Incompressible Fluid Flow Around a Plate

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■ Impressum

This Mathematica-Notebook is part of the book entitled
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■ General Description

This Notebook implements with *Mathematica* the analytical solution (4.2.29), (4.2.30) of the problem of a circulatory planar stationary flow of an incompressible fluid around a plate..

The formulation of this problem and the discussion of its solution may be found in Section 4.2.3 of the above book.

■ User's Guide

■ Step 1

Load and compile the program file beginning with the line

```
ClearAll[y, w, Plate];
```

(see the Section "Program Listing")

■ Step 2

Specify the input data by entering them in the line (see also Section "Examples of the Input Data" below)

```
PlateFlow[1, 1, p / 4, -4, 4, -3, 3, 40]
```

Then click in this line and wait for the result of numerical computation. The meaning of the input parameters is explained in the Section "Parameters Used in Program prog4-4.nb".

■ Program Listing

```
ClearAll[y, w, Plate];
(* --- The complex potential w(z) --- *)
w[z_] := 1/2*v1*(ca - I*sa)*(z + Sqrt[z^2 - a1^2]) +
  1/2* v1*(ca + I*sa)*(z - Sqrt[z^2 - a1^2]) +
  (G/(2*Pi*I)) * Log[(z + Sqrt[z^2 - a1^2])/a1];

y[x_, y_] := (z1 = w[z] /. z -> x + I y;
ComplexExpand[Im[z1]] // Expand // Simplify );

PlateFlow[a_, v_, a_, xl_, xr_, yb_, yt_, npp_] :=
( ca = Cos[ a]; sa = Sin[ a];
  a1 = a; v1 = v; G = -2*Pi*a1* v1*sa;
Print["y(x, y) = ", TraditionalForm[y[x, y]]];
lis = {{-a, -0.05 a}, {-a, 0.05 a}, {a, 0.05 a}, {a, -0.05 a},
       {-a, -0.05 a}};
obj = Polygon[lis];
streamlin1 = ContourPlot[y[x, -y], {x, xl, -0.01}, {y, yb, yt},
         PlotPoints -> npp,
         Contours -> {-1.5, -1, -0.5, -0.25, 0, 0.25, 0.5, 1, 1.5},
         ContourShading -> False,
         ContourSmoothing -> Automatic,
         DisplayFunction -> Identity];
streamlin2 = ContourPlot[y[x, y], {x, 0.01, xr}, {y, yb, yt},
       PlotPoints -> npp,
       Contours -> {-1.5, -1, -0.5, -0.25, 0, 0.25, 0.5, 1, 1.5},
       ContourShading -> False,
       ContourSmoothing -> Automatic,
       DisplayFunction -> Identity];
   plate = Graphics[{RGBColor[0.1, 1.0, 1.0], obj}];
bound = ListPlot[lis, PlotJoined -> True,
                      DisplayFunction -> Identity];
gr = Show[streamlin1, streamlin2, plate, bound,
          AspectRatio -> Automatic,
          Axes -> True,
          AxesLabel -> {"x", "y"},
          DisplayFunction -> $DisplayFunction];  )
```

■ Parameters Used in Program prog4-4.nb

Parameter	Description
a	2a is the plate length; the plate lies in the interval -a <= x <=a, a > 0;
v	the magnitude of the freestream velocity, v > 0;
a	the angle between the vector of freestream velocity and the positive direction of the x axis; the value of a is specified in radians;
xl	the abscissaa of the left boundary of a rectangular region D in the (x,y) plane, in which the fluid flow is to be determined; |xl| > a ;
xr	the abscissa of the right boundary of region D, xr > a ;
yb	the ordinate of the lower (bottom) boundary of region D, yb < 0;
yt	the ordinate of the upper boundary of region D, yt > 0;
npp	the number of points of each streamline for the graphics function ContourPlot, npp > 10.

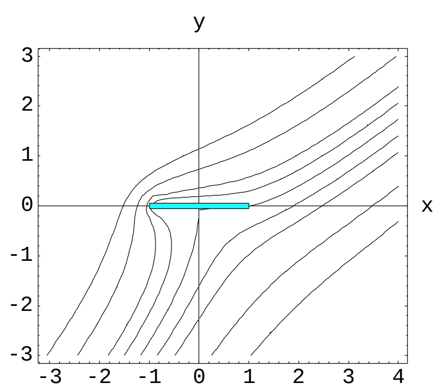
■ Examples of the Input Data

■ Example 1

```
PlateFlow[1, 1, p/4, -4, 4, -3, 3, 40]
```

$$y(x,\ y) = \frac{1}{\sqrt{2}}$$

$$\left(y - \sqrt{|(x + \text{‰} y)^2 - 1|}\ \cos\left(\frac{1}{2}\ \mathrm{Arg}((x + \text{‰} y)^2 - 1)\right) + \log\left(\left|x + \text{‰} y + \sqrt{(x + \text{‰} y)^2 - 1}\right|\right)\right)$$



■ Example 2

```
PlateFlow[1, 1, p/5, -4, 4, -3, 3, 40]
```

■ Example 3

```
PlateFlow[1, 1, p / 3, -4, 4, -3, 3, 40]
```

■ The Structure of the Output

The result of the work of the above program is the graphics picture showing both the plate and the fluid streamlines.

To resize an individual picture obtained by *Mathematica*, please

(i) Click anywhere inside the cell, but not the cell bracket itself. A bounding box with small handles appears around the graphic image.

(ii) Drag one of the handles to adjust the size and shape of the bounding box. In this way it is possible to resize the height and width of the graphic image.