

# Table of Contents

<b>1. Introduction</b>	1
1.1 Some Issues in Linear Computation	7
1.2 Three Examples of Linear Computation	13
1.2.1 Gargantuan Liquids, Inc.	13
1.2.2 Oil Refineries, bpd.	15
1.2.3 Save Berlin, usw.	20
<b>2. The Linear Programming Problem</b>	25
2.1 Standard and Canonical Forms	26
2.2 Matrices, Vectors, Scalars	27
<b>3. Basic Concepts</b>	33
3.1 A Fundamental Theorem	36
3.2 Notational Conventions and Illustrations	39
<b>4. Five Preliminaries</b>	43
4.1 Bases and Basic Feasible Solutions	43
4.2 Detecting Optimality	43
4.3 Detecting Unboundedness	44
4.4 A Rank-One Update	45
4.5 Changing Bases	45
<b>5. Simplex Algorithms</b>	49
5.1 Notation, Reading Instructions, Updating	50
5.2 Big M or How to Get Started	54
5.3 Selecting a Pivot Row and Column	56
5.4 Data Structures, Tolerances, Product Form	58
5.5 Equation Format and Cycling	63
5.6 Finiteness of a Simplex Algorithm	69
5.7 Canonical Form	71
5.7.1 A Worst-Case Example for a Simplex Algorithm	75
5.8 Block Pivots and Structure	77



5.8.1	A Generalized Product Form	79
5.8.2	Upper Bounds	82
<b>6.</b>	<b>Primal-Dual Pairs</b>	<b>87</b>
6.1	Weak Duality	89
6.2	Strong Duality	91
6.2.1	Economic Interpretation and Applications	94
6.3	Solvability, Redundancy, Separability	97
6.4	A Dual Simplex Algorithm	103
6.4.1	Correctness, Finiteness, Initialization	105
6.5	Post-Optimality	109
6.6	A Dynamic Simplex Algorithm	114
<b>7.</b>	<b>Analytical Geometry</b>	<b>121</b>
7.1	Points, Lines, Subspaces	124
7.2	Polyhedra, Ideal Descriptions, Cones	131
7.2.1	Faces, Valid Equations, Affine Hulls	134
7.2.2	Facets, Minimal Complete Descriptions, Quasi-Uniqueness	138
7.2.3	Asymptotic Cones and Extreme Rays	141
7.2.4	Adjacency I, Extreme Rays of Polyhedra, Homogenization	144
7.3	Point Sets, Affine Transformations, Minimal Generators	147
7.3.1	Displaced Cones, Adjacency II, Images of Polyhedra	150
7.3.2	Carathéodory, Minkowski, Weyl	155
7.3.3	Minimal Generators, Canonical Generators, Quasi-Uniqueness	157
7.4	Double Description Algorithms	165
7.4.1	Correctness and Finiteness of the Algorithm	168
7.4.2	Geometry, Euclidean Reduction, Analysis	173
7.4.3	The Basis Algorithm and All-Integer Inversion	180
7.4.4	An All-Integer Algorithm for Double Description	183
7.5	Digital Sizes of Rational Polyhedra and Linear Optimization	188
7.5.1	Facet Complexity, Vertex Complexity, Complexity of Inversion	190
7.5.2	Polyhedra and Related Polytopes for Linear Optimization	194
7.5.3	Feasibility, Binary Search, Linear Optimization	197
7.5.4	Perturbation, Uniqueness, Separation	202
7.6	Geometry and Complexity of Simplex Algorithms	207
7.6.1	Pivot Column Choice, Simplex Paths, Big M Revisited	208
7.6.2	Gaussian Elimination, Fill-In, Scaling	212



