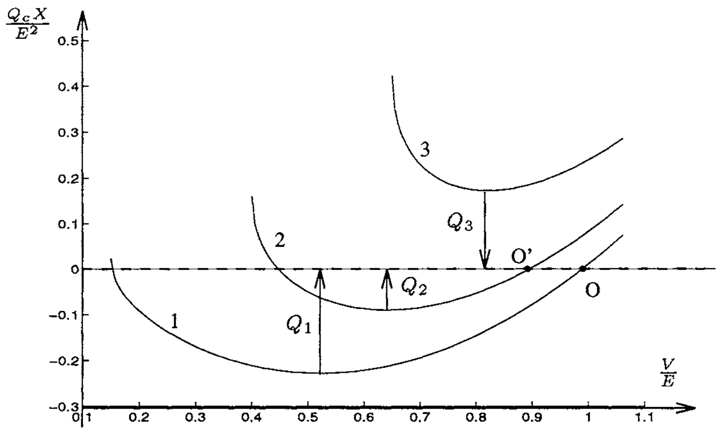


Figure 2.19 VQ curves

be seen, the VQ curve does not depart very much from a straight line around this point. Curve 2 refers to a more loaded situation. The operating point without compensation is O', where the curvature of the VQ curve is more pronounced. The Q_1 and Q_2 values shown in the figure are reactive power margins with respect to the loss of an operating point. These correspond to the minimum amount of reactive load increase (or equivalently generation decrease) for which there is no operating point any more. Finally curve 3 corresponds to a situation where the system cannot operate without reactive power injection. It might result from a severe disturbance that increases X. The shown margin Q_3 is negative and provides a measure of the Mvar distance to system operability.

VQ curves can help determining the amount of shunt compensation needed to either restore an operating point or obtain a desired voltage. We start for instance from Curve 3 of Fig. 2.19 and consider how an operating point can be restored using either a shunt capacitor or an SVC.

The case of introducing a shunt capacitor is shown in Fig. 2.20. The parabola $Q_c = BV^2$ corresponds to the minimal compensation needed to restore an operating point (denoted O) while the parabola $Q_c = B'V^2$ corresponds to the compensation needed to get the desired voltage V_d (point O'). In the latter case, the figure shows the reactive power reserve made available by the larger compensation used.

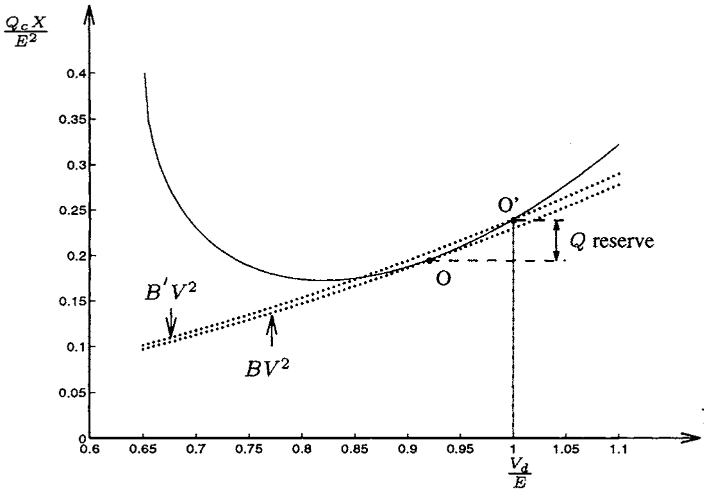


Figure 2.20 Shunt capacitor sizing based on VQ curves

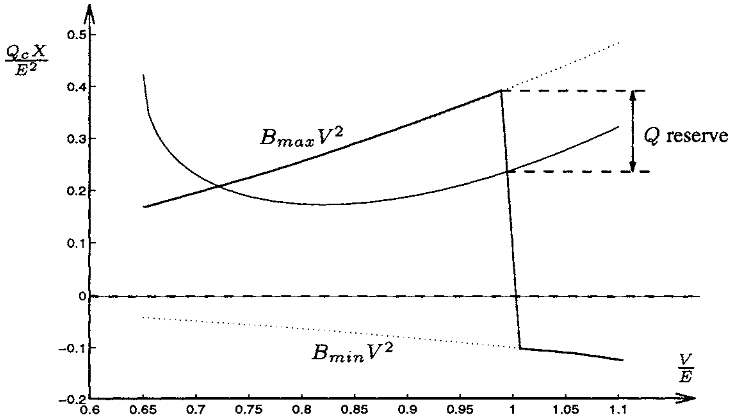


Figure 2.21 SVC sizing based on VQ curves

Note that when the reactive power source available is not producing a constant amount of Mvars the relation between Q_c and V must be taken into account to establish the reactive reserve. Thus, in the shunt capacitor case, point O does not correspond to the

