The Mathematica Functions

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■ Impressum

This Mathematica-Notebook is part of the book entitled
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■ General Description

In this Notebook we show at 47 examples how to use those built-in *Mathematica* functions, which we apply in the above book for the solution of various analytical or numerical tasks.

■ Example 1

This example illustrates the use of the *Mathematica* function Abs[arg].

```
Abs[-3.5]
```

```
3.5
```

■ Example 2

This example illustrates the use of the *Mathematica* function AppendTo[list, element].

```
xx = {x1, x2, x3}; AppendTo[xx, a^2]
```

```
{x1, x2, x3, a^2}
```

■ Example 3

This example illustrates the use of the *Mathematica* function ArcSin[arg].

```
ArcSin[1 / 2]
```

$$\frac{\pi}{6}$$

■ Example 4

This example illustrates the use of the *Mathematica* function ArcTan[arg].

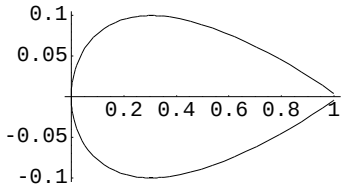
```
ArcTan[1]
```

$$\frac{\pi}{4}$$

■ Example 5

This example illustrates the use of the *Mathematica* options AspectRatio, $DisplayFunction, and the functions Plot[...], Show[...].

```
a1 = 1.4779155; a2 = -0.624424; a3 = -1.727016; a4 = 1.384087;
a5 = -0.489769; t1 = 0.2;
body[z_] := t1 (a1 Sqrt[z] + z (a2 + z (a3 + z (a4 + a5 z))));
g1 = Plot[body[x], {x, 0, 1}, DisplayFunction -> Identity];
g2 = Plot[-body[x], {x, 0, 1}, DisplayFunction -> Identity];
Show[g1, g2, DisplayFunction -> $DisplayFunction]
```

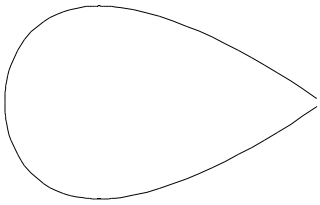


Ö Graphics Ö

■ Example 6

This example illustrates the use of the *Mathematica* option Axes.

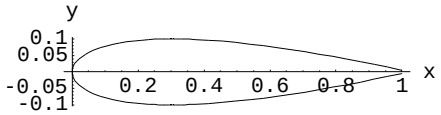
```
a1 = 1.4779155; a2=-0.624424; a3 = -1.727016; a4 = 1.384087;
a5=-0.489769; t1=0.2;
body[z_]:=t1 (a1 Sqrt[z]+z (a2+z (a3 + z (a4+a5 z))));
g1 = Plot[body[x], {x,0,1},DisplayFunction->Identity];
g2 = Plot[-body[x],{x,0,1},DisplayFunction->Identity];
Show[g1,g2,Axes->False,DisplayFunction->$DisplayFunction]
```



■ Example 7

This example illustrates the use of the *Mathematica* options AxesLabel and AspectRatio.

```
a1 = 1.4779155; a2 = -0.624424; a3 = -1.727016; a4 = 1.384087;
a5 = -0.489769; t1 = 0.2;
body[z_] := t1 (a1 Sqrt[z] + z (a2 + z (a3 + z (a4 + a5 z))));
g1 = Plot[body[x], {x, 0, 1}, DisplayFunction -> Identity];
g2 = Plot[-body[x], {x, 0, 1}, DisplayFunction -> Identity];
Show[g1, g2, AxesLabel -> {"x", "y"}, AspectRatio -> Automatic,
 DisplayFunction -> $DisplayFunction]
```



Ö Graphics Ö

■ Example 8

This example illustrates the use of the *Mathematica* function ClearAll[args].

```
x1 = 2 Cos[t]
 ClearAll[x1]
x1
```

```
2 Cos[t]
```

```
x1
```

■ Example 9

This example illustrates the use of the *Mathematica* function ComplexExpand[expr].

