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<b>2</b>	<b>Hall conductivity [2]</b>	<b>901</b>
<b>3</b>	<b>Magnetization and persistent currents [3]</b>	<b>904</b>

<i>Seminars by participants</i>		<i>911</i>
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Aspects topologiques de la physique en basse dimension. Topological aspects of low dimensional systems

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