Metric Tensor Components in the Oblique-Angled Coordinates

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■ Impressum

This Mathematica-Notebook is part of the book entitled
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■ General Description

This Notebook enables the user to compute the analytic expressions for the metric tensor components g_{ij} as well as the expressions for the covariant basis vectors e^1 and e^2 in terms of the contravariant basis vectors e_1 and e_2 in the oblique-angled coordinate system.

■ User's Guide

■ Step 1

Compile the program cell entitled "TheDefinition of the Oblique-Angled Coordinates" (see the Section "Program Listing").

■ Step 2

Compile the program cell entitled "The Partial Derivatives $\partial_x x_m$, $i, m = 1, 2$." (see the Section "Program Listing").

Step 3

Compile the program cell entitled "The Expressions for Metric Tensor Components"
(see the Section "Program Listing").

Step 4

Compile the program cell entitled "The Covariant Components g^{ij}, i,j = 1,2"
(see the Section "Program Listing").

Step 5

Compile the program cell entitled "The Expressions for the Covariant Basis Vectors e^1 and e^2
in Terms of the Contravariant Basis Vectors e_1 and e_2".

Program Listing

The Definition of the Oblique-Angled Coordinates

The quantities x[1] and x[2] are the Cartesian coordinates;
the quantities k[1] and k[2] are the oblique-angled coordinates.

```
x[1] = k[1] + k[2] Cos[j]; x[2] = k[2] Sin[j] ;
```

The Partial Derivatives $\partial_{\xi}\, x_m$, $i, m = 1, 2.$

```
g = IdentityMatrix[2];
Do[ Do[ g0 = Sum[D[x[m],k[i]]*D[x[m],k[j]],{m, 2}];
g = ReplacePart[g, g0, {i,j}], {i, 2}], {j, 2}];
```

The Expressions for Metric Tensor Components

```
Do[ Do[gj = g[[i, j]] /. {k[1] -> z1, k[2] -> z2};
gj = TrigExpand[TrigReduce[gj]];
 g = ReplacePart[g, Expand[gj], {i, j}], {j, 2}], {i, 2}];
Print["g = ", TraditionalForm[ MatrixForm[g]]];
```

$$g = \begin{pmatrix} 1 & \cos(j) \\ \cos(j) & 1 \end{pmatrix}$$

■ The Covariant Components g^{ij}, i,j = 1,2

```
ginv = Inverse[g];
Print[TraditionalForm[MatrixForm[ginv]]];
```

$$\begin{pmatrix} \frac{1}{1-\cos^2(j)} & -\frac{\cos(j)}{1-\cos^2(j)} \\ -\frac{\cos(j)}{1-\cos^2(j)} & \frac{1}{1-\cos^2(j)} \end{pmatrix}$$

■ The Expressions for the Covariant Basis Vectors $\vec{e}^{\,1}$ and $\vec{e}^{\,2}$ in Termsof the Contravariant Basis Vectors $\vec{e}_1$ and $\vec{e}_2$

```
ee = {};
Do[ei = Sum[ginv[[i, m]] * B[m], {m, 2}];
AppendTo[ee, ei], {i, 2}];
ClearAll[e];
  Do[ei = ee[[i]] /. {B[1] -> e⃗1, B[2] -> e⃗2};
 ei = ei /. {Cos[j]^2 -> 1 - Sin[j]^2};
  Print["e^", i, " = ", TraditionalForm[ei]], {i, 2}];
```

e^1 = $\csc^2(j)\,\vec{e}_1 - \cot(j)\csc(j)\,\vec{e}_2$

e^2 = $\csc^2(j)\,\vec{e}_2 - \cot(j)\csc(j)\,\vec{e}_1$

■ The Structure of the Output

The following results are obtained with the aid of the above program:

(1) the expression for the metric tensor g in the form of a 2x2 matrix;
(2) the expression for the covariant components of the metric tensor g in the form of a 2x2 matrix;
(3) the expressions for the covariant basis vectors e^1 and e^2 in terms of the contravariant basis vectors e_1 and e_2.