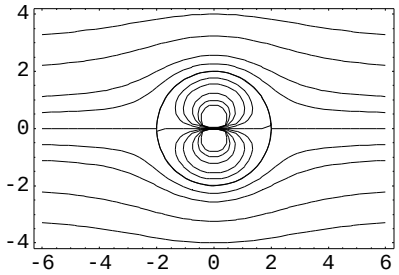
```
ComplexExpand[Im[Cos[z^2] /. z -> x + I y]] // TraditionalForm
```

$$-\sin(x^2 - y^2)\sinh(2xy)$$

■ Example 10

This example illustrates the use of the *Mathematica* function ContourPlot[...].

```
y[x_, y_] := y (1 - rc^2 / (x^2 + y^2));
rc = 2;
streamlines = ContourPlot[y[x, y], {x, -6, 6}, {y, -4, 4},
      PlotPoints -> 40,
      Contours -> {-4, -3, -2, -1, -0.5, 0, 0.5, 1, 2, 3, 4},
      ContourShading -> False,
      ContourSmoothing -> Automatic,
      DisplayFunction -> Identity];
bound = Graphics[Circle[{0, 0}, rc]];
Show[streamlines, bound, AspectRatio -> Automatic,
DisplayFunction -> $DisplayFunction];
```



■ Example 11

This example illustrates the use of the *Mathematica* function Cos[arg].

```
TraditionalForm[Cos[Pi / 10]]
```

$$\frac{1}{2}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}$$

■ Example 12

This example illustrates the use of the *Mathematica* function D[...].

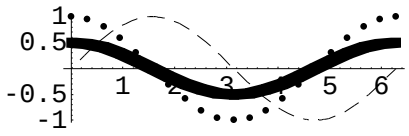
```
x = k[1] Sin[k[2]] Sin[k[3]]; D[x, k[2]]
```

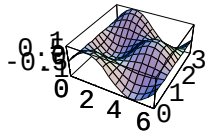
```
Cos[k[2]] k[1] Sin[k[3]]
```

■ Example 13

This example illustrates the use of the *Mathematica* functions Dashing[...], GraphicsArray, ListPlot, MapThread, Plot3D, Show , Table.

```
g1 = Plot[Sin[x], {x, 0, 2 p},
          PlotStyle -> {Dashing[{0.05, 0.03}]},
          DisplayFunction -> Identity];
xx = Table[N[(k - 1) p / 10], {k, 21}];
yy = Table[N[Cos[xx[[k]]]], {k, 21}];
g2 = ListPlot[MapThread[List, {xx, yy}],
              PlotStyle -> {PointSize[0.02]},
              DisplayFunction -> Identity];
g3 = ListPlot[MapThread[List, {xx, 0.5 yy}],
              PlotJoined -> True,
              PlotStyle -> {Thickness[0.03]},
               DisplayFunction -> Identity];
A2 = Plot3D[Sin[x] Cos[2 y], {x, 0, 2 Pi}, {y, 0, Pi},
            DisplayFunction -> Identity];
A1 = Show[g2, g1, g3, AspectRatio -> Automatic,
                      DisplayFunction -> Identity];
Show[GraphicsArray[{A1, A2}]];
```





■ Example 14

This example illustrates the use of the *Mathematica* function DSolve[...].

```
DSolve[{x2'[x] == -x2[x] x, x2[x_1] == x_2}, x2[x], x]
```

```
{{x2[x] → E^(-x^2/2 + x_1^2/2) x_2}}
```

■ Example 15

This example illustrates the use of the *Mathematica* functions Eigenvalues[matr] and Eigenvectors[matr].

```
A = {{-1, 0, 3}, {2, 1, 1}, {5, 1, 4}}; Eigenvalues[A]
Eigenvectors[A]
TraditionalForm[Inverse[A]]
```

$$\left\{-3, \frac{1}{2}\left(7-\sqrt{33}\right), \frac{1}{2}\left(7+\sqrt{33}\right)\right\}$$

$$\left\{\{-3, 1, 2\}, \left\{-\frac{6}{-9+\sqrt{33}}, \frac{2\left(9+2\sqrt{33}\right)}{-9+\sqrt{33}}, 1\right\}, \left\{\frac{6}{9+\sqrt{33}}, \frac{2\left(-9+2\sqrt{33}\right)}{9+\sqrt{33}}, 1\right\}\right\}$$

$$\begin{pmatrix} -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{19}{12} & -\frac{7}{12} \\ \frac{1}{4} & -\frac{1}{12} & \frac{1}{12} \end{pmatrix}$$

