Corrections for the first printing of *Spacetime*

I would like to thank those readers of the first edition who gave me feedback regarding related literature and errors in the text. I am especially thankful to Nico Giulini who found a serious error in the presentation of the fundamental theorem in affine geometry and who provided me with an extensive list of other errors and suggestions.

All errors known to me have been corrected in the second printing of the book.

This document has been prepared for owners of the first printing. It contains corrections of those errors which are not merely obvious typos and in particular a replacement for the text of Section 1.1.2 up to (and including) the proof of the fundamental theorem in affine geometry, Theorem 1.1.1.

Finally I would like to point to a review article on the material covered in chapter 8 and 9. This article provides many very illuminating examples of spacetimes as well as discussions which reinforce our sceptical approach towards the physical interpretation of singularity theorems:

Senovilla, J. M. M. (1998). Singularity Theorems and Their Consequences. *Gen. Rel. Grav. 30*, 701–848.

Hannover, 15th October 2001 M. Kriele

Corrections

Location	Error	Correction
Table of Contents: 2.7.1 and later	Levi-Cività	Levi-Civita
P. 19, line 6 from below	literarily	literally
P. 21, line 2 from below	intertial	inertial
Figure 1.4.7	*ell*$'$	ℓ'
Definition 2.1.2 (iii)	In the text the condition $\mathcal{U} \cup \mathcal{V} = M$ is missing	*A topological space* (M, τ) *is* connected *if* $\mathcal{U}, \mathcal{V} \in \tau$ *with* $\mathcal{U} \cap \mathcal{V} = \emptyset$ *and* $\mathcal{U} \cup \mathcal{V} = M$ *are necessarily of the form* $\mathcal{U} = M$, $\mathcal{V} = \emptyset$ *or* $\mathcal{V} = M$, $\mathcal{U} = \emptyset$.
Definition 2.1.2 (v)	$\mathcal{V}\bar{\mathcal{U}}$	$\mathcal{V} = \bar{\mathcal{U}}$
P.46-p.47		some $U, \hat{U}, U_i, U_j, V, W_i$ should read $\mathcal{U}, \hat{\mathcal{U}}, \mathcal{U}_i, \mathcal{U}_j, \mathcal{V}, \mathcal{W}$

continued on next page

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Location	Error	Correction
Definition 2.1.3		We assume $k \geq 1$ since we speak of C^k-diffeomorphisms. It is also possible to define C^0-manifolds where $\varphi_i \circ \varphi_j^{-1}$ are homeomorphisms. (This would be the approach taken in topology — but notice that C^0-manifolds are much more general then C^k-manifolds.)
Proof of Lemma 2.3.5	η_{lk}	n_{lk}
Theorem 2.4.1.	$f\colon \mathcal{J} \times \tilde{\mathcal{U}} \times \mathcal{V} \to \mathbb{K}^n, \quad (t,x,y) \mapsto f(t,x,y)$	$f\colon \mathcal{J} \times \mathcal{U} \times \mathcal{V} \to \mathbb{K}^n, \quad (t,x,y) \mapsto f(t,x,y)$
P. 118	Incomplete box	$\boxed{\begin{array}{c}\textbf{p. 79} \downarrow \\ [\downarrow \textbf{p. 121}]\end{array}}$
P. 124	Incomplete box	$\boxed{\begin{array}{c}\textbf{p. 155} \downarrow \\ [\downarrow \textbf{p. 125}]\end{array}}$
P. 155, line 9	$\hat{\Gamma}^a_{bc} = \frac{\partial htx^a}{\partial x^h} \frac{\partial^2 x^h}{\partial \hat{x}^b \partial \hat{x}^c}$	$\hat{\Gamma}^a_{bc} = \frac{\partial \hat{x}^a}{\partial x^h} \frac{\partial^2 x^h}{\partial \hat{x}^b \partial \hat{x}^c}$.
P. 266, line 12 from below	Astronomical observations seem to imply that $\|\Lambda\|$ is very small	I have been told that according to recent observations $\|\Lambda\|$ is not negligible. Note that the constant Λ is retained in most of the book.
P. 293	Hubble discovered that distant galaxies are moving away from us	Hubble merely discovered the the cosmological relation of red-shift and distance. Others have interpreted this observations as being due to cosmological expansion.