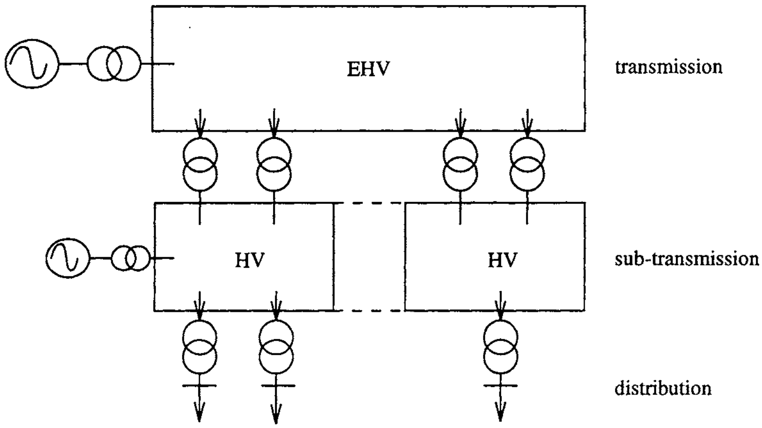


Figure 2.22 Two-level LTC structure

minimum of the VQ curve and the reactive reserve is not measured with respect to this minimum, but rather with respect to point O.

The use of an SVC is considered in Fig. 2.21. The steady-state characteristic of a device with production ($B = B_{max} > 0$) and absorption ($B = B_{min} < 0$) capabilities is shown with heavy lines. The chosen limit B_{max} leaves some reactive power reserve, as indicated in the figure.

2.8 EFFECT OF ADJUSTABLE TRANSFORMER RATIOS

Most contemporary power transmission systems are separated into different voltage levels. For instance a system may have a main transmission grid at EHV level, ranging typically from 220 to 735 kV, and a secondary transmission or sub-transmission network at HV level, with a nominal voltage from 60 to 150 kV. This two-level structure is sketched in Fig. 2.22.

It is common to have the transformers connecting the various levels equipped with Load Tap Changers (LTCs), i.e. devices which allow the turns ratio of the transformer to be adjusted without interrupting the power flow in the apparatus. Depending on the system, these can be found on :

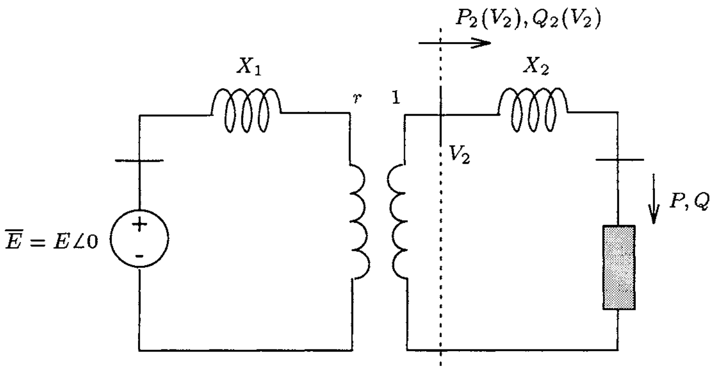


Figure 2.23 equivalent circuit showing the effect of transformer ratios

1. transformers feeding the distribution systems
2. transformers connecting sub-transmission to transmission
3. transformers connecting two transmission voltage levels
4. generator step-up transformers.

The first type of LTC is an important component of load dynamics and as such will be dealt with in Chapter 4. In this section we focus on the last three types and investigate their effects on network characteristics.

We first consider manual adjustments of the ratio, such as those performed remotely by control center operators.

Consider for this purpose the simple circuit of Fig. 2.23. The reactance X_1 on the "primary side" may represent either a transmission system equivalent reactance (Cases 2 and 3 above) or it may account for the generator voltage droop effect (Case 4). Similarly, X_2 may represent transmission and/or sub-transmission reactances. The transformer is assumed ideal, by incorporating its leakage reactance to X_2. In normal operating conditions, the ratio r is decreased (resp. increased) when an increase (resp. decrease) in voltage V_2 is sought.

