The Strain and RotationTensors

S.P. Kiselev, E.V. Vorozhtsov, and V.M. Fomin

■ Impressum

This Mathematica-Notebook is part of the book entitled
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■ General Description

This Notebook enables the user to find the analytic expressions for the strain tensor e and the rotation tensor w_{ij} for the case where the displacement vector components u_1, u_2, u_3 are given in Cartesian basis x_1, x_2, x_3.

The program also computes the analytic expressions for the principal strains e_i and the eigenvectors n_i.

This Notebook solves Problem 1.6, see Section 1.2.3 of the above book.

This Notebook can also be used for other displacement filelds. For this purpose it is sufficient to specify the expressions for the displacements u_1= u[1], u_2= u[2], u_3= u[3] in terms of the Cartesian coordinates x_1= k[1], x_2= k[2], x_3= k[3] in the cell entitled "The specification of the Displacement Vector Components u_1, u_2, u_3 in Cartesian Basis".

■ User's Guide

■ Step 1

Compile the program cell entitled "The Specification of the Displacement Vector Components u_1, u_2, u_3 inCartesianBasis" (see the Section "Program Listing").

■ Step 2

Compile the program cell entitled "Calculation of the Strain Tensor e_{ij}, i,j = 1,2,3"
(see the Section "Program Listing").

■ Step 3

Compile the program cell entitled "Calculation of the Rotation Tensor w_{ij}"
(see the Section "Program Listing").

■ Step 4

Compile the program cell entitled "The Principal Strains e_i and the Eigenvectors n_i"
(see the Section "Program Listing").

■ Step 5

Compile the program cell entitled "The Normalized Eigenvectors"
(see the Section "Program Listing").

■ Program Listing

■ The Specification of the Displacement Vector Components u_1, u_1, u_3 in Cartesian Basis

```
Clear[x];
u[1] =  4 x[1] - x[2] + 3 x[3]; u[2] = x[1] + 7 x[2];
u[3] = -3 x[1] + 4 x[2] + 4 x[3];
ru = {x[1] -> x1, x[2] -> x2, x[3]  -> x3};
u[1] /. ru
u[2] /. ru
u[3] /. ru
```

$4 x_1 - x_2 + 3 x_3$

$x_1 + 7 x_2$

$-3 x_1 + 4 x_2 + 4 x_3$

■ Calculation of the Strain Tensor ϵ_{ij}

```
e = IdentityMatrix[3];
Do[ Do[ g0 = (1 / 2) * (D[u[i], x[j]] + D[u[j], x[i]]);
 e  = ReplacePart[e , g0, {i, j}], {i, 3}], {j, 3}];
Print["e = ", MatrixForm[e ]];
```

$$e = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 7 & 2 \\ 0 & 2 & 4 \end{pmatrix}$$

■ Calculation of the Rotation Tensor ω_{ij}

```
w = IdentityMatrix[3];
Do[ Do[ g0 = (1 / 2) * (D[u[i], x[j]] - D[u[j], x[i]]);
 w  = ReplacePart[w , g0, {i, j}], {i, 3}], {j, 3}];
Print["w = ", MatrixForm[w ]];
```

$$w = \begin{pmatrix} 0 & -1 & 3 \\ 1 & 0 & -2 \\ -3 & 2 & 0 \end{pmatrix}$$

■ The Principal Strains and the Eigenvectors n_i

```
vals = Eigenvalues[e]
nvecs = Eigenvectors[e]
```

{3, 4, 8}

{{0, -1, 2}, {1, 0, 0}, {0, 2, 1}}

■ The Normalized Eigenvectors

```
n1 = nvecs[[1]];
n1 = Table[ n1[[i]] / Sqrt[n1[[1]]^2 + n1[[2]]^2 + n1[[3]]^2], {i, 3}]

n2 = nvecs[[2]];
n2 = Table[ n2[[i]] / Sqrt[n2[[1]]^2 + n2[[2]]^2 + n2[[3]]^2], {i, 3}]

n3 = nvecs[[3]];
n3 = Table[ n3[[i]] / Sqrt[n3[[1]]^2 + n3[[2]]^2 + n3[[3]]^2], {i, 3}]
```

$\left\{0, -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\}$

{1, 0, 0}

$$\left\{0, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\}$$

■ The Structure of the Output

The following results are obtained with the aid of the above program:

(1) the strain tensor e;
(2) the rotation tensor w;
(3) the principal strains e_i and the eigenvectors n_i;
(4) the normalized eigenvectors n_i, i = 1,2,3.