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Location	Error	Correction
P. 313		Figure 7.2.1 should be replaced by the figure on the cover of the book.
P. 422, line 13 from below.	Chruschiel	Chruściel

$1'.1'.2'$ Replacement for Section 1.1.2 up to (and including) the proof of the fundamental theorem in affine geometry

In this section we present some results of affine geometry which will be needed in the proof of Theorem 1.4.1. This section is very technical and should be omitted on first reading.

Let $o, x_1, \dots, x_k \in \mathbb{A}^n$ and $\alpha^1, \dots, \alpha^k \in \mathbb{R}$ such that $\sum_{i=1}^k \alpha^i = 1$. Then the *barycentre with masses* $\alpha^1, \dots, \alpha^k$,

$$\alpha^1 x_1 \dot{+} \alpha^2 x_2 \dot{+} \dots \dot{+} \alpha^k x_k := o + \sum_{i=1}^{k} \alpha^i (x_i - o),$$

is independent of o and therefore an affine invariant. The symbol $\dot{+}$ is defined via the right hand side and can only be applied to "linear combinations" where the real factors add to 1. An *affine subspace* B of $\mathbb{A}^n$ is a set of points $\{x = \alpha^1 x_1 \dot{+} \alpha^2 x_2 \dot{+} \dots \dot{+} \alpha^k x_k \big| \sum_{i=1}^k \alpha^i = 1\}$, where $x_1, \dots, x_k$ are pairwise different, fixed points. The *affine dimension* of B is $k-1$. It follows that an affine subspace is an affine space. An affine subspace of dimension 1 is called an *affine line*. We call points lying on a single line *collinear*. Observe that lines are the smallest sets which are invariant under parallel transport.

Lemma $1'.1'.1'$. *Let $x, y, z \in \mathbb{A}^n$. Then x, y, z lie on an affine line if and only if there exists a $\lambda \in \mathbb{R}$ such that $x = y + \lambda(z - y)$.*

Proof. x lies on the line generated by y, z if and only if there exists an $\beta \in \mathbb{R}$ with $x = \beta y \dot{+} (1-\beta) z = y + \beta(y - y) + (1-\beta)(z - y) = y + (1-\beta)(z - y)$. ∎

Definition $1'.1'.1'$. *An* affine map *is a map $f\colon \mathbb{A}^n \to \mathbb{A}^n$, $f(x) = A(x - o) + b$, where A is a linear map, $o \in \mathbb{A}^n$, and $b \in \mathbb{R}^n$. If A is bijective then f is called an* affine transformation*.*

A collineation *is a bijection $f\colon \mathbb{A}^n \to \mathbb{A}^n$ which maps any three collinear points into collinear points.*

Consider a line l and three points x_1, x_2, x_3 on l. Then the number λ given by $x_3 - x_1 = \lambda(x_2 - x_1)$ is denoted by

$$\frac{x_3 - x_1}{x_2 - x_1}.$$

The following lemma is the classical theorem of Thales. It will be used in the proof of the fundamental theorem in affine geometry (Theorem 1′.1′.1′ below).

Lemma 1′.1′.2′. *Let $H_1, H_2, H_3 \subset \mathbb{R}^n$ be parallel hypersurfaces and l be a line which intersects these hypersurfaces. Let $x_i(l) = H_i \cap l$. Then*

$$\frac{x_3(l) - x_1(l)}{x_2(l) - x_1(l)}.$$

does not depend on l.

