

Determination of Vortex Lines from a Given Velocity Field

S.P. Kiselev, E.V. Vorozhtsov, and V.M. Fomin

■ Impressum

This Mathematica-Notebook is part of the book entitled

S.P. Kiselev, E.V. Vorozhtsov, and V.M. Fomin
 Foundations of Fluid Mechanics with Applications
 Problem Solving Using *Mathematica*.
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■ General Description

This Notebook enables the user to find the analytic expressions for the vortex lines $x_1 = x_1$, $x_2 = f_2(x_1)$, $x_3 = f_3(x_1)$ from a velocity field given in a Cartesian basis $x_1 = x[1]$, $x_2 = x[2]$, $x_3 = x[3]$.

This program at first computes the components of the angular velocity $\mathbf{w} = (1/2) \text{rot}(\mathbf{v})$.

This Notebook solves Problem 1.7, see Section 1.2.3 of the above book.

This Notebook can also be used for other velocity fields given in a Cartesian basis. For this purpose it is sufficient to specify the expressions for the velocity components $v[1]$, $v[2]$, $v[3]$ in the cell entitled "The Specification of the Velocity Field".

The coordinates $(x_1, x_2(x_1), x_3(x_1))$ of vortex lines are found by the solution of the ordinary differential equations

$$dx_2 / dx_1 = w_2 / w_1, \quad dx_3 / dx_1 = w_3 / w_1. \quad (1)$$

It should be noted that the built-in *Mathematica* program `DSolve[...]` can fail in the case of too complex right-hand sides of equations (1).

■ User's Guide

■ Step 1

Compile the program cell entitled "The Specification of the Velocity Field" (see the Section "Program Listing").

■ Step 2

Compile the program cell entitled "Calculation of the Components of the Angular Velocity \mathbf{w}^A ". (see the Section "Program Listing").

■ Step 3

Compile the program cell entitled "Calculation of the Coordinates $x_1, x_2 (x_1), x_3 (x_1)$ of Vortex Lines" (see the Section "Program Listing").

■ Program Listing

■ The Specification of the Velocity Field

```

Clear[x];
v[1] = A x[3] - B x[2]; v[2] = B x[1] - C x[3];
v[3] = C x[2] - A x[1];
ru = {x[1] -> x1, x[2] -> x2, x[3] -> x3};
v[1] /. ru
v[2] /. ru
v[3] /. ru

- B x2 + A x3

B x1 - C x3

- A x1 + C x2

```

■ Calculation of the Components of the Angular Velocity $\vec{\omega}$

```

ru = {x[1] -> x, x[2] -> y, x[3] -> z};
w1 = (1/2) * (D[v[3], x[2]] - D[v[2], x[3]]); w1 = w1 /. ru; Print["w_1 = ", w1];
w2 = (1/2) * (D[v[1], x[3]] - D[v[3], x[1]]); w2 = w2 /. ru; Print["w_2 = ", w2];
w3 = (1/2) * (D[v[2], x[1]] - D[v[1], x[2]]); w3 = w3 /. ru; Print["w_3 = ", w3];

w_1 = C
w_2 = A
w_3 = B

```

■ Calculation of the Coordinates $x_1, x_2(x_1), x_3(x_1)$ of Vortex Lines

```

sol1 = DSolve[y'[x] == w2/w1, y[x], x];
sol2 = DSolve[z'[x] == w3/w1, z[x], x];
ur = {x -> x1, y -> x2, z -> x3};
x2 = y[x] /. sol1; x2 = x2 /. ur;
x3 = z[x] /. sol2; x3 = x3 /. ur;
Print["x_2 = ", x2[[1]]]; Print["x_3 = ", x3[[1]] /. C[1] -> C[2]];

x_2 = C[1] +  $\frac{2x_1^2}{C}$ 
x_3 = C[2] +  $\frac{3x_1^3}{C}$ 

```

■ The Structure of the Output

The following results are obtained with the aid of the above program:

- (1) the components w_1, w_2, w_3 of the angular velocity $\mathbf{w} = (1/2) \text{rot}(\mathbf{v})$;
- (2) the functions $x_2(x_1)$ and $x_3(x_1)$ describing the vortex lines by equations
 $x_1 = x_1, x_2 = x_2(x_1), x_3 = x_3(x_1)$.