The Thévenin equivalent as seen by the load has the following emf and reactance :

$$E_{th} = \frac{E}{r}$$

$$X_{th} = \frac{X_1}{r^2} + X_2$$

Replacing E by E_{th} and X by X_{th} into (2.6, 2.8) gives the maximum deliverable power (under power factor $\cos\phi$) :

$$P_{max} = \frac{1}{2}\frac{\cos\phi}{1+\sin\phi}\frac{E^2}{r^2X_2+X_1} \tag{2.23}$$

and the corresponding voltage :

$$V_{maxP} = \frac{E}{r\sqrt{2}\sqrt{1+\sin\phi}} \tag{2.24}$$

Comparing to the case without transformer, which corresponds to $r = 1$, we conclude that by decreasing r, so as to increase the secondary voltage V_2, more power can be delivered to the load. The larger the X_2/X_1 ratio, the more pronounced this effect. Formula (2.23) also shows that decreasing r is equivalent to decreasing the net impedance between the source and the load.

We consider now the case of an automatic LTC adjusting r in order to keep the secondary voltage V_2 equal to a setpoint value V_2^0. We neglect here the deadband and discrete step effects that characterize the real device and we ignore the limits on r. Conditions for steady-state operation of this system can be derived as follows.

The reactance X_2 together with the load make up a voltage sensitive load with power $P_2(V_2) + jQ(V_2)$ as shown in Fig. 2.23. By restoring the voltage V_2 to its setpoint V_2^0, the LTC restores the above power to the constant value:

$$\begin{aligned} P_2(V_2^0) &= P_2^0 \\ Q_2(V_2^0) &= Q_2^0 \end{aligned}$$

The same power enters the primary side of the (ideal) transformer, leading to the situation depicted by the left circuit in Fig. 2.24. This is possible only if the point (P_2^0, Q_2^0) lies within the corresponding feasible domain, which is limited by the parabola shown at the bottom left of Fig. 2.24 and derived as explained in Section 2.2.3.

Assuming that this condition is met, the voltage V_2 is equal to V_2^0 in steady-state. This is equivalent to replacing the transformer and its primary side by a voltage source V_2^0, as shown by the right circuit in Fig. 2.24. Again, the condition for this subsystem to operate is to have the point (P, Q) within the feasibility domain, which is limited by the parabola shown at the bottom right of Fig. 2.24.

As can be seen, the effect of the voltage controlling LTC is to "break" the electrical distance between the source and the load. Some systems have more than one level

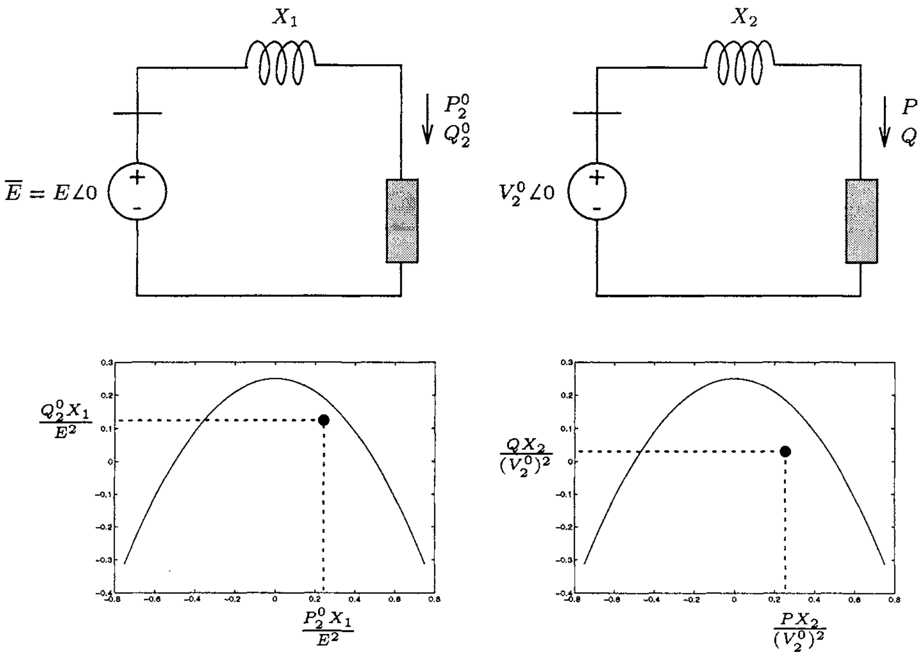


Figure 2.24 System of previous figure decomposed

of LTCs controlling voltages at intermediate points. The above reasoning applies to each level. The LTCs make it possible to operate the system with electrical distances between generators and loads that would otherwise not allow the power to be delivered to loads.

