Three-Dimensional Incompressible Fluid Flow Around the Body of Revolution. Method of Sources and Sinks

S.P. Kiselev, E.V. Vorozhtsov, and V.M. Fomin

■ Impressum

This Mathematica-Notebook is part of the book entitled
S.P. Kiselev, E.V. Vorozhtsov, and V.M. Fomin
Foundations of Fluid Mechanics with Applications
Problem Solving Using *Mathematica.*
Birkhauser Boston, Basel, 1999.

■ Copyright

■ General Description

This Notebook solves the problem of three-dimensional incompressible fluid flow around a body of revolution by the method of sources and sinks in the case where the freestream velocity is directed along a normal to the symmetry axis of the body.

In order to avoid a too big length of the list of the input parameters we have placed the specification of the initial points (z,y) of the streamlines in the body of the program ThreeDimFlow[...]. The user can introduce his changes in these coordinates by re-specifying the entries of the lists z0l and y0l.

The user can also re-specify the coordinates in the option ViewPoint -> {...}, see the body of our function grlin[j_].

The formulation of this problem and the discussion of its solution may be found in Section 4.3.4 of the above book.

■ User's Guide

■ Step 1

Load and compile the program file beginning with the line

```
ClearAll[body, uxyz, ThreeDimFlow];
```

(see the Section "Program Listing")

■ Step 2

Specify the input data by entering them in the line (see also Section "Examples of the Input Data" below)

```
ThreeDimFlow[1, 0.2, 15]
```

Then click in this line and wait for the result of symboli/ numerical computation. The meaning of the input parameters is explained in the Section "Parameters Used in Program prog4-6.nb".

