Axisymmetric Incompressible Fluid Flow Around the Body of Revolution. Method of Sources and Sinks

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■ Impressum

This Mathematica-Notebook is part of the book entitled
S.P. Kiselev, E.V. Vorozhtsov, and V.M. Fomin
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■ General Description

This Notebook solves the problem of incompressible fluid flow around a body of revolution by the method of sources and sinks in the case where the freestream velocity is directed along the body axis.

The formulation of this problem and the discussion of its solution may be found in Section 4.3.2 of the above book.

■ User's Guide

■ Step 1

Load and compile the program file beginning with the line

```
ClearAll[body, y, Axisym];
```

(see the Section "Program Listing")

■ Step 2

Specify the input data by entering them in the line (see also Section "Examples of the Input Data" below)

```
Axisym[1, 0.2, 12, 60]
```

Then click in this line and wait for the result of numerical computation. The meaning of the input parameters is explained in the Section "Parameters Used in Program prog4-5.nb".

■ Program Listing

```
ClearAll[body, y, Axisym];
(* --- The function x = body[z], the profile NACA 0020 --- *)
body := ( a1 = 1.4779155; a2 = -0.624424; a3 = -1.727016;
a4 = 1.384087; a5 = -0.489769;
(t1 * (a1 * Sqrt[#] + # * (a2 + # * (a3 + # * (a4 + a5 * #)))))& );

y[z_, x_] := -1 / 2 * v∞ * x^2 +
             1 / (4 Pi) * Sum[m[[i]] * (z - (i - 0.5) * h) * h /
              Sqrt[x^2 + (z - (i - 0.5) * h) ^2], {i, n}];

Axisym[v0_, tprf_, n_, npp_] :=
( h = 1 / n; v∞ = v0; t1 = tprf;
b = Table[N[1 / 2 * v∞ * body[k * h] ^2], {k, n}];
A = N[Table[(1 / (4 Pi)) * (k - i + 0.5) * h * h /
   Sqrt[body[k * h] ^2 + ((k - i + 0.5) * h) ^2], {k, n}, {i, n}]];
m = LinearSolve[A, b];

Print["y[z, x] = ", y[z, x]];
m = 51; h1 = 1 / (m - 1);
xx1 = Table[N[body[(k - 1) * h1]], {k, m}];
xx2 = Table[N[-body[(m - k) * h1]], {k, m}]; xx = Join[xx1, xx2];
zz1 = Table[N[(k - 1) * h1], {k, m}];
zz2 = Table[N[(m - k) * h1], {k, m}]; zz = Join[zz1, zz2];
lis = MapThread[List, {zz, xx}]; obj = Polygon[lis];
prof = Graphics[{RGBColor[0.1, 1.0, 1.0], obj}];

streamlines = ContourPlot[y[z, x], {z, -1, 2}, {x, -1, 1},
      PlotPoints -> npp,
      Contours -> {-0.12, -0.08, -0.05, -0.03, -0.02, -0.012,
                   -0.006, -0.003, -0.001,
                    0.001, 0.003, 0.006, 0.012, 0.02, 0.03,
                    0.05, 0.08, 0.12},
      ContourShading -> False,
      ContourSmoothing -> Automatic,
       DisplayFunction -> Identity];

bound = ListPlot[lis, PlotJoined -> True,
                       DisplayFunction -> Identity];
gr = Show[streamlines, prof, bound,
```

```
AspectRatio -> Automatic,
 Axes -> False,
 AxesLabel -> {"z", "x"},
 DisplayFunction -> $DisplayFunction]; )
```

■ Parameters Used in Program prog4-5.nb

Parameter	Description
v0	the magnitude of the freestream velocity, v0 > 0;
tprf	the relative thickness of the body profile, that is tprf is the ratio of the profile thickness and chord length; 0 < tprf < 1;
n	the number of sources and sinks; n is a positive integer;
npp	the number of points of each streamline for the buit-in *Mathematica* function ContourPlot; npp > 10.