■ Example 16

This example illustrates the use of the *Mathematica* function Expand[arg].

```
TraditionalForm[Expand[(a - 3 b)^5]]
```

$$a^5 - 15\, b\, a^4 + 90\, b^2\, a^3 - 270\, b^3\, a^2 + 405\, b^4\, a - 243\, b^5$$

■ Example 17

This example illustrates the use of the *Mathematica* function First[arg].

```
lis = {a1, Sin[t], p / 8}; First[lis]
```

```
a1
```

■ Example 18

This example illustrates the use of the *Mathematica* function Flatten[args].

```
Flatten[{a1, b2, {{2, 3}, {a4, s5}}}]
```

```
{a1, b2, 2, 3, a4, s5}
```

■ Example 19

This example illustrates the use of the *Mathematica* function Floor[arg].

```
Floor[-2.4]
Floor[2.4]
```

```
-3
```

```
2
```

■ Example 20

This example illustrates the use of the *Mathematica* function IdentityMatrix[arg].

```
TraditionalForm[IdentityMatrix[3]]
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

■ Example 21

This example illustrates the use of the *Mathematica* function If[args].

```
b = 5; c = Sin[t]; d = t^2; If[b < 4, c = Cos[t]; d = 5 t,
 c = Cosh[t]; d = 4 t];
Print["c = ", c, "; d = ", d];
```

```
c = Cosh[t]; d = 4 t
```

■ Example 22

This example illustrates the use of the *Mathematica* function Join[args].

```
ClearAll[b];
ls1 = {x₁, x₂}; ls2 = {Sqrt[1 + b^2], Sin[t]};
TraditionalForm[Join[ls1, ls2]]
```

$$\left\{x_1, x_2, \sqrt{b^2+1}, \sin(t)\right\}$$

■ Example 23

This example illustrates the use of the *Mathematica* function Length[arg].

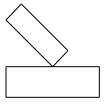
```
Length[{a, x^3, 1998}]
```

```
3
```

■ Example 24

This example illustrates the use of the *Mathematica* function Line[arg], Graphics[...], and Show[...].

```
Show[Graphics[Line[{{4, 3}, {5, 4}, {2, 7}, {1, 6}, {4, 3}, {1, 3},
                   {1, 1}, {7, 1}, {7, 3}, {4, 3}}]],
                   AspectRatio -> Automatic]
```



```
Ö Graphics Ö
```

■ Example 25

This example illustrates the use of the *Mathematica* function LinearSolve[args].

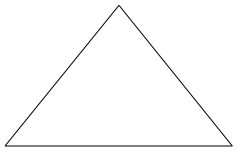
```
n = 3; b = Table[k, {k, n}];
A = Table[k / (i + k), {i, n}, {k, n}];
LinearSolve[A, b]
```

```
{132, -300, 180}
```

■ Example 26

This example illustrates the use of the *Mathematica* function ListPlot[arg].

```
ListPlot[{{0, 1}, {2, 1}, {1, 3}, {0, 1}}, Axes -> False,
         PlotJoined -> True]
```



```
Ö Graphics Ö
```

■ Example 27

This example illustrates the use of the *Mathematica* function Log[arg].

```
N[Log[10]]
```

```
2.30259
```

■ Example 28

This example illustrates the use of the Mathematica function MapThread[...].

```
xx = {x₁, x₂, x₃}; yy = {y₁, y₂, y₃}; MapThread[List, {xx, yy}]
```

```
{{x₁, y₁}, {x₂, y₂}, {x₃, y₃}}
```

■ Example 29

This example illustrates the use of the Mathematica function MatrixForm[arg].

