

Contents

Preface	ix
Chapter I Review	1
I.1 Introduction	1
I.2 Notation	3
I.3 The Theorem of Stone–Weierstraß	8
I.4 The Riemann Integral	17
I.5 An Integrability Criterion	21
I.6 The Permanence Properties	23
I.7 Seminorms	28
Chapter II Extension of the Integral	31
II.1 Σ -additivity	33
II.2 Elementary Integrals	35
II.3 The Daniell Mean	42
II.4 Negligible Functions and Sets	48
II.5 Integrable Functions	52
II.6 Extending the Integral	58
II.7 Integrable Sets	61
II.8 Example of a Non-Integrable Function	66
Chapter III Measurability	69
III.1 Littlewood’s Principles	70
III.2 The Permanence Properties	73
III.3 The Integrability Criterion	75
III.4 Measurable Sets	81
III.5 Baire and Borel Functions	85
III.6 Further Properties of Daniell’s Mean	92
III.7 The Procedures of Lebesgue and Carathéodory	95

Chapter IV The Classical Banach Spaces 101

IV.1 The p -norms 102

IV.2 The $\mathcal{L}^p$ -spaces 105

IV.3 The L^p -spaces 108

IV.4 Linear Functionals 112

IV.5 The Dual of L^p 118

IV.6 The Hilbert space L^2 122

Chapter V Operations on Measures 125

V.1 Products of Elementary Integrals 125

V.2 The Theorems of Fubini and Tonelli 130

V.3 An Application: Convolution 133

V.4 An Application: Marcinkiewicz Interpolation 134

V.5 Signed Measures 137

V.6 The Space of Measures 144

V.7 Measures with Densities 151

V.8 The Radon–Nikodym Theorem 154

V.9 An Application: Conditional Expectation 155

V.10 Differentiation 157

Appendix A Answers to Selected Problems 165

References 180

Index of Notations 181

Index 184



<http://www.springer.com/978-3-0348-0054-9>

Integration - A Functional Approach

Bichteler, K.

1998, VIII, 197 p., Softcover

ISBN: 978-3-0348-0054-9

A product of Birkhäuser Basel