Proof. Denote by $\vec{H}$ the subspace of $\mathbb{R}^n$ which is the associated vector space to the affine space H_1 (and since H_1, H_2, H_3 are parallel also to H_2, H_3). We consider the quotient space $\mathbb{A}^n/\vec{H}$ defined by

$$x \sim y \quad \text{if and only if} \quad y - x \in \vec{H}.$$

This space has a natural affine structure with associated vector space $\mathbb{R}^n/\vec{H}$ given by $\pi(x) - \pi(z) = \vec{\pi}(x - z)$ where $\pi, \vec{\pi}$ denote the projections to the equivalence classes. We have

$$\begin{aligned} \pi(x_3(l)) - \pi(x_1(l)) &= \vec{\pi}(x_3(l) - x_1(l)) \\ &= \vec{\pi}\left(\frac{x_3(l) - x_1(l)}{x_2(l) - x_1(l)}\,(x_2(l) - x_1(l))\right) \\ &= \frac{x_3(l) - x_1(l)}{x_2(l) - x_1(l)}\vec{\pi}\,(x_2(l) - x_1(l)) \\ &= \frac{x_3(l) - x_1(l)}{x_2(l) - x_1(l)}\,(\pi(x_2(l)) - \pi(x_1(l))) \end{aligned}$$

which implies that

$$\frac{x_3(l) - x_1(l)}{x_2(l) - x_1(l)} = \frac{\pi(x_3(l)) - \pi(x_1(l))}{\pi(x_2(l)) - \pi(x_1(l))}$$

only depends on the projected values. Now it is sufficient to observe that $\pi(x_i(l))$ is independent of l since all points in H_i are equivalent: $x, y \in H_i \Rightarrow \pi(x) = \pi(y)$. ∎

It is easy to see that all bijective, affine maps are collineations. Conversely, the fundamental theorem in affine geometry asserts that any collineation must be affine:

Theorem 1′.1′.1′. *Let $\mathbb{A}^n$ be an affine space over $\mathbb{R}$ with $n \geq 2$ and fix $o \in \mathbb{A}$. Let $f\colon \mathbb{A}^n \to \mathbb{A}^n$ be a bijection which takes each three collinear points into collinear points. Then there exists a point $b \in \mathbb{A}^n$ and an invertible linear map $\vec{f}$ such that $f(x) = \vec{f}(x - o) + b$ for all $x \in \mathbb{A}^n$.*

The proof is elementary but lengthy and requires some preparatory lemmas. We will follow (Berger 1987, p. 52–55) where one can also find a version of this theorem which holds in the complex case. Observe that the following proof makes heavy use of the assumption $n \geq 2$. The theorem does not hold for $n = 1$ since in this case any map maps collinear points into collinear points.

Lemma 1′.1′.3′. *Let $o, x_1, \ldots, x_k \in \mathbb{A}^n$, f be a collineation, $\lambda^1, \ldots, \lambda^k \in \mathbb{R}$, and*

$$x = o + \sum_{i=1}^{k} \lambda^i \,(x_i - o) \in \mathbb{A}^n.$$

Then there exist $\mu^1, \ldots, \mu^k \in \mathbb{R}$ such that

$$f(x) = f(o) + \sum_{i=1}^{k} \mu^i \,(f(x_i) - f(o))\,.$$

Proof. For $k = 1$ the claim is clear by the definition of a collineation. Assume now, the assertion is true for all $m \in \{1, \ldots, k-1\}$. For

$$x = o + \sum_{i=1}^{m+1} \lambda^i \,(x_i - o) \qquad \text{let} \qquad x' = o + \sum_{i=1}^{m} \lambda^i \,(x_i - o)\,.$$

Then we have

$$x = x' + \lambda^{m+1}(x_{m+1} - o) \tag{1′.1′.1}$$

and by induction hypothesis there are real numbers ${\mu'}^1, \ldots, {\mu'}^m$ with $f(x') - f(o) = \sum_{i=1}^{m} {\mu'}^i (f(x_i) - f(o))$. We define also

