

Table of Contents

Introduction

Part I. Geodesic Metric Spaces	1
<hr/>	
1. Basic Concepts	2
Metric Spaces	2
Geodesics	4
Angles	8
The Length of a Curve	12
2. The Model Spaces M_κ^n	15
Euclidean n -Space \mathbb{E}^n	15
The n -Sphere \mathbb{S}^n	16
Hyperbolic n -Space \mathbb{H}^n	18
The Model Spaces M_κ^n	23
Alexandrov's Lemma	24
The Isometry Groups $\text{Isom}(M_\kappa^n)$	26
Approximate Midpoints	30
3. Length Spaces	32
Length Metrics	32
The Hopf-Rinow Theorem	35
Riemannian Manifolds as Metric Spaces	39
Length Metrics on Covering Spaces	42
Manifolds of Constant Curvature	45
4. Normed Spaces	47
Hilbert Spaces	47
Isometries of Normed Spaces	51
ℓ^p Spaces	53
5. Some Basic Constructions	56
Products	56
κ -Cones	59

Spherical Joins	63
Quotient Metrics and Gluing	64
Limits of Metric Spaces	70
Ultralimits and Asymptotic Cones	77
6. More on the Geometry of M_κ^n	81
The Klein Model for \mathbb{H}^n	81
The Möbius Group	84
The Poincaré Ball Model for \mathbb{H}^n	86
The Poincaré Half-Space Model for \mathbb{H}^n	90
Isometries of \mathbb{H}^2	91
M_κ^n as a Riemannian Manifold	92
7. M_κ-Polyhedral Complexes	97
Metric Simplicial Complexes	97
Geometric Links and Cone Neighbourhoods	102
The Existence of Geodesics	105
The Main Argument	108
Cubical Complexes	111
M_κ -Polyhedral Complexes	112
Barycentric Subdivision	115
More on the Geometry of Geodesics	118
Alternative Hypotheses	122
Appendix: Metrizing Abstract Simplicial Complexes	123
8. Group Actions and Quasi-Isometries	131
Group Actions on Metric Spaces	131
Presenting Groups of Homeomorphisms	134
Quasi-Isometries	138
Some Invariants of Quasi-Isometry	142
The Ends of a Space	144
Growth and Rigidity	148
Quasi-Isometries of the Model Spaces	150
Approximation by Metric Graphs	152
Appendix: Combinatorial 2-Complexes	153
<hr/>	
Part II. CAT(κ) Spaces	157
<hr/>	
1. Definitions and Characterizations of CAT(κ) Spaces	158
The CAT(κ) Inequality	158
Characterizations of CAT(κ) Spaces	161
CAT(κ) Implies CAT(κ') if $\kappa \leq \kappa'$	165
Simple Examples of CAT(κ) Spaces	167

Historical Remarks	168
<i>Appendix: The Curvature of Riemannian Manifolds</i>	169
2. Convexity and Its Consequences	175
Convexity of the Metric	175
Convex Subspaces and Projection	176
The Centre of a Bounded Set	178
Flat Subspaces	180
3. Angles, Limits, Cones and Joins	184
Angles in $\text{CAT}(\kappa)$ Spaces	184
4-Point Limits of $\text{CAT}(\kappa)$ Spaces	186
Cones and Spherical Joins	188
The Space of Directions	190
4. The Cartan-Hadamard Theorem	193
Local-to-Global	193
An Exponential Map	196
Alexandrov's Patchwork	199
Local Isometries and π_1 -Injectivity	200
Injectivity Radius and Systole	202
5. M_κ-Polyhedral Complexes of Bounded Curvature	205
Characterizations of Curvature $\leq \kappa$	206
Extending Geodesics	207
Flag Complexes	210
Constructions with Cubical Complexes	212
Two-Dimensional Complexes	215
Subcomplexes and Subgroups in Dimension 2	216
Knot and Link Groups	220
From Group Presentations to Negatively Curved 2-Complexes	224
6. Isometries of $\text{CAT}(0)$ Spaces	228
Individual Isometries	228
On the General Structure of Groups of Isometries	233
Clifford Translations and the Euclidean de Rham Factor	235
The Group of Isometries of a Compact Metric Space of Non-Positive Curvature	237
A Splitting Theorem	239
7. The Flat Torus Theorem	244
The Flat Torus Theorem	244
Cocompact Actions and the Solvable Subgroup Theorem	247
Proper Actions That Are Not Cocompact	250
Actions That Are Not Proper	254
Some Applications to Topology	254

