Numerical Solution of a Problem on the Boundary Layer of a Flat Plate

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■ Impressum

This Mathematica-Notebook is part of the book entitled
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■ General Description

This Notebook enables the user to solve numerically the problem of a boundary layer of a flat plate. The problem under consideration is the initial-value problem (5.3.18), (5.3.19). It is solved numerically with the aid of the standard fourth-order Runge-Kutta method.

The formulation of this problem and the discussion of its solution may be found in Section 5.3 of the above book.

■ User's Guide

■ Step 1

Load and compile the program file beginning with the line

```
ClearAll[fyz, Blayer];
```

(see the Section "Program Listing")

■ Step 2

Specify the input data by entering them in the line (see also Section "Examples of the Input Data" below)

```
Blayer[10, 0.02]
```

Then click in this line and wait for the result of numerical computation.
The meaning of the input parameters is explained in the Section "Parameters Used in Program prog5-2.nb".

■ Program Listing

```
ClearAll[fyz, Blayer];
fyz[f_, y_, z_] := {y, z, z^2 / y - f y};
Blayer[xmax_, hmin_] :=
((*  Initial conditions  *)
f0 = 0; y0 = 1; z0 = 0;
ff = {f0}; yy = {y0}; zz = {z0};
h = hmin; x = 0; xx = {0};

While[x < xmax, {df0 , dy0, dz0} = N[fyz[f0, y0, z0] * h];
{df1, dy1, dz1} = N[h * fyz[f0 + df0 / 2, y0 + dy0 / 2, z0 + dz0 / 2]];
{df2, dy2, dz2} = N[h * fyz[f0 + df1 / 2, y0 + dy1 / 2, z0 + dz1 / 2]];
{df3, dy3, dz3} = N[h * fyz[f0 + df2, y0 + dy2, z0 + dz2]];
  fn = N[f0 + (df0 + 2 * (df1 + df2) + df3) / 6];
  yn = N[y0 + (dy0 + 2 * (dy1 + dy2) + dy3) / 6];
  zn = N[z0 + (dz0 + 2 * (dz1 + dz2) + dz3) / 6];
  f0 = fn; y0 = yn; z0 = zn; x = x + h; h = 1.05 h;
 AppendTo[ff, fn]; AppendTo[xx, x]];
n = Length[ff]; c0 = ff[[n]];
 c1 = N[2 Sqrt[2] (1 / c0)^(3/2)];
Print["c1 = ", c1];
 lis = MapThread[List, {xx, ff}];
ListPlot[lis, PlotJoined -> True,
              AxesLabel -> {"x", "F(x)"} ]; )
```

■ Parameters Used in Program prog5-2.nb

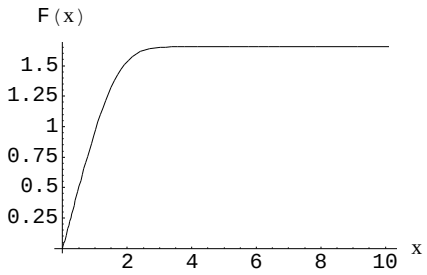
Parameter	Description
xmax	the abscissa of the right end of the integration interval 0<= x <= xmax, xmax > 0;
hmin	the start value of the integration stepsize along the x axis, 0 < hmin < 1.

■ Examples of the Input Data

■ Example 1

```
Blayer[10, 0.02]
```

```
c1 = 1.32822
```



■ Example 2

```
Blayer[5, 0.01]
```

■ The Structure of the Output

The result of the work of the above program is the graphics picture showing the solution profile F(x).

To resize an individual picture obtained by *Mathematica*, please

(i) Click anywhere inside the cell, but not the cell bracket itself. A bounding box with small handles appears around the graphic image.

(ii) Drag one of the handles to adjust the size and shape of the bounding box. In this way it is possible to resize the height and width of the graphic image.