The Shock Wave Structure

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■ Impressum

This Mathematica-Notebook is part of the book entitled

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■ General Description

The program prog6-2.nb, which follows below, enables one to determine the form of a curve V=V(x) in the shock transition in a viscous, non-heat-conducting compressible gas.

The formulation of this problem and the discussion of its solution can be found in Section 6.2 of the above mentioned book.

■ User's Guide

■ Step 1

Load and compile the program file beginning with the line

```
Struct[V1_, V2_, g_, h_, xl_, xr_, e_, np_] :=
```

(see the Section "Program Listing")

■ Step 2

Specify the input data by entering them in the line (see also Section "Examples of the Input Data" below)

```
Struct[3.0, 1.0, 1.4, 1.3, -9, 9, 10^-4, 40]
```

Then click in this line and wait for the result of symbolic/numerical computation.
The meaning of the input parameters is explained in the Section "Parameters Used in Program prog6-2.nb".

■ Program Listing

```
Struct[V1_, V2_, g_, h_, xl_, xr_, e_, np_] :=
 (g2 = g + 1; ck = 2 h / g2;
  p1 = V1 / (V1 - V2); p2 = - V2 / (V1 - V2);
  f[V_, x0_] := N[x0 - ck * (p1 Log[V1 - V] + p2 Log[V - V2]), 15];
  dx = N[(xr - xl) / np]; xx = {}; yy = {}; a0 = (1 + 1 / 10^5) V2;
  b0 = (1 - 1 / 10^5) V1; x0 = xl;
  Do[x0 = xl + (i - 1) dx; a = a0; b = b0;
   While[b - a > e, fa = f[a, x0]; fb = f[b, x0];
    isl = Sign[fa]; isr = Sign[fb]; c = N[(a + b) / 2];
    fc = f[c, x0]; isc = Sign[fc]; If[isc == isr, b = c, a = c]];
   x = N[x0]; AppendTo[xx, x]; AppendTo[yy, c], {i, np + 1}];
  lc = MapThread[List, {xx, yy}];
  ListPlot[lc, PlotJoined → True,
               PlotRange → All,
               PlotStyle → {Thickness[0.02]},
               AxesLabel → {"x", "V"}]; )
```

■ Parameters Used in Program prog6-2.nb

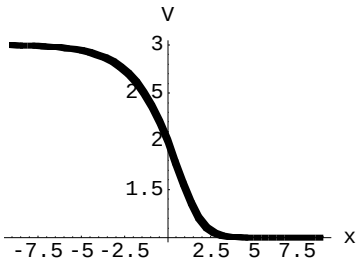
Parameter	Description
V1	the value of the gas specific volume ahead of the shock wave; V1 > 0
V2	the value of the gas specific volume behind the shock wave; 0 < V2 < V1
g	the ratio of the gas specific heats; g > 0
h	the nondimensionalized gas viscosity coefficient; h > 0
xl	the abscissa of the left end of the integration interval on the x axis; xl < 0
xr	the abscissa of the right end of the integration interval on the x axis; xr > xl
e	the accuracy of the computation of V(x) by the bisection method for a given value of x
np	the number of uniform grid nodes in the interval [xl, xr]; np is a positive integer

■ Examples of the Input Data

■ Example 1

h = 1.3

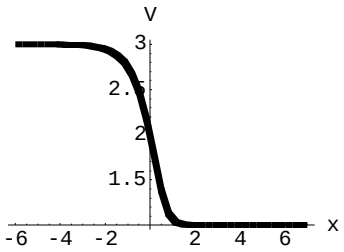
```
Struct[3.0, 1.0, 1.4, 1.3, -9, 9, 10^-4, 40]
```



■ Example 2

h = 0.5

```
Struct[3, 1, 1.4, 0.5, -6, 7, 10^-4, 40]
```



■ The Structure of the Output

The output of the above program is the curve V = V(x)

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