

Table of Contents

1. Vector Algebra	1
1.1 Vectors and scalars	1
1.1.1 Definition of a vector and a scalar	1
1.1.2 Addition of vectors	2
1.1.3 Components of a vector	3
1.2 Dot product	4
1.2.1 Applications of the dot product	7
1.3 Cross product	9
1.3.1 Applications of the cross product	11
1.4 Scalar triple product	14
1.5 Vector triple product	16
1.6 Scalar fields and vector fields	17
2. Line, Surface and Volume Integrals	21
2.1 Applications and methods of integration	21
2.1.1 Examples of the use of integration	21
2.1.2 Integration by substitution	22
2.1.3 Integration by parts	23
2.2 Line integrals	25
2.2.1 Introductory example: work done against a force	25
2.2.2 Evaluation of line integrals	26
2.2.3 Conservative vector fields	28
2.2.4 Other forms of line integrals	30
2.3 Surface integrals	31
2.3.1 Introductory example: flow through a pipe	31
2.3.2 Evaluation of surface integrals	33
2.3.3 Other forms of surface integrals	38
2.4 Volume integrals	39

2.4.1	Introductory example: mass of an object with variable density	39
2.4.2	Evaluation of volume integrals	40
3.	Gradient, Divergence and Curl	45
3.1	Partial differentiation and Taylor series	45
3.1.1	Partial differentiation	45
3.1.2	Taylor series in more than one variable	47
3.2	Gradient of a scalar field	48
3.2.1	Gradients, conservative fields and potentials	51
3.2.2	Physical applications of the gradient	52
3.3	Divergence of a vector field	53
3.3.1	Physical interpretation of divergence	56
3.3.2	Laplacian of a scalar field	56
3.4	Curl of a vector field	58
3.4.1	Physical interpretation of curl	60
3.4.2	Relation between curl and rotation	61
3.4.3	Curl and conservative vector fields	61
4.	Suffix Notation and its Applications	65
4.1	Introduction to suffix notation	65
4.2	The Kronecker delta δ_{ij}	68
4.3	The alternating tensor ϵ_{ijk}	70
4.4	Relation between ϵ_{ijk} and δ_{ij}	72
4.5	Grad, div and curl in suffix notation	74
4.6	Combinations of grad, div and curl	76
4.7	Grad, div and curl applied to products of functions	78
5.	Integral Theorems	83
5.1	Divergence theorem	83
5.1.1	Conservation of mass for a fluid	85
5.1.2	Applications of the divergence theorem	87
5.1.3	Related theorems linking surface and volume integrals ..	88
5.2	Stokes's theorem	91
5.2.1	Applications of Stokes's theorem	93
5.2.2	Related theorems linking line and surface integrals	95
6.	Curvilinear Coordinates	99
6.1	Orthogonal curvilinear coordinates	99
6.2	Grad, div and curl in orthogonal curvilinear coordinate systems	104
6.2.1	Gradient	104
6.2.2	Divergence	105

6.2.3	Curl	106
6.3	Cylindrical polar coordinates	107
6.4	Spherical polar coordinates	110
7.	Cartesian Tensors	115
7.1	Coordinate transformations	115
7.2	Vectors and scalars	117
7.3	Tensors	119
7.3.1	The quotient rule	120
7.3.2	Symmetric and anti-symmetric tensors	122
7.3.3	Isotropic tensors	123
7.4	Physical examples of tensors	126
7.4.1	Ohm's law	126
7.4.2	The inertia tensor	127
8.	Applications of Vector Calculus	131
8.1	Heat transfer	132
8.2	Electromagnetism	134
8.2.1	Electrostatics	135
8.2.2	Electromagnetic waves in a vacuum	137
8.3	Continuum mechanics and the stress tensor	140
8.4	Solid mechanics	143
8.5	Fluid mechanics	145
8.5.1	Equation of motion for a fluid	146
8.5.2	The vorticity equation	147
8.5.3	Bernoulli's equation	149
	Solutions	153
	Index	181



<http://www.springer.com/978-3-540-76180-8>

Vector Calculus

Matthews, P.C.

1998, X, 182 p. 1 illus., Softcover

ISBN: 978-3-540-76180-8