The Lagrangian $(\overset{0}{\epsilon})_{ij}$ and Eulerian ϵ_{ij} Strain Tensors

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■ Impressum

This Mathematica-Notebook is part of the book entitled
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■ General Description

This Notebook enables the user to find the analytic expressions for the components of the Lagrangian $\left(\overset{0}{e}\right)_{ij}$ and Eulerian e_{ij} strain tensors for a displacement field $u(u_1, u_2, u_3)$ given in the Cartesian basis (x_1, x_2, x_3), that is $u_j = u_j(x_1, x_2, x_3)$, j = 1,2,3.

This Notebook solves Problem 1.5, see Section 1.2 of the above book.

This Notebook can also be used for other displacement filelds. For this purpose, it is sufficient to specify the expressions for the displacements u_1= u[1], u_2= u[2], u_3= u[3] in terms of the Cartesian coordinates x_1= k[1], x_2= k[2], x_3= k[3] in the cell entitled "The specification of the displacement vector components".

■ User's Guide

■ Step 1

Compile the program cell entitled "The Specification of the Displacement Vector Components u_1, u_2, u_3" (see the Section "Program Listing").

■ Step 2

Compile the program cell entitled "Calculation of the Components $(e^0)_{ij}$, i,j = 1,2,3"
(see the Section "Program Listing").

■ Step 3

Compile the program cell entitled "Calculation of the Components e_{ij} by Almansi Formula"
(see the Section "Program Listing").

■ Step 4

Compile the program cell entitled "The Calculation of Displacements u_i in Eulerian Variables"
(see the Section "Program Listing").

■ Step 5

Compile the program cell entitled "The Almansi Formulas"
(see the Section "Program Listing").

■ Program Listing

■ The Specification of the Displacement Vector Components u_1, u_1, u_3

```
Clear[k, x];
u[1] = k[2] - k[3]; u[2] = k[2] + k[3]; u[3] = -k[1];
```

■ Calculation of the Components $(\overset{o}{\epsilon})_{ij}$, $i, j = 1, 2, 3.$

```
e̊ = IdentityMatrix[3];
Do[ Do[ g0 = Sum[D[u[m], k[i]] * D[u[m], k[j]], {m, 3}];
eij = (1/2) * (D[u[j], k[i]] + D[u[i], k[j]] + g0);
 e̊ = ReplacePart[e̊, eij, {i, j}], {i, 3}], {j, 3}];
Print["e^o = ", MatrixForm[e̊]];
```

$$e\text{^}o = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -1 \\ \frac{1}{2} & 2 & \frac{1}{2} \\ -1 & \frac{1}{2} & 1 \end{pmatrix}$$

■ The Calculation of the Components ϵ_{ij} by Almansi Formula

```
xx = Table[k[i] + u[i], {i, 3}]
sol = Solve[{xx[[1]] == x[1], xx[[2]] == x[2], xx[[3]] == x[3]},
            {k[1], k[2], k[3]}]
k[1] = First[k[1] /. sol]
k[2] = First[k[2] /. sol]
k[3] = First[k[3] /. sol]
```

```
{k[1] + k[2] - k[3], 2 k[2] + k[3], -k[1] + k[3]}

{{k[1] Æ -2 x[1] + x[2] - 3 x[3], k[2] Æ x[1] + x[3], k[3] Æ -2 x[1] + x[2] - 2 x[3]}}

-2 x[1] + x[2] - 3 x[3]

x[1] + x[3]

-2 x[1] + x[2] - 2 x[3]
```

■ The Calculation of Displacements u_i in Eulerian Variables

```
u[1] = u[1]
u[2] = u[2]
u[3] = u[3]
```

```
3 x[1] - x[2] + 3 x[3]

-x[1] + x[2] - x[3]

2 x[1] - x[2] + 3 x[3]
```

■ The Almansi Formulas

```
e = IdentityMatrix[3];
Do[Do[ g0 = Sum[D[u[m], x[i]] * D[u[m], x[j]], {m, 3}];
eij = (1 / 2) * (D[u[j], x[i]] + D[u[i], x[j]] - g0);
 e  = ReplacePart[e , eij, {i, j}], {i, 3}], {j, 3}];
Print["e = ", MatrixForm[e ]];
```

$$e = \begin{pmatrix} -4 & 2 & -\frac{11}{2} \\ 2 & -\frac{1}{2} & \frac{5}{2} \\ -\frac{11}{2} & \frac{5}{2} & -\frac{13}{2} \end{pmatrix}$$

■ The Structure of the Output

The following results are obtained with the aid of the above program:

(1) the components $(e^0)_{ij}$, i,j = 1,2,3, of the Lagrangian strain tensor;
(2) the expressions for the displacements u_i, i= 1,2,3, in Eulerian variables;
(3) the components e_{ij}, i,j = 1,2,3, of the Eulerian strain tensor.