V-Shaped Wing

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■ Impressum

This Mathematica-Notebook is part of the book entitled
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■ General Description

The program prog6-4.nb, which follows below, enables the user to construct the solution of a problem of three-dimensional supersonic inviscid compressible gas flow around a V-shaped wing. The functions uxyz[...] and P[...] presented in this program enable the user to compute the components of the velocity vector and the gas pressure at any point (x,y,z) of a spatial region containing the V-shaped wing.

It is assumed in the present program that the dependent variables u, v, w, P, r are nondimensional. Let us denote the dimensional variables by u, v, w, P, r, and denote the corresponding nondimensional variables by $\bar{u}$, $\bar{v}$, $\bar{w}$, $\bar{P}$, $\bar{r}$. Then the following relations are valid:

$$\bar{u} = u / u_{\text{ref}},\ \bar{v} = v / u_{\text{ref}},\ \bar{w} = w / u_{\text{ref}},\ \bar{P} = P / P_\infty,\ \ \bar{r} = r / r_\infty,$$

where P_∞ and r_∞ are the dimensional freestream values of the pressure and density, respectively; $u_{\text{ref}} = c_\infty / \sqrt{g}$, c_∞ is thedimensional freestream sound speed, $c_\infty = \sqrt{gP_\infty / r_\infty}$.

We use in the present program the built-in *Mathematica* function PlotVectorField3D[...]. It assumes that the z axis is the vertical axis. However, it is adopted in Section 6.5 of the above cited book that the z axis lies in the wing plane, and the y axis is the vertical axis. For the convenience of the use of the *Mathematica* function PlotVectorFiled3D it is assumed in the below program that the x axis is directed along the external rib of the wing (as in Section 6.5 of the book), however, the z axis is the vertical axis lying in the symmetry plane of the wing. Thus, the y axis is directed in our program along the wing span plane in such a way that the coordinate system Oxyz forms a right coordinate system.

The formulation of this problem and the discussion of its solution can be found in Section 6.5 of the above mentioned book.

■ User's Guide

■ Step 1

Load and compile the program file beginning with the line

```
uxyz[x_, y_, z_] := ( ux = u1; uy = 0; uz = 0;
```

(see the Section "Program Listing")

■ Step 2

Specify the input data by entering them in the line (see also Section "Examples of the Input Data" below)

```
Wing[p / 4, p / 4, 2.2, 1.4, -0.2, 1.0, 0.4]
```

Then click in this line and wait for the result of numerical computation.
The meaning of the input parameters is explained in the Section "Parameters Used in Program prog6-4.nb".

