

Contents

Introduction	v
1. Preliminaries	1
1.1. Notation and conventions	1
1.2. Basic results concerning weights	2
1.2.1. General weights	2
1.2.2. A_p weights	3
1.2.3. Doubling weights	6
1.2.4. A_∞ weights	7
1.2.5. Proof of Muckenhoupt's maximal theorem	10
1.2.6. Boundedness of singular integrals	12
1.2.7. Two theorems by Muckenhoupt and Wheeden	12
2. Sobolev spaces	15
2.1. The Sobolev space $W_w^{m,p}(\Omega)$	16
2.1.1. Approximation results	17
2.1.2. Extension theorems	19
2.1.3. An interpolation inequality	23
2.2. The Sobolev space $V_w^{m,p}(\Omega)$	25
2.3. Hausdorff measures	37
2.4. Isoperimetric inequalities	40
2.4.1. Preliminary lemmas	41
2.4.2. Extensions of some results by David and Semmes	44
2.4.3. Isoperimetric inequalities involving lower Minkowski content	47
2.4.4. Isoperimetric inequalities with Hausdorff measures	49
2.4.5. A boxing inequality	53
2.5. Some Sobolev type inequalities	54
2.6. Embeddings into $L_\mu^q(\Omega)$	58
2.6.1. Introduction	59
2.6.2. Embedding theorems	62

3. Potential theory	69
3.1. Norm inequalities for fractional integrals and maximal functions	70
3.1.1. Proof of the main inequality and some corollaries	70
3.1.2. An inequality for Bessel potentials	75
3.2. Meyers' theory for L^p -capacities	77
3.2.1. Outline of Meyers' theory	77
3.2.2. Capacitary measures and capacitary potentials	80
3.3. Bessel and Riesz capacities	88
3.3.1. Basic properties	88
3.3.2. Adams' formula for the capacity of a ball	92
3.4. Hausdorff capacities	97
3.4.1. Basic properties	97
3.4.2. The capacity of a ball	99
3.4.3. Non-triviality of $\mathcal{H}_{w,\infty}^{N-\alpha}$	104
3.4.4. Local equivalence between $\mathcal{H}_{w,p}^{N-\alpha}$ and $\mathcal{H}_{w,\infty}^{N-\alpha}$	108
3.4.5. Continuity properties	110
3.4.6. Frostman's lemma	113
3.5. Variational capacities	115
3.5.1. The case $1 < p < \infty$	115
3.5.2. The case $p = 1$	117
3.5.3. An embedding theorem	120
3.6. Thinness: The case $1 < p < \infty$	121
3.6.1. Preliminary considerations	122
3.6.2. A Wolff type inequality	124
3.6.3. Proof of the Kellogg property	127
3.6.4. A concept of thinness based on a condenser capacity	131
3.7. Thinness: The case $p = 1$	134
4. Applications of potential theory to Sobolev spaces	141
4.1. Quasicontinuity	141
4.1.1. The case $1 < p < \infty$	142
4.1.2. The case $p = 1$	144
4.2. Measures in the dual of $W_w^{m,p}(\Omega)$	148
4.2.1. The case $1 < p < \infty$	148
4.2.2. The case $p = 1$	149
4.3. Poincaré type inequalities	151
4.3.1. The case $1 < p < \infty$	151
4.3.2. The case $p = 1$	155
4.4. Spectral synthesis	156
4.4.1. The case $1 < p < \infty$	157
4.4.2. The case $p = 1$	160
References	163
Index	171



<http://www.springer.com/978-3-540-67588-4>

Nonlinear Potential Theory and Weighted Sobolev
Spaces

Turesson, B.O.

2000, XII, 180 p., Softcover

ISBN: 978-3-540-67588-4