$$y = o + \lambda^{m+1}(x_{m+1} - o), \tag{1′.1′.2}$$

$$z = \frac{1}{2} y + \frac{1}{2} x'. \tag{1′.1′.3}$$

The triples $\{z, x', y\}$, $\{y, o, x_{m+1}\}$, and $\{z, o, x\}$ consist each of collinear points. This is clear for the first triple and follows from Lemma 1′.1′.1 for the second triple. To see this for the third triple observe that $y - o = x - x'$. $z = \frac{1}{2} y + \frac{1}{2} x'$ is the centre of the parallelogram defined by o, y, x, x' and therefore the intersection of the line connecting y with x' and the line connecting o with x. Since each of these three triples consists of collinear points there exist α, β, γ such that

$$\begin{aligned} f(z) &= \alpha f(x') + (1-\alpha) f(y), \\ f(x) &= \beta f(o) + (1-\beta) f(z), \\ f(y) &= f(o) + \gamma(f(x_{m+1}) - f(o)). \end{aligned}$$

This implies

$$\begin{aligned} f(x) &= \beta f(o) + (1-\beta) f(z) \\ &= \beta(f(o) - f(o)) + (1-\beta)(f(z) - f(o)) + f(o) \\ &= (1-\beta)((\alpha f(x') + (1-\alpha) f(y)) - f(o)) + f(o) \\ &= (1-\beta)(\alpha(f(x') - f(o)) + (1-\alpha)(f(y))) + f(o) \\ &= (1-\beta)(\alpha \sum_{i=1}^{m} {\mu'}^{i} \left(f(x_i) - f(o)\right) \\ &\quad + (1-\alpha)\gamma(f(x_{m+1}) - f(o))) + f(o) \\ &= \sum_{i=1}^{m+1} \mu^i \left(f(x_i) - f(o)\right) + f(o). \end{aligned}$$

∎

Lemma 1′.1′.4′. *Let $o, x_1, \ldots, x_n \in \mathbb{A}^n$ such that $\{x_1 - o, \ldots, x_n - o\}$ is a basis of $\mathbb{R}^n$. If f is a collineation then $\{f(x_1) - f(o), \ldots, f(x_n) - f(o)\}$ is also a basis of $\mathbb{R}^n$.*

Proof. Let $\tilde{x} \in \mathbb{A}^n$ be any point and let $x = f^{-1}(\tilde{x})$. Since $\{x_1 - o, \ldots, x_n - o\}$ is a basis of $\mathbb{R}^n$ there exist $\xi^i \in \mathbb{R}$ such that $x - o = \sum_{i=1}^n \xi^i (x_i - o)$. Lemma $1'.1'.3'$ implies that there exist $\mu^1, \ldots, \mu^n \in \mathbb{R}$ such that

$$\tilde{x} - f(o) = f(x) - f(o) = \sum_{i=1}^{k} \mu^i \left(f(x_i) - f(o)\right).$$

Since $\tilde{x}$ was arbitrary the assertion follows. ∎

Lemma 1′.1′.5′. *A bijection f is a collineation if and only if it maps affine lines onto affine lines.*

Proof. Let $x, y \in \mathbb{A}^n$ and denote by l the line spanned by these points. Let $\hat{z}$ be a point on the line spanned by $f(x)$, $f(y)$. We have to show that $z = f^{-1}(\hat{z}) \in l$. If this was not true than the vectors $z - x, y - x$ would be linearly independent. But then Lemma $1'.1'.4'$ would imply that $f(z) - f(x)$, $f(y) - f(x)$ where linearly independent as well. Contradiction to the construction of $\hat{z} = f(z)$ ∎

Lemma 1′.1′.6′. *Let f be a collineation. Then f maps parallel lines into parallel lines.*

Proof. Let l, $\tilde{l}$ be two parallel lines (which do not coincide — otherwise there would be nothing to prove). Since they are parallel they span a plane P rather than a 3-dimensional subspace of $\mathbb{A}^n$.

This plane is mapped into a plane P'. In order to see this consider a line $\hat{l}$ such that the lines $l, \hat{l}$ intersect and span P. It is clear that any line which intersects both l and $\hat{l}$ is contained in P. Moreover, any point $y \in P$ lies on a line $\bar{l}$ which intersects both l and $\hat{l}$. Let P' be the plane generated by the (intersecting) lines $f(l)$ and $f(\hat{l})$. $f(y)$ lies on the line $f(\bar{l})$ which intersects $f(l)$ and $f(\hat{l})$. Hence $f(\bar{l})$ (and therefore $f(y)$) lies in P'.