■ Program Listing

```
 ClearAll[body, uxyz, ThreeDimFlow];
(* --- The function x = body[z], the profile NACA 0020 --- *)
body[z5_] := ( a1 = 1.4779155; a2 = -0.624424; a3 = -1.727016;
a4 = 1.384087; a5 = -0.489769;
t1 * (a1 * Sqrt[z5] + z5 * (a2 + z5 * (a3 + z5 * (a4 + a5 * z5)))) );

uxyz[x5_, y5_, z5_] := ( r2 = x5^2 + y5^2; r = Sqrt[r2];
vr = v∞ * x5 / r - (x5 / (4 Pi * r)) * Sum[m[[m]] * h / ((Sqrt[r2 +
            (z5 - (m - 0.5) * h) ^2]) ^3) - 3 * m[[m]] * r2 * h /
             ((Sqrt[r2 + (z5 - (m - 0.5) * h) ^2]) ^5), {m, n5}];
uz = x5 / (4 Pi) * Sum[m[[m]] * 3 * (z5 - (m - 0.5) * h) * h /
           (( Sqrt[r2 + (z5 - (m - 0.5) * h) ^2]) ^5), {m, n5}];
vq  = -v∞ * y5 / r + (y5 / (4 * Pi * r)) * Sum[m[[m]] * h / ((Sqrt[r2 +
            (z5 - (m - 0.5) * h) ^2]) ^3), {m, n5}];
  ux = (x5 * vr - y5 * vq) / r; uy = (y5 * vr + x5 * vq) / r;
If[(z5 > 0 && z5 < 1) && r2 < body[z5]^2, ux = v∞; uy = 0; uz = 0];
{ux, uy, uz} );

grlin[j_] := ( dt = t[[j]]; xu = xul[[j]];
(* The initial condition for a streamline *)
 x0 = x0l[[j]]; y0 = y0l[[j]]; z0 = z0l[[j]];
  xx = {x0}; yy = {y0};  zz = {z0};
While[x0 < xu, {dx0 , dy0, dz0} = N[uxyz[x0, y0, z0] * dt];
{dx1, dy1, dz1} = N[dt * uxyz[x0 + dx0 / 2, y0 + dy0 / 2, z0 + dz0 / 2]];
{dx2, dy2, dz2} = N[dt * uxyz[x0 + dx1 / 2, y0 + dy1 / 2, z0 + dz1 / 2]];
{dx3, dy3, dz3} = N[dt * uxyz[x0 + dx2, y0 + dy2, z0 + dz2]];
   xn = N[x0 + (dx0 + 2 * (dx1 + dx2) + dx3) / 6];
   yn = N[y0 + (dy0 + 2 * (dy1 + dy2) + dy3) / 6];
   zn = N[z0 + (dz0 + 2 * (dz1 + dz2) + dz3) / 6];
  x0 = xn; y0 = yn; z0 = zn; AppendTo[xx, xn]; AppendTo[yy, yn];
   AppendTo[zz, zn]];
np = Length[xx];

xp[t_] := (n = Min[Floor[t], np - 1];
        xj = (xx[[n + 1]] - xx[[n]]) * (t - n) + xx[[n]]; xj );

yp[t_] := (n = Min[Floor[t], np - 1];
        yj = (yy[[n + 1]] - yy[[n]]) * (t - n) + yy[[n]]; yj );
```

```
zp[t_] := (n = Min[Floor[t], np - 1];
        zj = (zz[[n + 1]] - zz[[n]]) * (t - n) + zz[[n]]; zj );
 streamlin = ParametricPlot3D[{yp[t], zp[t], xp[t]}, {t, 1, np},
                          (* ViewPoint -> {2.59, -0.864, 2},*)
                           ViewPoint -> {0.8, - 1.3, 0.4},
                           Boxed -> False,
                           DisplayFunction -> Identity];
  Print["Streamline No. ", j, " is ready."];
  streamlin );

ThreeDimFlow[v0_, tprf_, n_, vpts_] :=
( h = 1 / n; v∞ = v0; t1 = tprf; n5 = n;
(* -- Numerical solution of the system Am = b -- *)
  b = Table[N[v∞ ], {k, n}];
dd = Table[Sqrt[body[k * h] ^2 + ((k - i + 0.5) * h) ^2], {i, n}, {k, n}]
A = N[Table[body'[k * h] / (4 Pi) * 3 * body[k * h] * (k - i + 0.5) * h * h /
     (dd[[k, i]] ^5) + 1 / (4 Pi) * (h / (dd[[k, i]] ^3) -
      3 * body[k * h] ^2 * h / (dd[[k, i]] ^5)), {i, n}, {k, n}]];
m = LinearSolve[A, b];
 Print["m = ", m];
(* -- The approximate analytic formulas for the velocity
   components vr (x,y,z), vq (x,y,z), and vz (x,y,z) -- *)
 {u1, u2, u3} = uxyz[x, y, z];
(*  Print["ux(x, y, z) = ", u1]; Print["uy(x, y, z) = ", u2]; *)
 Print["uz(x, y, z) = ", u3];
(* The specification of the coordinates z,y of the initial
    points of of the streamlines *)
z0l = {0.2, 0.2, 0.5, 0.5, 0.8, 0.8};
y0l = {-0.05, 0.05, -0.05, 0.05, -0.05, 0.05};
nxl = Length[z0l];
Print["The number of streamlines = ", nxl];
x0l = Table[-0.4, {i, nxl}]; xul = Table[0.3, {i, nxl}];
t = {0.02, 0.02, 0.02, 0.02, 0.02, 0.02, 0.02, 0.02, 0.02, 0.02,
     0.02, 0.02, 0.03, 0.03};

  (* -- The picture of streamlines -- *)
 streamlines = Table[grlin[j], {j, nxl}];
bound = ParametricPlot3D[{body[z] * Sin[u], z, body[z] * Cos[u]},
          {z, 0, 1}, {u, 0, 2 Pi},
          AspectRatio -> Automatic,
          Boxed -> False,
          Axes -> False,
          ViewPoint -> {0.8, - 1.3, 0.4},
```

```
              DisplayFunction -> Identity];
     gr = Show[streamlines, bound,
              AspectRatio -> Automatic,
              PlotRange -> All,
              Axes -> False,
              DisplayFunction -> $DisplayFunction];
    (* -- The 3 D plot of the velocity vectors field -- *)
     bound = ParametricPlot3D[{body[z] * Cos[u], body[z] * Sin[u], z},
              {z, 0, 1}, {u, 0, 2 Pi},
              AspectRatio -> Automatic,
              Boxed -> False,
              Axes -> False,
              DisplayFunction -> Identity];
      xvl = -t1 - 0.1; xvr = t1 + 0.11;
      g5 = PlotVectorField3D[uxyz[x, y, z], {x, xvl, xvr},
           {y, xvl, xvr}, {z, -0.2, 1.1},
           PlotPoints -> vpts,
           VectorHeads -> True, DisplayFunction -> Identity];
      Show[g5, bound, AspectRatio -> Automatic,
                      ViewVertical -> {1, 0, 0},
                      PlotRange -> All,
                      Axes -> False,
                      DisplayFunction -> $DisplayFunction] )
```

