

Contents

Selected Solutions of Einstein's Field Equations: Their Role in General Relativity and Astrophysics

<i>Jiří Bičák</i>	1
1 Introduction and a Few Excursions	1
1.1 A Word on the Role of Explicit Solutions in Other Parts of Physics and Astrophysics	3
1.2 Einstein's Field Equations	5
1.3 "Just So" Notes on the Simplest Solutions: The Minkowski, de Sitter, and Anti-de Sitter Spacetimes	8
1.4 On the Interpretation and Characterization of Metrics	11
1.5 The Choice of Solutions	15
1.6 The Outline	17
2 The Schwarzschild Solution	19
2.1 Spherically Symmetric Spacetimes	19
2.2 The Schwarzschild Metric and Its Role in the Solar System . .	20
2.3 Schwarzschild Metric Outside a Collapsing Star	21
2.4 The Schwarzschild–Kruskal Spacetime	25
2.5 The Schwarzschild Metric as a Case Against Lorentz-Covariant Approaches	28
2.6 The Schwarzschild Metric and Astrophysics	29
3 The Reissner–Nordström Solution	31
3.1 Reissner–Nordström Black Holes and the Question of Cosmic Censorship	32
3.2 On Extreme Black Holes, d -Dimensional Black Holes, String Theory and "All That"	39
4 The Kerr Metric	42
4.1 Basic Features	42
4.2 The Physics and Astrophysics Around Rotating Black Holes .	47
4.3 Astrophysical Evidence for a Kerr Metric	50
5 Black Hole Uniqueness and Multi-black Hole Solutions	52
6 On Stationary Axisymmetric Fields and Relativistic Disks	55
6.1 Static Weyl Metrics	55
6.2 Relativistic Disks as Sources of the Kerr Metric and Other Stationary Spacetimes	57

6.3	Uniformly Rotating Disks	59
7	Taub-NUT Space	62
7.1	A New Way to the NUT Metric	62
7.2	Taub-NUT Pathologies and Applications	64
8	Plane Waves and Their Collisions	66
8.1	Plane-Fronted Waves	66
8.2	Plane-Fronted Waves: New Developments and Applications	71
8.3	Colliding Plane Waves	72
9	Cylindrical Waves	77
9.1	Cylindrical Waves and the Asymptotic Structure of 3-Dimensional General Relativity	78
9.2	Cylindrical Waves and Quantum Gravity	82
9.3	Cylindrical Waves: a Miscellany	85
10	On the Robinson–Trautman Solutions	86
11	The Boost-Rotation Symmetric Radiative Spacetimes	88
12	The Cosmological Models	93
12.1	Spatially Homogeneous Cosmologies	95
12.2	Inhomogeneous Cosmologies	102
13	Concluding Remarks	105
	References	108

The Cauchy Problem for the Einstein Equations

	<i>Helmut Friedrich, Alan Rendall</i>	127
1	Introduction	127
2	Basic Observations and Concepts	131
2.1	The Principal Symbol	132
2.2	The Constraints	135
2.3	The Bianchi Identities	137
2.4	The Evolution Equations	137
2.5	Assumptions and Consequences	146
3	PDE Techniques	147
3.1	Symmetric Hyperbolic Systems	147
3.2	Symmetric Hyperbolic Systems on Manifolds	157
3.3	Other Notions of Hyperbolicity	159
4	Reductions	164
4.1	Hyperbolic Systems from the ADM Equations	167
4.2	The Einstein–Euler System	173
4.3	The Initial Boundary Value Problem	185
4.4	The Einstein–Dirac System	193
4.5	Remarks on the Structure of the Characteristic Set	200
5	Local Evolution	201
5.1	Local Existence Theorems for the Einstein Equations	201
5.2	Uniqueness	204
5.3	Cauchy Stability	206
5.4	Matter Models	207