```
Print["g = ", TraditionalForm[MatrixForm[IdentityMatrix[3]]]]
```

$$g = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

■ Example 30

This example illustrates the use of the Mathematica function N[arg].

```
N[Sqrt[2], 45]
```

```
1.41421356237309504880168872420969807856967188
```

■ Example 31

This example illustrates the use of the Mathematica function NIntegrate[arg].

```
NIntegrate[Sqrt[Sin[p / 2] - Sin[x]], {x, 0, p / 2}]
Integrate[Sqrt[Sin[p / 2] - Sin[x]], {x, 0, p / 2}]
N[%]
```

```
0.828427
```

$$-2+2\sqrt{2}$$

```
0.828427
```

■ Example 32

This example illustrates the use of the Mathematica function ParametricPlot3D[args].

```
ParametricPlot3D[{2 t, Sin[3 t], Cos[3 t]}, {t, 0, 5 Pi},
                 Boxed -> False, Axes -> False]
```

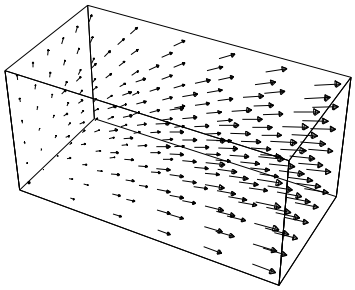
Ö Graphics3D Ö

■ Example 33

This example illustrates the use of the Mathematica function PlotVectorField3D[args].

```
(* If you want to run this Example after the foregoing Example
   in the same Mathematica session, then you will fail.
   Please at first exit Mathematica, and then begin the
   Mathematica session, and run this Example as the
   first Example in your new Mathematica session  *)

 << Graphics`PlotField3D`
PlotVectorField3D[{2 x, y, z}, {x, 0, 2}, {y, 0, 1}, {z, 0, 1},
                  PlotPoints -> 6,
                  VectorHeads -> True]
```

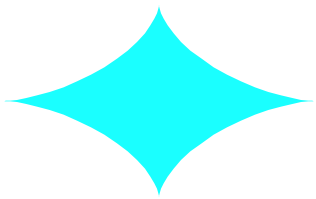


Ö Graphics3D Ö

■ Example 34

This example illustrates the use of the Mathematica function Polygon[arg].

```
Show[Graphics[{RGBColor[0.1, 1.0, 1.0],
   Polygon[Table[{2 Cos[n p/40]^3, 2 Sin[n p/40]^3}, {n, 80}]]}]]
```



Ö Graphics Ö

■ Example 35

This example illustrates the use of the Mathematica function PowerExpand[arg].

```
a = r^4 Sin[j]^2; TraditionalForm[Sqrt[a]]
TraditionalForm[PowerExpand[a^1/2]]
```

$$\sqrt{r^4 \sin^2(j)}$$

$$r^2 \sin(j)$$

■ Example 36

This example illustrates the use of the Mathematica function ReplacePart[arg].

```
ReplacePart[IdentityMatrix[3], Sin[j], {3, 1}] //
 TraditionalForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \sin(j) & 0 & 1 \end{pmatrix}$$

■ Example 37

This example illustrates the use of the Mathematica function Simplify[expr].

```
Simplify[x^2 / 3 + x y / (5 - x)]
```

$$\frac{1}{3} x \left(x - \frac{3 y}{-5 + x}\right)$$

■ Example 38

This example illustrates the use of the Mathematica function Sign[arg].

```
Sign[-1.35]
Sign[0]
Sign[0.005]
```

```
-1

0

1
```

