Determination of Streamines and Vortex Lines from a Given Velocity Field

S.P. Kiselev, E.V. Vorozhtsov, and V.M. Fomin

■ Impressum

This Mathematica-Notebook is part of the book entitled
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■ General Description

This Notebook enables the user to find the analytic expressions for the vortex lines $x_1 = x_1$, $x_2 = f_2(x_1)$, $x_3 = f_3(x_1)$ from a velocity field given in a Cartesian basis x_1= x[1], x_2= x[2], x_3=x[3].

This program at first computes the components of the angular velocity w = (1/2) rot(v).

This Notebook solves Problem 1.8, see Section 1.2.3 of the above book.

This Notebook can also be used for other velocity filelds given in a Cartesian basis. For this purpose it is sufficient to specify the expressions for the velocity components v[1], v[2], v[3] in the cell entitled "The Specification of the Velocity Field".

The coordinates (x_1, x_2 (x_1), x_3 (x_1)) of vortex lines are found by the solution of the ordinary differential equations

$$\mathrm{dx}_2 / \mathrm{dx}_1 = \mathrm{w}_2 / \mathrm{w}_1, \ \mathrm{dx}_3 / \mathrm{dx}_1 = \mathrm{w}_3 / \mathrm{w}_1. \qquad (1)$$

It should be noted that the built-in *Mathematica* program DSolve[...] can fail in the case of too complex right-hand sides of equations (1).

■ User's Guide

■ Step 1

Compile the program cell entitled "The Specification of the Velocity Field"
(see the Section "Program Listing").

■ Step 2

Compile the program cell entitled "Calculation of the Components of the Angular Velocity w^Æ".
(see the Section "Program Listing").

■ Step 3

Compile the program cell entitled "Calculation of the Coordinates x_1, x_2 (x_1), x_3 (x_1) of Vortex Lines"
(see the Section "Program Listing").

■ Program Listing

■ The Specification of the Velocity Field

```
Clear[x];
v[1] = C x[2] ; v[2] = 0; v[3] = 0;
ru = {x[1] -> x1, x[2] -> x2, x[3] -> x3};
v[1] /. ru
v[2] /. ru
v[3] /. ru
```

```
C x2

0

0
```

■ Calculation of the Components of the Angular Velocity $\vec{\omega}$

```
ru = {x[1] -> x, x[2] -> y, x[3] -> z};
w1 = (1 / 2) * (D[v[3], x[2]] - D[v[2], x[3]]); w1 = w1 /. ru; Print["w_1 = ", w1];
w2 = (1 / 2) * (D[v[1], x[3]] - D[v[3], x[1]]); w2 = w2 /. ru; Print["w_2 = ", w2];
w3 = (1 / 2) * (D[v[2], x[1]] - D[v[1], x[2]]); w3 = w3 /. ru; Print["w_3 = ", w3];
```

```
w_1 = 0

w_2 = 0

w_3 = -C/2
```

■ Calculation of the Coordinates x_1, x_2 (x_1), x_3 (x_1) of Vortex Lines

```
ur = {x -> x₁, y -> x₂, z -> x₃};
If[w₂ == 0, x₂ = k₂];
If[w₂ != 0, sol1 = DSolve[y'[x] == w2 / w1, y[x], x]; x₂ = y[x] /. sol1; x₂ = x₂ /. ur];
If[w₁ == 0, x₃ = k₃];
If[w₁ != 0, sol2 = DSolve[z'[x] == w3 / w1, z[x], x]; x₃ = z[x] /. sol2; x₃ = x₃ /. ur];
Print["x_2 = ", x₂]; Print["x_3 = ", x₃ /. C[1] -> C[2]];
```

```
x_2 = k₂

x_3 = k₃
```

■ The Structure of the Output

The following results are obtained with the aid of the above program:

(1) the components w_1, w_2, w_3 of the angular velocity w =(1/2) rot(v);
(2) the functions x_2 (x_1) and x_3 (x_1) describing the vortex lines by equations
$x_1 = x_1$, $x_2 = x_2(x_1)$, $x_3 = x_3(x_1)$.