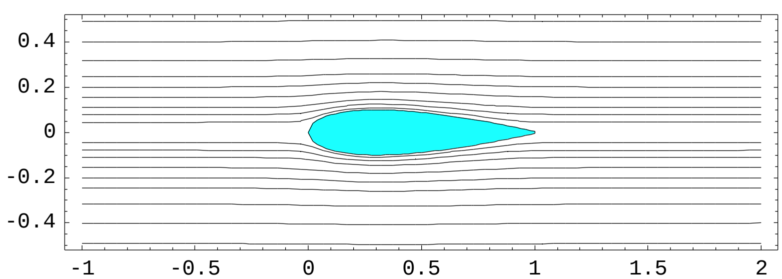
■ Examples of the Input Data

■ Example 1

```
Axisym[1, 0.2, 12, 60]
```

y[z, x] =

$$-\frac{x^2}{2}+\frac{1}{4\,p}\left(-\frac{0.00136163\,(-0.958333+z)}{\sqrt{x^2+(-0.958333+z)^2}}-\frac{0.00299313\,(-0.875+z)}{\sqrt{x^2+(-0.875+z)^2}}-\frac{0.00431155\,(-0.791667+z)}{\sqrt{x^2+(-0.791667+z)^2}}-\frac{0.00528399\,(-0.708333+z)}{\sqrt{x^2+(-0.708333+z)^2}}-\frac{0.00587169\,(-0.625+z)}{\sqrt{x^2+(-0.625+z)^2}}-\frac{0.00597752\,(-0.541667+z)}{\sqrt{x^2+(-0.541667+z)^2}}-\frac{0.00537849\,(-0.458333+z)}{\sqrt{x^2+(-0.458333+z)^2}}-\frac{0.00381788\,(-0.375+z)}{\sqrt{x^2+(-0.375+z)^2}}-\frac{0.000567995\,(-0.291667+z)}{\sqrt{x^2+(-0.291667+z)^2}}+\frac{0.00423871\,(-0.208333+z)}{\sqrt{x^2+(-0.208333+z)^2}}+\frac{0.0139702\,(-0.125+z)}{\sqrt{x^2+(-0.125+z)^2}}+\frac{0.0174533\,(-0.0416667+z)}{\sqrt{x^2+(-0.0416667+z)^2}}\right)$$

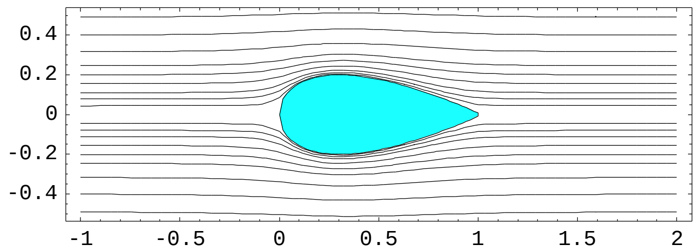


■ Example 2

```
Axisym[1, 0.4, 12, 60]
```

y[z, x] =

$$-\frac{x^2}{2}+\frac{1}{4\,p}\left(-\frac{0.00416104\,(-0.958333+z)}{\sqrt{x^2+(-0.958333+z)^2}}-\frac{0.0104159\,(-0.875+z)}{\sqrt{x^2+(-0.875+z)^2}}-\frac{0.016253\,(-0.791667+z)}{\sqrt{x^2+(-0.791667+z)^2}}-\frac{0.0214297\,(-0.708333+z)}{\sqrt{x^2+(-0.708333+z)^2}}-\frac{0.025599\,(-0.625+z)}{\sqrt{x^2+(-0.625+z)^2}}-\frac{0.0298286\,(-0.541667+z)}{\sqrt{x^2+(-0.541667+z)^2}}-\frac{0.0308775\,(-0.458333+z)}{\sqrt{x^2+(-0.458333+z)^2}}-\frac{0.0329781\,(-0.375+z)}{\sqrt{x^2+(-0.375+z)^2}}-\frac{0.0214472\,(-0.291667+z)}{\sqrt{x^2+(-0.291667+z)^2}}-\frac{0.00035384\,(-0.208333+z)}{\sqrt{x^2+(-0.208333+z)^2}}+\frac{0.18333\,(-0.125+z)}{\sqrt{x^2+(-0.125+z)^2}}+\frac{0.0103131\,(-0.0416667+z)}{\sqrt{x^2+(-0.0416667+z)^2}}\right)$$



■ The Structure of the Output

The following information is printed as the result of the work of the above program :
(1) the analytical expression for the stream function y(z,x);
(2) graphics picture showing both the body of revolution and the fluid streamlines.

To resize an individual picture obtained by *Mathematica*, please

(i) Click anywhere inside the cell, but not the cell bracket itself. A bounding box with small handles appears around the graphic image.

(ii) Drag one of the handles to adjust the size and shape of the bounding box. In this way it is possible to resize the height and width of the graphic image.