■ Example 39

This example illustrates the use of the Mathematica function Sin[arg].

```
TraditionalForm[Sin[Pi / 10]]
```

$$\frac{1}{4}\left(-1+\sqrt{5}\right)$$

■ Example 40

This example illustrates the use of the Mathematica function Solve[args].

```
ComplexExpand[Solve[x^3 - 1 / 3 x - 1 == 0, x]]
```

$$\left\{\left\{x \to \frac{1}{9}\left(\frac{729}{2}-\frac{135\sqrt{29}}{2}\right)^{1/3}+\frac{1}{3}\left(\frac{1}{2}\left(27+5\sqrt{29}\right)\right)^{1/3}\right\},\right.$$
$$\left\{x \to -\frac{1}{18}\left(1+i\sqrt{3}\right)\left(\frac{729}{2}-\frac{135\sqrt{29}}{2}\right)^{1/3}-\frac{1}{6}\left(1-i\sqrt{3}\right)\left(\frac{1}{2}\left(27+5\sqrt{29}\right)\right)^{1/3}\right\},$$
$$\left.\left\{x \to -\frac{1}{18}\left(1-i\sqrt{3}\right)\left(\frac{729}{2}-\frac{135\sqrt{29}}{2}\right)^{1/3}-\frac{1}{6}\left(1+i\sqrt{3}\right)\left(\frac{1}{2}\left(27+5\sqrt{29}\right)\right)^{1/3}\right\}\right\}$$

■ Example 41

This example illustrates the use of the Mathematica function Sum[args].

```
Simplify[Sum[(-1)^(j+1) Cos[j x], {j, 4}]] // TraditionalForm
```

$$\cos(x) - \cos(2x) + \cos(3x) - \cos(4x)$$

■ Example 42

This example illustrates the use of the Mathematica function Table[arg].

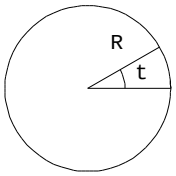
```
ClearAll[a, b];
TraditionalForm[Table[a^i b^j, {i, 3}, {j, 3}]]
```

$$\begin{pmatrix} ab & ab^2 & ab^3 \\ a^2 b & a^2 b^2 & a^2 b^3 \\ a^3 b & a^3 b^2 & a^3 b^3 \end{pmatrix}$$

■ Example 43

This example illustrates the use of the Mathematica function Text[arg], Graphics[...], Line[...], Circle[...], Show[...].

```
rc = 2; bound = Graphics[Circle[{0, 0}, rc]];
arc = Graphics[Circle[{0, 0}, 0.9, {0, Pi / 6}]];
lin = Graphics[Line[{{rc, 0}, {0, 0}, {rc Cos[Pi/6], rc Sin[Pi/6]}}]];
gR = Graphics[Text["R", {0.7, 1.1}]];
gt = Graphics[Text["t", {1.3, 0.37}]];
Show[bound, arc, lin, gR, gt, AspectRatio -> Automatic]
```



Ö Graphics Ö

■ Example 44

This example illustrates the use of the Mathematica function Transpose[list].

```
A = {{a, b, c}, {1, 2, 3}}; TraditionalForm[A]
TraditionalForm[Transpose[A]]
```

$$\begin{pmatrix} a & b & c \\ 1 & 2 & 3 \end{pmatrix}$$

$$\begin{pmatrix} a & 1 \\ b & 2 \\ c & 3 \end{pmatrix}$$

■ Example 45

This example illustrates the use of the Mathematica function TrigExpand[arg].

```
TraditionalForm[TrigExpand[Cos[3 x] Sin[4 y]]]
```

$$4\cos^3(x)\sin(y)\cos^3(y) - 12\cos(x)\sin^2(x)\sin(y)\cos^3(y) - 4\cos^3(x)\sin^3(y)\cos(y) + 12\cos(x)\sin^2(x)\sin^3(y)\cos(y)$$

■ Example 46

This example illustrates the use of the Mathematica function TrigReduce[arg].

```
TraditionalForm[TrigReduce[Cos[x]^5 + Sin[x]^5]]
```

$$\frac{1}{16}(10\cos(x) + 5\cos(3\,x) + \cos(5\,x) + 10\sin(x) - 5\sin(3\,x) + \sin(5\,x))$$

■ Example 47

This example illustrates the use of the Mathematica function While[test, body].

```
x = 0; y0 = 0; y = {y0}; While[x < 1, x = x + 0.1; y0 = y0 + 0.1 x^2;
AppendTo[y, y0]]; y
```

```
{0, 0.001, 0.005, 0.014, 0.03, 0.055, 0.091, 0.14, 0.204, 0.285, 0.385, 0.506}
```