8. The Boundary at Infinity of a CAT(0) Space	260
Asymptotic Rays and the Boundary ∂X	260
The Cone Topology on $\bar{X} = X \cup \partial X$	263
Horofunctions and Busemann Functions	267
Characterizations of Horofunctions	271
Parabolic Isometries	274
9. The Tits Metric and Visibility Spaces	277
Angles in \bar{X}	278
The Angular Metric	279
The Boundary $(\partial X, \angle)$ is a CAT(1) Space	285
The Tits Metric	289
How the Tits Metric Determines Splittings	291
Visibility Spaces	294
10. Symmetric Spaces	299
Real, Complex and Quaternionic Hyperbolic n -Spaces	300
The Curvature of $\mathbb{K}H^n$	304
The Curvature of Distinguished Subspaces of $\mathbb{K}H^n$	306
The Group of Isometries of $\mathbb{K}H^n$	307
The Boundary at Infinity and Horospheres in $\mathbb{K}H^n$	309
Horocyclic Coordinates and Parabolic Subgroups for $\mathbb{K}H^n$	311
The Symmetric Space $P(n, \mathbb{R})$	314
$P(n, \mathbb{R})$ as a Riemannian Manifold	314
The Exponential Map $\exp: M(n, \mathbb{R}) \rightarrow GL(n, \mathbb{R})$	316
$P(n, \mathbb{R})$ is a CAT(0) Space	318
Flats, Regular Geodesics and Weyl Chambers	320
The Iwasawa Decomposition of $GL(n, \mathbb{R})$	323
The Irreducible Symmetric space $P(n, \mathbb{R})_1$	324
Reductive Subgroups of $GL(n, \mathbb{R})$	327
Semi-Simple Isometries	331
Parabolic Subgroups and Horospherical Decompositions of $P(n, \mathbb{R})$	332
The Tits Boundary of $P(n, \mathbb{R})_1$ is a Spherical Building	337
$\partial_T P(n, \mathbb{R})$ in the Language of Flags and Frames	340
Appendix: Spherical and Euclidean Buildings	342
11. Gluing Constructions	347
Gluing CAT(κ) Spaces Along Convex Subspaces	347
Gluing Using Local Isometries	350
Equivariant Gluing	355
Gluing Along Subspaces that are not Locally Convex	359
Truncated Hyperbolic Spaces	362
12. Simple Complexes of Groups	367
Stratified Spaces	368

Group Actions with a Strict Fundamental Domain	372
Simple Complexes of Groups: Definition and Examples	375
The Basic Construction	381
Local Development and Curvature	387
Constructions Using Coxeter Groups	391
Part III. Aspects of the Geometry of Group Actions	397
H. δ-Hyperbolic Spaces	398
1. Hyperbolic Metric Spaces	399
The Slim Triangles Condition	399
Quasi-Geodesics in Hyperbolic Spaces	400
k -Local Geodesics	405
Reformulations of the Hyperbolicity Condition	407
2. Area and Isoperimetric Inequalities	414
A Coarse Notion of Area	414
The Linear Isoperimetric Inequality and Hyperbolicity	417
Sub-Quadratic Implies Linear	422
More Refined Notions of Area	425
3. The Gromov Boundary of a δ-Hyperbolic Space	427
The Boundary ∂X as a Set of Rays	427
The Topology on $X \cup \partial X$	429
Metrizing ∂X	432
I. Non-Positive Curvature and Group Theory	438
1. Isometries of CAT(0) Spaces	439
A Summary of What We Already Know	439
Decision Problems for Groups of Isometries	440
The Word Problem	442
The Conjugacy Problem	445
2. Hyperbolic Groups and Their Algorithmic Properties	448
Hyperbolic Groups	448
Dehn's Algorithm	449
The Conjugacy Problem	451
Cone Types and Growth	455
3. Further Properties of Hyperbolic Groups	459
Finite Subgroups	459
Quasiconvexity and Centralizers	460
Translation Lengths	464
Free Subgroups	467
The Rips Complex	468