5.5	An Example of an Ill-Posed Initial Value Problem	214
5.6	Symmetries	216
6	Outlook	217
	References	219

Post-Newtonian Gravitational Radiation

<i>Luc Blanchet</i>		225
1	Introduction	225
1.1	On Approximation Methods in General Relativity	225
1.2	Field Equations and the No-Incoming-Radiation Condition	228
1.3	Method and General Physical Picture	231
2	Multipole Decomposition	233
2.1	The Matching Equation	233
2.2	The Field in Terms of Multipole Moments	236
2.3	Equivalence with the Will–Wiseman Multipole Expansion	238
3	Source Multipole Moments	240
3.1	Multipole Expansion in Symmetric Trace-Free Form	240
3.2	Linearized Approximation to the Exterior Field	241
3.3	Derivation of the Source Multipole Moments	242
4	Post-Minkowskian Approximation	244
4.1	Multipolar Post-Minkowskian Iteration of the Exterior Field	244
4.2	The “Canonical” Multipole Moments	246
4.3	Retarded Integral of a Multipolar Extended Source	247
5	Radiative Multipole Moments	248
5.1	Definition and General Structure	249
5.2	The Radiative Quadrupole Moment to 3PN Order	250
5.3	Tail Contributions in the Total Energy Flux	251
6	Post-Newtonian Approximation	253
6.1	The Inner Metric to 2.5PN Order	254
6.2	The Mass-Type Source Moment to 2.5PN Order	256
7	Point-Particles	258
7.1	Hadamard Partie Finie Regularization	259
7.2	Multipole Moments of Point-Mass Binaries	261
7.3	Equations of Motion of Compact Binaries	263
7.4	Gravitational Waveforms of Inspiralling Compact Binaries	265
8	Conclusion	267

Duality and Hidden Symmetries in Gravitational Theories

<i>Dieter Maison</i>		273
1	Introduction	273
2	Electromagnetic Duality	277
3	Duality in Kaluza–Klein Theories	279
3.1	Dimensional Reduction from D to d Dimensions	280
3.2	Reduction to $d = 4$ Dimensions	282
3.3	Reduction to $d = 3$ Dimensions	285

3.4	Reduction to $d = 2$ Dimensions	290
4	Geroch Group	292
5	Stationary Black Holes	302
5.1	Spherically Symmetric Solutions	306
5.2	Uniqueness Theorems for Static Black Holes	312
5.3	Stationary, Axially Symmetric Black Holes	314
6	Acknowledgments	316
7	Non-linear σ -Models and Symmetric Spaces	316
7.1	Non-compact Riemannian Symmetric Spaces	316
7.2	Pseudo-Riemannian Symmetric Spaces	319
7.3	Consistent Truncations	319
8	Structure of the Lie Algebra	319

Time-Independent Gravitational Fields

	<i>Robert Beig, Bernd Schmidt</i>	325
1	Introduction	325
2	Field Equations	327
2.1	Generalities	327
2.2	Axial Symmetry	333
2.3	Asymptotic Flatness: Lichnerowicz Theorems	334
2.4	Newtonian Limit	339
2.5	Existence Issues and the Newtonian Limit	340
3	Far Fields	341
3.1	Far-Field Expansions	341
3.2	Conformal Treatment of Infinity, Multipole Moments	344
4	Global Rotating Solutions	350
4.1	Lindblom's Theorem	350
4.2	Existence of Stationary Rotating Axi-symmetric Fluid Bodies	353
4.3	The Neugebauer–Meinel Disk	357
5	Global Non-rotating Solutions	360
5.1	Elastic Static Bodies	360
5.2	Are Perfect Fluids $O(3)$ -Symmetric?	362
5.3	Spherically Symmetric, Static Perfect Fluid Solutions	365
5.4	Spherically Symmetric, Static Einstein–Vlasov Solutions	370

Gravitational Lensing from a Geometric Viewpoint

	<i>Volker Perlick</i>	373
1	Introduction	373
2	Some Basic Notions of Spacetime Geometry	375
3	Gravitational Lensing in Arbitrary Spacetimes	378
3.1	Conjugate Points and Cut Points	381
3.2	The Geometry of Light Cones	385
3.3	Criteria for Multiple Imaging	391
3.4	Fermat's Principle	396

3.5	Morse Index Theory for Fermat's Principle	399
4	Gravitational Lensing in Globally Hyperbolic Spacetimes	403
4.1	Criteria for Multiple Imaging in Globally Hyperbolic Spacetimes	405
4.2	Morse Theory in Globally Hyperbolic Spacetimes	408
5	Gravitational Lensing in Asymptotically Simple and Empty Spacetimes	414
	References	422
	Jürgen Ehlers – Bibliography	427

Einstein's Field Equations and Their Physical
Implications

Selected Essays in Honour of Jürgen Ehlers

Schmidt, B.G. (Ed.)

2000, XIII, 433 p., Hardcover

ISBN: 978-3-540-67073-5