7.6.3	Iterative Step I, Pivot Choice, Cholesky Factorization .	216
7.6.4	Cross Multiplication, Iterative Step II, Integer Factorization . . . . .	219
7.6.5	Division Free Gaussian Elimination and Cramer's Rule	221
7.7	Circles, Spheres, Ellipsoids . . . . .	229
<b>8.</b>	<b>Projective Algorithms . . . . .</b>	<b>239</b>
8.1	A Basic Algorithm . . . . .	243
8.1.1	The Solution of the Approximate Problem . . . . .	245
8.1.2	Convergence of the Approximate Iterates . . . . .	246
8.1.3	Correctness, Finiteness, Initialization . . . . .	250
8.2	Analysis, Algebra, Geometry . . . . .	253
8.2.1	Solution to the Problem in the Original Space . . . . .	254
8.2.2	The Solution in the Transformed Space . . . . .	260
8.2.3	Geometric Interpretations and Properties . . . . .	264
8.2.4	Extending the Exact Solution and Proofs . . . . .	268
8.2.5	Examples of Projective Images . . . . .	271
8.3	The Cross Ratio . . . . .	274
8.4	Reflection on a Circle and Sandwiching . . . . .	278
8.4.1	The Iterative Step . . . . .	283
8.5	A Projective Algorithm . . . . .	288
8.6	Centers, Barriers, Newton Steps . . . . .	292
8.6.1	A Method of Centers . . . . .	296
8.6.2	The Logarithmic Barrier Function . . . . .	298
8.6.3	A Newtonian Algorithm . . . . .	303
8.7	Coda . . . . .	308
<b>9.</b>	<b>Ellipsoid Algorithms . . . . .</b>	<b>309</b>
9.1	Matrix Norms, Approximate Inverses, Matrix Inequalities . . .	316
9.2	Ellipsoid "Halving" in Approximate Arithmetic . . . . .	320
9.3	Polynomial-Time Algorithms for Linear Programming . . . . .	328
9.3.1	Linear Programming and Binary Search . . . . .	336
9.4	Deep Cuts, Sliding Objective, Large Steps, Line Search . . . . .	339
9.4.1	Linear Programming the Ellipsoidal Way: Two Examples . . . . .	344
9.4.2	Correctness and Finiteness of the DCS Ellipsoid Algorithm . . . . .	348
9.5	Optimal Separators, Most Violated Separators, Separation . . .	352
9.6	$\varepsilon$ -Solidification of Flats, Polytopal Norms, Rounding . . . . .	356
9.6.1	Rational Rounding and Continued Fractions . . . . .	361
9.7	Optimization and Separation . . . . .	368
9.7.1	$\varepsilon$ -Optimal Sets and $\varepsilon$ -Optimal Solutions . . . . .	371



9.7.2	Finding Direction Vectors in the Asymptotic Cone . . .	373
9.7.3	A CCS Ellipsoid Algorithm . . . . .	375
9.7.4	Linear Optimization and Polyhedral Separation . . . . .	378
<b>10.</b>	<b>Combinatorial Optimization: An Introduction . . . . .</b>	<b>387</b>
10.1	The Berlin Airlift Model Revisited . . . . .	389
10.2	Complete Formulations and Their Implications . . . . .	396
10.3	Extremal Characterizations of Ideal Formulations . . . . .	405
10.3.1	Blocking and Antiblocking Polyhedra . . . . .	414
10.4	Polyhedra with the Integrality Property . . . . .	417
 <b>Appendices</b>		
<b>A.</b>	<b>Short-Term Financial Management . . . . .</b>	<b>423</b>
<b>B.</b>	<b>Operations Management in a Refinery . . . . .</b>	<b>427</b>
<b>C.</b>	<b>Automatized Production: PCBs and Ulysses' Problem . . . .</b>	<b>441</b>
<b>References . . . . .</b>		<b>457</b>
<b>Bibliography . . . . .</b>		<b>479</b>
<b>Index . . . . .</b>		<b>495</b>





<http://www.springer.com/978-3-540-65833-7>

Linear Optimization and Extensions

Padberg, M.

1999, XXI, 501 p., Hardcover

ISBN: 978-3-540-65833-7