The Motion Trajectories

S.P. Kiselev, E.V. Vorozhtsov, and V.M. Fomin

■ Impressum

This Mathematica-Notebook is part of the book entitled
S.P. Kiselev, E.V. Vorozhtsov, and V.M. Fomin
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Problem Solving Using *Mathematica.*
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■ General Description

This Notebook enables the user to find:

(1) the motion trajectories;
(2) the components of velocity and acceleration;
(3) the streamlines

for a given velocity field $v_1 = v_1(x_1)$, $v_2 = v_2(x_2)$, $v_3 = v_3(x_3)$, where x_1(t), x_2(t), and x_3(t) are the coordinates of a material point moving in the (x_1, x_2, x_3) space.

The formulation of this problem and the discussion of its solution can be found in Section 1.2 of the above mentioned book, see Problem 1.4.

This Notebook can be used to solve a similar task for other velocity fields. For this purpose please specify the functions v_1, v_2, and v_3 in the cell entitled "The velocity Field". Please keep in mind that *Mathematica* tries to solve analytically the ordinary differential equations (O.D.E.'s) x_j'(t) = v_j (x_j(t),t), j= 1,2,3. Such closed-form solutions can be found only for certain classes of O.D.E.'s.

■ User's Guide

■ Step 1

Compile the program cell entitled "The Velocity Field"
(see the Section "Program Listing").

■ Step 2

Compile the program cell entitled "Determination of the Motion Trajectories."
(see the Section "Program Listing").

■ Step 3

Compile the program cell entitled "The The Velocities v_i (x_i, t), i = 1,2,3, in the Lagrangian Coordinates"
(see the Section "Program Listing").

■ Step 4

Compile the program cell entitled "The Accelerations in Lagrangian Coordinates"
(see the Section "Program Listing").

■ Step 5

Compile the program cell entitled "The Accelerations in Eulerian Coordinates"
(see the Section "Program Listing").

■ Step 6

Compile the program cell entitled "The Streamlines" (see the Section "Program Listing").

■ Step 7

Compile the program cell entitled "The Picture of Trajectories"
(see the Section "Program Listing").

■ Program Listing

■ The Velocity Field

```
v1 = x1[t] / (1 + t); v2 = 2 x2[t] / (1 + t); v3 = 3 x3[t] / (1 + t);
v10 = v1 /. x1[t] -> x1
v20 = v2 /. x2[t] -> x2
v30 = v3 /. x3[t] -> x3
```

$\frac{x_1}{1+t}$

$\frac{2 x_2}{1+t}$

$\frac{3 x_3}{1+t}$

■ Determination of the Motion Trajectories

```
sol1 = DSolve[{x1'[t] == v1, x1[0] == ξ1}, x1[t], t]
x1 = x1[t] /. sol1[[1]]
 sol2 = DSolve[{x2'[t] == v2, x2[0] == ξ2}, x2[t], t]
 x2 = x2[t] /. sol2[[1]]
sol3 = DSolve[{x3'[t] == v3, x3[0] == ξ3}, x3[t], t]
x3 = x3[t] /. sol3[[1]]
```

{{x1[t] Æ (1 + t) ξ1}}

(1 + t) ξ1

{{x2[t] Æ (1 + t)^2 ξ2}}

(1 + t)^2 ξ2

{{x3[t] Æ (1 + t)^3 ξ3}}

(1 + t)^3 ξ3

■ The Velocities v_i (ξ_i, t), i = 1,2,3, in the Lagrangian Coordinates

```
sol = Flatten[{sol1, sol2, sol3}, 2]
v1L = v1 /. sol
v2L = v2 /. sol
v3L = v3 /. sol
```

{x1[t] Æ (1 + t) ξ1, x2[t] Æ (1 + t)^2 ξ2, x3[t] Æ (1 + t)^3 ξ3}

ξ1

$2\,(1+t)\,x_2$

$3\,(1+t)^2\,x_3$

■ The Accelerations in Lagrangian Coordinates

```
a1L = D[v1L, t]
a2L = D[v2L, t]
a3L = D[v3L, t]
```

0

$2\,x_2$

$6\,(1+t)\,x_3$

■ The Accelerations in Eulerian Coordinates

```
a1E = D[v10, t] + v10 D[v10 , x1] + v20 D[v10 , x2] + v30 D[v10 , x3 ]
a2E = D[v20, t] + v10 D[v20 , x1] + v20 D[v20 , x2] + v30 D[v20 , x3 ]
a1E = D[v30, t] + v10 D[v30 , x1] + v20 D[v30 , x2] + v30 D[v30 , x3 ]
```

0

$\frac{2\,x_2}{(1+t)^2}$

$\frac{6\,x_3}{(1+t)^2}$

■ The Streamlines

```
v1x = v10 /. x1 -> x
v2x = v20 /. x2 -> x2[x]
v3x = v30 /. x3 -> x3[x]
sol4 = DSolve[{x2'[x] == v2x / v1x, x2[x1] == x2}, x2[x], x]
sol5 = DSolve[{x3'[x] == v3x / v1x, x3[x1] == x3}, x3[x], x]
```

$\frac{x}{1+t}$

$\frac{2\,x_2[x]}{1+t}$

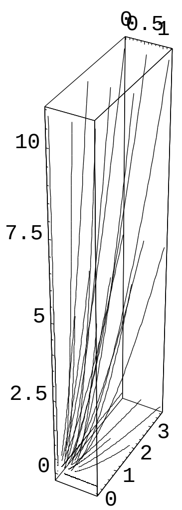
$\frac{3\,x_3[x]}{1+t}$

$\{\{x_2[x]\ \text{Æ}\ \frac{x^2\,x_2}{x_1^2}\}\}$

$\{\{x_3[x]\ \text{Æ}\ \frac{x^3\,x_3}{x_1^3}\}\}$

■ The Picture of Trajectories

```
 curves = {};
Do[x1i = (i - 1) * 0.2;
  Do[x2j = (j - 1) * 0.2;
    Do[x3k = (k - 1) * 0.2;
 y1 = x1 /. x1 -> x1i;
 y2 = x2 /. x2 -> x2j;
 y3 = x3 /. x3 -> x3k;
   cj = ParametricPlot3D[{y1, y2, y3}, {t, 0, 2},
        DisplayFunction -> Identity];
   AppendTo[curves, cj], {k, 3}], {j, 3}], {i, 3}];
gr = Show[curves,
     AspectRatio -> Automatic,
     DisplayFunction -> $DisplayFunction];
```



■ The Structure of the Output

The following results are obtained with the aid of the above program:

(1) the motion trajectories in the form of the functions x_1(t), x_2(t), x_3(t);
(2) the velocities v_i (x_i, t), i = 1,2,3, in the Lagrangian coordinates;
(3) the accelerations in the Lagrangian coordinates;
(4) the accelerations in the Eulerian coordinates;
(5) the expressions for the functions x_2(x) and x_3(x) describing the streamlines;
(6) the picture of particles trajectories.

To resize an individual picture obtained by *Mathematica*, please

(i) Click anywhere inside the cell, but not the cell bracket itself. A bounding box with small handles appears around the graphic image.

(ii) Drag one of the handles to adjust the size and shape of the bounding box. In this way it is possible to resize the height and width of the graphic image.