The dynamic interaction between various levels of LTCs in cascade will be discussed in Section 4.4.

2.9 PROBLEMS

2.1. Show that the extremum given by (2.2a,b) is indeed a maximum of P with respect to R_ℓ and X_ℓ.

2.2. Referring to Fig. 2.4, show that the value of the load resistance R_ℓ which maximizes the active power P also corresponds to the intersection of the IX/E and

Table 2.1 Typical data for transmission lines (2 conductors per phase)

nominal frequency	50 Hz	60 Hz
nominal voltage	380 kV	500 kV
X (see Fig. 2.13)	0.3 Ω/km	0.37 Ω/km
B_l (see Fig. 2.13)	1.5 μS/km	2.05 μS/km

V/E curves. Assume a lossless system and a constant load power factor, as in the figure.

2.3. In the (P, Q) plane of Fig. 2.5, determine the locus of operating points characterized by a given voltage magnitude V at the load bus. Show that, for $V > 0.5E$, this locus is tangent to the parabola of the figure at two points; determine these points. *Hint*: the second part does not require any calculation !

2.4. Derive the load flow equations of the 2-bus system when the transmission resistance R is not neglected. Determine the condition of existence of a solution. Draw the corresponding boundary in the (P, Q) plane.

2.5. With the load flow equations (2.10a,b) written in matrix form:

$$\mathbf{f}(\mathbf{x}) = 0 \qquad \text{with} \;\; \mathbf{x} = \begin{bmatrix} V \\ \theta \end{bmatrix}$$

the load flow Jacobian $\mathbf{J}$ is defined as the matrix of partial derivatives of $\mathbf{f}$ with respect to $\mathbf{x}$.
Show that this matrix is singular under maximum power conditions, for any load power factor (or equivalently, the Jacobian is singular at any point of the parabola of Fig. 2.5).

2.6. Consider the typical line characteristics given in Table 2.1. Assuming a 1 pu sending-end voltage and a unity power factor load, compute the maximum deliverable power as a function of line length. The correction for long lines will be neglected in a first approximation.
Compare with the line surge impedance loading.
Repeat for a 1.05 pu sending-end voltage.

2.7. Using the same data as in the previous problem, with a 1.05 pu sending-end voltage, determine the load power which results in a receiving-end voltage V=0.95 pu as a function of line length.
Repeat for various decreasing values of V.

2.8. Consider a lossless transmission line with a 1 pu sending-end voltage and shunt compensation at its receiving end so that the load voltage is always equal to 1 pu. Furthermore assume that the load dynamics is such that operation on the lower part of PV curve is unstable. What is the maximum admissible load power ?

2.9. Consider the 4-bus system shown in Fig. 3.20 (see Section 3.6). Its load flow data (on 100-MVA base) are as follows:

line A-B:	$X = 0.056250$ pu
	R and B_l neglected
line B-L:	$X = 0.005625$ pu
	R and B_l neglected
step-up transformer:	$X = 0.032$ pu
	open-circuit ratio $V_B/V_G = 1.04$ pu
	series resistance and shunt susceptance neglected
shunt at bus L:	$B_c = 0.25$ pu
generator G:	nominal apparent power = 250 MVA

Run a base case load flow corresponding to the following operating point:

load L:	$P = 2$ pu, $Q = 0.5$ pu
generator G:	$P = 2$ pu, $V = 1$ pu
generator G_∞:	slack-bus with $V = 1.04$ pu

Assuming that generator G (resp. G_∞) can be represented by a PV (resp. a slack-) bus and ignoring (presently: see Problem 3.6) any reactive power limit, obtain the PV curve at bus L using repeated load flow calculations. The load power factor will be kept constant and the slack-bus will compensate for the active power.

Hints: Use small enough steps to approach the maximum power point with reasonable accuracy. If your load flow allows for constant impedance load modelling, use this option to produce the lower part of the PV curve. The solution is the solid curve of Fig. 3.21, when using the generator equilibrium equations instead of the PV-bus approximation.

2.10. For the same system, draw the VQ curve at bus L for a load of resp. 200, 700 and 1400 MW (under constant power factor). Use the fictitious generator technique of Section 2.7, with Q_c corresponding to the reactive power injected by the shunt capacitor. Determine the amount of compensation needed to restore (i) a 1 pu voltage under the 700 MW load; (ii) an operating point at the 1400 MW level.

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