■ Parameters Used in Program prog4-6.nb

Parameter	Description
v0	the magnitude of the freestream velocity, v0 > 0;
tprf	the relative thickness of the body profile, that is tprf is the ratio of the profile thickness and chord length; 0 < tprf < 1;
n	the number of sources and sinks; n is a positive integer;
vpts	the number of mesh points in each of the spatial directions x,y,z for the plot of the velocity vectors; vpts > 1

■ Examples of the Input Data

■ Example 1

```
ThreeDimFlow[1, 0.2, 20, 7]
```

```
m = {-0.00346033, -0.0235973, -0.0400043, -0.0502304, -0.0571718, -0.0597075, -0.060343,
  -0.0579864, -0.0548961, -0.0497748, -0.0448252, -0.038448, -0.0330143, -0.0264969,
  -0.0217828, -0.0161306, -0.01384, -0.0109999, -0.0258113, 0.00647203}
```

$$
\begin{aligned}
uz(x,\ y,\ z) = \frac{1}{4\,p}\Bigg(x\Bigg(&\frac{0.000970804\,(-0.975+z)}{(x^2+y^2+(-0.975+z)^2)^{5/2}} - \frac{0.00387169\,(-0.925+z)}{(x^2+y^2+(-0.925+z)^2)^{5/2}} - \\
&\frac{0.00164998\,(-0.875+z)}{(x^2+y^2+(-0.875+z)^2)^{5/2}} - \frac{0.00207601\,(-0.825+z)}{(x^2+y^2+(-0.825+z)^2)^{5/2}} - \frac{0.00241959\,(-0.775+z)}{(x^2+y^2+(-0.775+z)^2)^{5/2}} - \\
&\frac{0.00326743\,(-0.725+z)}{(x^2+y^2+(-0.725+z)^2)^{5/2}} - \frac{0.00397453\,(-0.675+z)}{(x^2+y^2+(-0.675+z)^2)^{5/2}} - \frac{0.00495214\,(-0.625+z)}{(x^2+y^2+(-0.625+z)^2)^{5/2}} - \\
&\frac{0.0057672\,(-0.575+z)}{(x^2+y^2+(-0.575+z)^2)^{5/2}} - \frac{0.00672378\,(-0.525+z)}{(x^2+y^2+(-0.525+z)^2)^{5/2}} - \frac{0.00746622\,(-0.475+z)}{(x^2+y^2+(-0.475+z)^2)^{5/2}} - \\
&\frac{0.00823441\,(-0.425+z)}{(x^2+y^2+(-0.425+z)^2)^{5/2}} - \frac{0.00869796\,(-0.375+z)}{(x^2+y^2+(-0.375+z)^2)^{5/2}} - \frac{0.00905145\,(-0.325+z)}{(x^2+y^2+(-0.325+z)^2)^{5/2}} - \\
&\frac{0.00895613\,(-0.275+z)}{(x^2+y^2+(-0.275+z)^2)^{5/2}} - \frac{0.00857577\,(-0.225+z)}{(x^2+y^2+(-0.225+z)^2)^{5/2}} - \frac{0.00753456\,(-0.175+z)}{(x^2+y^2+(-0.175+z)^2)^{5/2}} - \\
&\frac{0.00600065\,(-0.125+z)}{(x^2+y^2+(-0.125+z)^2)^{5/2}} - \frac{0.0035396\,(-0.075+z)}{(x^2+y^2+(-0.075+z)^2)^{5/2}} - \frac{0.000519049\,(-0.025+z)}{(x^2+y^2+(-0.025+z)^2)^{5/2}}\Bigg)\Bigg)
\end{aligned}
$$

```
The number of streamlines = 6

Streamline No. 1 is ready.

Streamline No. 2 is ready.

Streamline No. 3 is ready.

Streamline No. 4 is ready.

Streamline No. 5 is ready.

Streamline No. 6 is ready.

Ö Graphics3D Ö
```

■ Example 2

```
ThreeDimFlow[1, 0.4, 20, 8]
```

■ The Structure of the Output

The following information is printed as the result of the work of the above program :
(1) the list of the numerical values of the source strengths m(z_i), i = 1,...,n;
(2) the analytical expression for the velocity component u_z(x,y,z);
(3) graphics picture showing both the body of revolution and the fluid streamlines.

To resize an individual picture obtained by *Mathematica*, please

(i) Click anywhere inside the cell, but not the cell bracket itself. A bounding box with small handles appears around the graphic image.

(ii) Drag one of the handles to adjust the size and shape of the bounding box. In this way it is possible to resize the height and width of the graphic image.