Having established that $f(P)$ is a subset of a plane we only have to show that $f(l) \cap f(\tilde{l}) = \emptyset$. If there was a point $z \in f(l) \cap f(\tilde{l})$ then $f^{-1}(z)$ would lie in both l and l' which is impossible since both lines are parallel. ∎

Lemma 1′.1′.7′. *Let $k\colon \mathbb{R} \to \mathbb{R}$ an automorphism, i.e., $k(\alpha\beta) = k(\alpha)k(\beta)$ and $k(\alpha+\beta) = k(\alpha) + k(\beta)$ for all real numbers α, β. If $k \neq 0$ then $k = \mathrm{id}$*

Proof. $k(0) = k(0+0) = k(0)+k(0)$ implies $k(0) = 0$. Assume, there is an $\alpha \neq 0$ with $k(\alpha) = 0$. Then $k(\beta) = k(\alpha)k(\beta/\alpha) = 0$ for all β and k must vanish. Hence $k(\alpha) \neq 0 \quad \forall \alpha \neq 0$. $k(1) = k(1 \cdot 1) = k(1)k(1)$ implies $k(1) = 1$. By induction we obtain $k(n) = n$ for all natural numbers. $k(-n) = k(0-n) = k(0) - k(n) = -k(n)$. Similarly, we have $k(1/n) = 1/k(n) = 1/n$. For $n, m \in \mathbb{Z}$ we have now $k(n/m) = n/m$ and the lemma is proved for all rational numbers. $\alpha \leq \beta$ implies $k(\alpha) \leq k(\beta)$ since for any positive number γ^2 we have $k(\gamma^2) = k(\gamma)k(\gamma) \geq 0$. Let now γ be any number. Then there exists a monotonically increasing sequence $\alpha_i \to \gamma$ of rational numbers and likewise a monotonically decreasing sequence of rational numbers $\beta_i \to \gamma$. Hence $\alpha_i = k(\alpha_i) \leq k(\gamma) \leq k(\beta_i) = \beta_i$ which implies $k(\gamma) = \gamma$. ∎

Observe that this lemma would be false if we had replaced $\mathbb{R}$ by $\mathbb{C}$ as $z \mapsto \bar{z}$ would be a counter example. This is why theorem 1′.1′.1′ (as stated above) is not true for affine spaces over the field $\mathbb{C}$.

Proof of Theorem 1′.1′.1′. Let $\vec{f}\colon \mathbb{R}^n \to \mathbb{R}^n$, $v \mapsto \vec{f}(v) = f(o+v) - f(o)$. The idea of proof is to construct an automorphism $k\colon \mathbb{R} \to \mathbb{R}$ such that $\vec{f}(\lambda v + \mu w) = k(\lambda)\vec{f}(v) + k(\mu)\vec{f}(w)$ holds for all $\lambda, \mu \in \mathbb{R}$ and $v, w \in \mathbb{R}^n$. We will use constructions based on parallel lines in order to represent vectors such as $v + w$, $(\lambda + \mu)v$, $\lambda\mu v$. Since f maps parallel lines into parallel lines (Lemma 1′.1′.6′) these constructions will be preserved by f

and can therefore be used in order to prove linearity and multiplicativity of $\vec{f}, k$.

We will first show that $\vec{f}$ is additive.