4. Semihyperbolic Groups	471
Definitions	471
Basic Properties of Semihyperbolic Groups	473
Subgroups of Semihyperbolic Groups	475
5. Subgroups of Cocompact Groups of Isometries	481
Finiteness Properties	481
The Word, Conjugacy and Membership Problems	487
Isomorphism Problems	491
Distinguishing Among Non-Positively Curved Manifolds	494
6. Amalgamating Groups of Isometries	496
Amalgamated Free Products and HNN Extensions	497
Amalgamating Along Abelian Subgroups	500
Amalgamating Along Free Subgroups	503
Subgroup Distortion and the Dehn Functions of Doubles	506
7. Finite-Sheeted Coverings and Residual Finiteness	511
Residual Finiteness	511
Groups Without Finite Quotients	514
C. Complexes of Groups	519
1. Small Categories Without Loops (Scwols)	520
Scwols and Their Geometric Realizations	521
The Fundamental Group and Coverings	526
Group Actions on Scwols	528
The Local Structure of Scwols	531
2. Complexes of Groups	534
Basic Definitions	535
Developability	538
The Basic Construction	542
3. The Fundamental Group of a Complex of Groups	546
The Universal Group $FG(\mathcal{Y})$	546
The Fundamental Group $\pi_1(G(\mathcal{Y}), \sigma_0)$	548
A Presentation of $\pi_1(G(\mathcal{Y}), \sigma_0)$	549
The Universal Covering of a Developable Complex of Groups	553
4. Local Developments of a Complex of Groups	555
The Local Structure of the Geometric Realization	555
The Geometric Realization of the Local Development	557
Local Development and Curvature	562
The Local Development as a Scwol	564
5. Coverings of Complexes of Groups	566
Definitions	566

The Fibres of a Covering	568
The Monodromy	572
A Appendix: Fundamental Groups and Coverings of Small Categories	573
Basic Definitions	574
The Fundamental Group	576
Covering of a Category	579
The Relationship with Coverings of Complexes of Groups	583
\mathcal{G}. Groupoids of local isometries	584
1. Orbifolds	585
Basic Definitions	585
Coverings of Orbifolds	589
Orbifolds with Geometric Structures	591
2. Étale Groupoids, Homomorphisms and Equivalences	594
Étale Groupoids	594
Equivalences and Developability	597
Groupoids of Local Isometries	601
Statement of the Main Theorem	603
3. The Fundamental Group and Coverings of Étale Groupoids	604
Equivalence and Homotopy of \mathcal{G} -Paths	604
The Fundamental Group $\pi_1((\mathcal{G}, X), x_0)$	607
Coverings	609
4. Proof of the Main Theorem	613
Outline of the Proof	613
\mathcal{G} -Geodesics	614
The Space \hat{X} of \mathcal{G} -Geodesics Issuing from a Base Point	616
The Space $\tilde{X} = \hat{X}/\mathcal{G}$	617
The Covering $p : \tilde{X} \rightarrow X$	618
References	620
Index	637

Metric Spaces of Non-Positive Curvature

Bridson, M.R.; Häfliger, A.

1999, XXI, 643 p., Hardcover

ISBN: 978-3-540-64324-1