

Preface

This book reviews equivariant localization techniques for the evaluation of Feynman path integrals. It develops systematic geometric methods for studying the semi-classical properties of phase space path integrals for dynamical systems, emphasizing the relations with integrable and topological quantum field theories. Beginning with a detailed review of the relevant mathematical background – equivariant cohomology and the Duistermaat-Heckman theorem, it demonstrates how the localization ideas are related to classical integrability and how they can be formally extended to derive explicit localization formulas for path integrals in special instances using BRST quantization techniques. Various loop space localizations are presented and related to notions in quantum integrability and topological field theory. The book emphasizes the common symmetries that such localizable models always possess and uses these symmetries to discuss the range of applicability of the localization formulas. A number of physical and mathematical applications are presented in connection with elementary quantum mechanics, Morse theory, index theorems, character formulas for semi-simple Lie groups, quantization of spin systems, unitary integrations in matrix models, modular invariants of Riemann surfaces, supersymmetric quantum field theories, two-dimensional Yang-Mills theory, conformal field theory, cohomological field theories and the loop expansion in quantum field theory. Some modern techniques of path integral quantization, such as coherent state methods, are also discussed. The relations between equivariant localization and other ideas in topological field theory, such as the Batalin-Fradkin-Vilkovisky and Mathai-Quillen formalisms, are presented and used to discuss the general relationship between topological field theories and more conventional physical models.

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