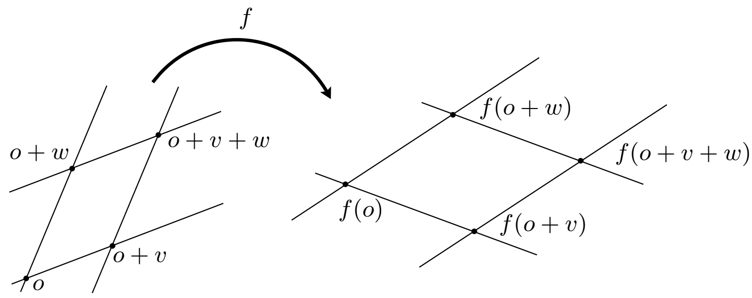

Fig. 1′.1′.1′. Additivity of f

Let $v, w \in \mathbb{R}^n$ and consider the lines l_v, l_w spanned by $o, o+v$ and $o, o+w$. The point $o+v+w$ is the intersection of the parallel translation of l_w that contains $o+v$ and of l_v that contains $o+w$ (cf. Figure 1′.1′.1′). Since parallel lines are mapped into parallel lines we know that $f(o+v+w)$ is constructed analogously from $f(o)$, $f(o+v)$, $f(o+w)$. Hence $\vec{f}(v+w) = f(o+v+w) - f(o) = f(o+v+w) - f(o+v) + f(o+v) - f(o) = f(o+w) - f(o) + f(o+v) - f(o) = \vec{f}(w) + \vec{f}(v)$. Here we have used the fact that the vectors connecting $f(o)$ with $f(o+w)$ and $f(o+v)$ with $f(o+v+w)$ are identical since they correspond to opposite sides of a parallelogram in a plane.

Now we show that there is a well defined automorphism $k\colon R \to \mathbb{R}$ such that $\vec{f}(\lambda v) = k(\lambda)\vec{f}(v)$ for all $v \in \mathbb{R}^n$ and $\lambda \in \mathbb{R}$. We first fix a vector v and consider the line l through o spanned by v. Denote by $g_l\colon l \to \mathbb{R}$ the map $o + \lambda v \mapsto \lambda$ and by $g_{f(l)}$ the map $f(o) + \mu\vec{f}(v) \mapsto \mu$. Since f maps the line through o which is spanned by v into the line through $f(o)$ which is spanned by $f(o+v) - f(o)$ the map $k\colon \mathbb{R} \to \mathbb{R}$ is well defined through the relationship $\vec{f}(\lambda v) = k(\lambda)\vec{f}(v)$. From

$$f(o) + k(\lambda)\vec{f}(v) = f(o) + \vec{f}(\lambda v) = f(o + \lambda v) = f(g_l^{-1}(\lambda))$$

we see that k is given by $k(\lambda) = g_{f(l)} \circ f \circ g_l^{-1}(\lambda)$.

In order to prove additivity of k we use the fact that $(\lambda+\mu)v = \lambda v + \mu v$ can be constructed using parallel lines (cf. Figure 1′.1′.2′) Let $w \in \mathbb{R}^n$ be linearly independent from v and consider the triangle defined by the points o, $o+w$, $o+\lambda v$. This triangle can be parallely translated so that the point o is mapped into $o + \mu v$. (We simply parallely translate the lines generated by its sides as indicated in the figure). Since this translation preserves the vectors defined by the sides of the triangle we

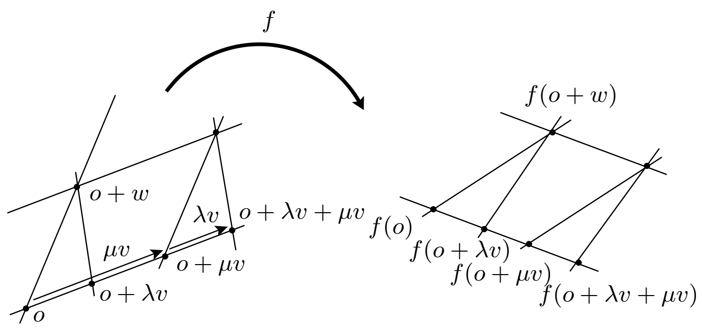

Fig. 1′.1′.2′. Additivity of k

have obtained a geometric construction of the point $o + \lambda v + \lambda w$. Since this construction only employs intersection points and parallel lines it is preserved by the map f. Hence we obtain $\vec{f}((\lambda+\mu)v) = \vec{f}(\lambda v) + \vec{f}(\mu v) = k(\lambda)\vec{f}(v) + k(\mu)\vec{f}(v)$ and therefore