■ Program Listing

```
uxyz[x_, y_, z_] := ( ux = u1; uy = 0; uz = 0;
zw = x k2 - Abs[y] k3; zs = - x k4;
If[(zs < z < zw) && Abs[y] < k x, ux = u2s; uy = 0;
    uz = w2s]; {ux, uy, uz});

P[x_, y_, z_] := (p1 = 1;
zw = x k2 - Abs[y] k3; zs = - x k4;
If[(zs < z && z < zw) && Abs[y] < k x, p1 = P2]; p1 );

cl2[f_] := Hue[1.2 * (Af * f + Bf)];

Wing[b_, w_, M1_, g_, xl_, xr_, h0_] :=
(    (* --- The determination of q --- *)
t1 = 1 / (Tan[b] ((g + 1) * M1^2 / (2 * (M1^2 * Sin[b]^2 - 1)) - 1));
 q = - ArcTan[t1];
M2 = N[Sqrt[(2 + (g - 1) M1^2) / (2 g M1^2 Sin[b]^2 - (g - 1)) +
     2 M1^2 Cos[b]^2 / ((g - 1) M1^2 Sin[b]^2)]];
   P2 = 1 + (2 g / (g + 1)) * (M1^2 Sin[b]^2 - 1);
  If[P2 < 1, Print["P2 is less than 1"];
Print["Please re-specify beta or M1"]; Interrupt[]];
   r2 = 1 + 2 (M1^2 Sin[b]^2 - 1) / ((g - 1) * M1^2 * Sin[b]^2 + 2);
   c2 = √(g P2 / r2); q2 = M2 c2;
  Print["theta = ", q, ", M2 = ", M2, ", P2 = ", P2];
     u2s = q2 Cos[q]; w2s = q2 Sin[q]; u1 = M1 * Sqrt[g];
  Print["u2s = ", u2s, ", w2s = ", w2s];
   (* -- The 3 D plot of the wing surface --- *)
  k = Tan[b] Tan[w]; k1 = Cot[w]; k4 = Tan[b];
  yr = 1.2 * xr * k; yl = -yr;
  gu = ParametricPlot3D[{x, x u, -Abs[x u] k1}, {x, 0, xr},
              {u, -k, k},
              PlotPoints Æ 11,
              Boxed Æ False,
              Axes Æ False,
              DisplayFunction Æ Identity];
  k2 = -t1; k3 = k1 (1 + k2 / k); zb = -1.2 xr k4;
  gb = ParametricPlot3D[{x, x u, x k2 - Abs[x u] k3}, {x, 0, xr},
```

```
                        {u, -k, k},
              PlotPoints Æ 11,
              Boxed Æ False, Axes Æ False,
              DisplayFunction -> Identity];
     surf = Show[gu, gb, DisplayFunction -> Identity];
        << Graphics`PlotField3D`;
       g5 = PlotVectorField3D[uxyz[x5, y5, z5], {x5, xl, xr},
            {y5, yl, yr}, {z5, zb, 0.3},
            PlotPoints -> 5,
            VectorHeads -> True, DisplayFunction -> Identity];
      Show[g5, surf, DisplayFunction -> $DisplayFunction];
      (* The color map of the pressure field in section y = 0 *)
      zwr = -t1*xr; zsr = -k4*xr; ywr = k*xr;
        Af = h0/(1 - P2); Bf = -Af*P2;
       obj = Polygon[{{0, 0}, {xr, 0}, {xr, zwr}, {0, 0}}];
       wall = Graphics[{RGBColor[0.3, 0.9, 1], obj}];
       gp1 = Graphics[{cl2[1], Rectangle[{xl, zsr}, {xr, 0.3}]}];
       gp2 = Graphics[{cl2[P2], Polygon[{{0, 0}, {xr, zwr},
             {xr, zsr}, {0, 0}}]}];
        Show[gp1, gp2, wall];
       (* The color map of the pressure field in section x = xr *)
      wall = Graphics[{RGBColor[0.3, 0.9, 1], Polygon[{{-ywr, zsr},
            {0, 0}, {ywr, zsr}, {0, zwr}, {-ywr, zsr}}]}];
       gp1 = Graphics[{cl2[1], Rectangle[{-ywr, zsr}, {ywr, 0.3}]}];
       gp2 = Graphics[{cl2[P2], Polygon[{{-ywr, zsr}, {0, zwr},
                       {ywr, zsr}, {-ywr, zsr}}]}];
       Show[gp1, wall, gp2];)
```

■ Parameters Used in Program prog6-4.nb

Parameter	Description
b	the angle between the shock wave and the x-axis; the angle b > 0 and is specified in radians
w	the angle between the upper wing facet and the z axis, 0 < w < p /2
M1	the freestream Mach number; M1 > 1
g	the ratio of the gas specific heats; g > 0
xl	the x-coordinate of the left end of the interval on the x-axis, in which the gas flow is considered; xl < 0 (at x = 0, the wing nose tip is located)
xr	the x-coordinate of the right end of the interval on the x-axis, in which the gas

	flow is considered; xr >0
h0	the number in the interval 0 < h0 <= 1 to specify the color corresponding to the minimum value of the function $P(x, y)/P_1$ in the spatial region under consideration