$$\begin{aligned} k(\lambda + \mu) &= g_{f(l)} \circ f \circ g_l^{-1}(\lambda + \mu) = g_{f(l)} \circ f(o + (\lambda + \mu)v) \\ &= g_{f(l)}(f(o) + \vec{f}((\lambda + \mu)v)) \\ &= g_{f(l)}(f(o) + k(\lambda)\vec{f}(v) + k(\mu)\vec{f}(v)) \\ &= g_{f(l)}(f(o) + (k(\lambda) + k(\mu))\vec{f}(v)) = k(\lambda) + k(\mu). \end{aligned}$$

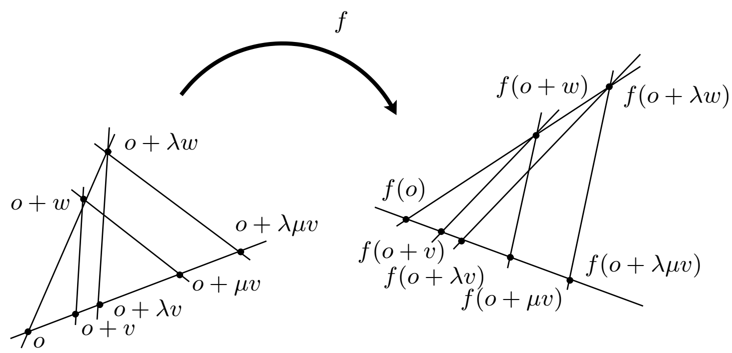

Fig. 1′.1′.3′. Multiplicativity of k

The proof of multiplicativity is similar and employs a slightly different geometrical construction (cf. Figure 1′.1′.3′) which is justified by Lemma 1′.1′.2′. The configuration in the first part of Figure 1′.1′.3′ lies in a plane whence hypersurfaces are simply lines. Denote by H_2 the line which connects $o + v$ with $o + w$, by H_1 its parallel translation through o, and by H_3 its parallel translation through $o + \lambda v$. Further denote the line through o and $o+v$ by l and the line which connects o with $o+w$ by l'. Using the notation of Lemma $1'.1'.2'$ we have

$$\lambda = \frac{(o+\lambda v) - o}{(o+v) - o} = \frac{x_3(l) - x_1(l)}{x_2(l) - x_1(l)}.$$

Hence Lemma $1'.1'.2'$ implies that the intersection of H_3 and l' is really $o+\lambda w$ as depicted in the figure. We apply this lemma a second time where the three parallel hypersurfaces H_2', H_1', H_3' are now given by the line connecting $o+\mu v$ with $o+w$, its parallel translation through o, and its parallel translation through $o+\lambda w$. It follows that the intersection of H_3' with l is $o+\mu(\lambda v) = o+\lambda\mu v$. Since this construction only employs intersections and parallel lines it is preserved by f and we obtain $\vec{f}(\lambda\mu v) = k(\lambda)k(\mu)\vec{f}(v)$. This implies

$$\begin{aligned} k(\lambda\mu) &= g_{f(l)} \circ f(o+\lambda\mu v) = g_{f(l)}(f(o) + \vec{f}(\lambda\mu v)) \\ &= g_{f(l)}(f(o) + k(\lambda)k(\mu)\vec{f}(v)) = k(\lambda)k(\mu). \end{aligned}$$

Hence k is really an automorphism of the real line. One can geometrically show that this automorphism neither depends on v nor on o. However in our case this automorphism is trivially well defined since we already know that the only non-zero automorphism of $\mathbb{R}$ is the identity. This also implies $\vec{f}(\lambda v) = \lambda\vec{f}(v)$ for all $\lambda \in \mathbb{R}$, $v \in \mathbb{R}^n$. Hence the theorem is proved. ∎

The text continues now with Definition 1.1.3 in the main text.

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Kriele, M.
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