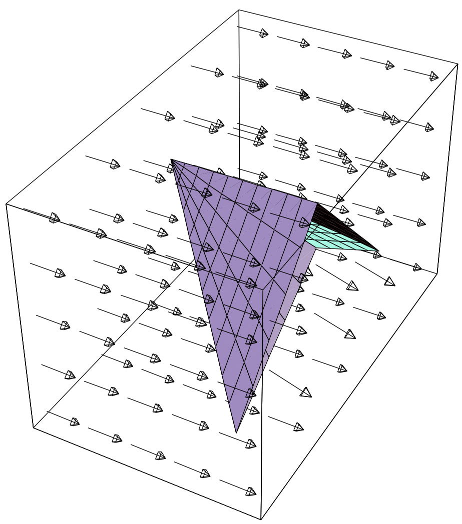
■ Examples of the Input Data

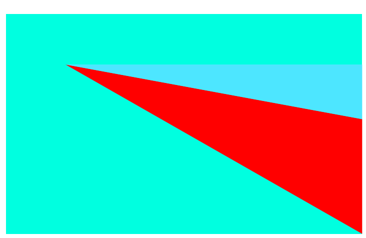
■ Example 1

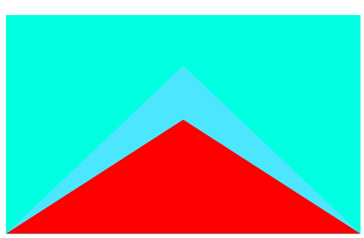
```
Wing[p/4, p/4, 2.2, 1.4, -0.2, 1.0, 0.4]
```

```
theta = -0.312974, M2 = 2.37009, P2 = 2.65667

u2s = 3.10877, w2s = -1.00603
```





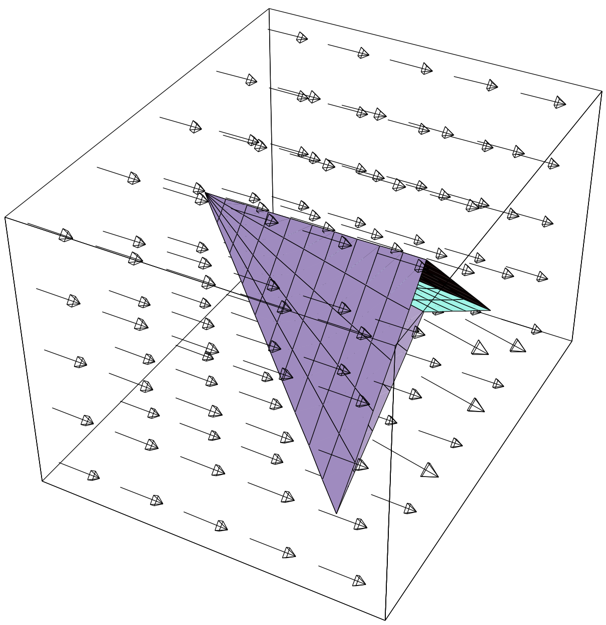


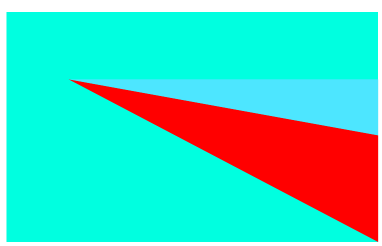
■ Example 2

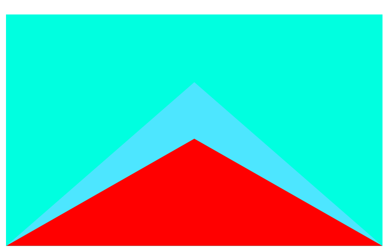
```
Wing[p / 5, p / 4, 2.5, 1.4, -0.2, 1.0, 0.4]
```

```
theta = -0.246537, M2 = 3.20455, P2 = 2.35254

u2s = 4.19244, w2s = -1.05505
```







■ The Structure of the Output

The following information is printed out in the process of the work of the program:

1. The angle q between the rib of the lower wing surface and the x-axis; the Mach number M_2, and the pressure P_2 behind the oblique shock wave front.

2. The field of the velocity vectors in the flow around the V-shaped wing.
 degrees;

3. The color map of the pressure field $P(x, 0, z)$;

4. The color map of the pressure field $P(\mathrm{xl}, y, z)$.

To resize an individual picture obtained by *Mathematica*, please

(i) Click anywhere inside the cell, but not the cell bracket itself. A bounding box with small handles appears around the graphic image.

(ii) Drag one of the handles to adjust the size and shape of the bounding box. In this way it is possible to resize the height and width of the graphic image.