

Contents

1. Introduction	1
1.1 The Aim of Computable Analysis	1
1.2 Why a New Introduction?	2
1.3 A Sketch of TTE	3
1.3.1 A Model of Computation	3
1.3.2 A Naming System for Real Numbers	4
1.3.3 Computable Real Numbers and Functions	4
1.3.4 Subsets of Real Numbers	7
1.3.5 The Space $C[0; 1]$ of Continuous Functions	8
1.3.6 Computational Complexity of Real Functions	9
1.4 Prerequisites and Notation	10
2. Computability on the Cantor Space	13
2.1 Type-2 Machines and Computable String Functions	14
2.2 Computable String Functions are Continuous	27
2.3 Standard Representations of Sets of Continuous String Functions	33
2.4 Effective Subsets	43
3. Naming Systems	51
3.1 Continuity and Computability Induced by Naming Systems	51
3.2 Admissible Naming Systems	62
3.3 Constructions of New Naming Systems	75
4. Computability on the Real Numbers	85
4.1 Various Representations of the Real Numbers	85
4.2 Computable Real Numbers	101
4.3 Computable Real Functions	108
5. Computability on Closed, Open and Compact Sets	123
5.1 Closed Sets and Open Sets	123
5.2 Compact Sets	143

6. Spaces of Continuous Functions	153
6.1 Various representations	153
6.2 Computable Operators on Functions, Sets and Numbers	163
6.3 Zero-Finding	173
6.4 Differentiation and Integration	182
6.5 Analytic Functions	190
7. Computational Complexity	195
7.1 Complexity of Type-2 Machine Computations	195
7.2 Complexity Induced by the Signed Digit Representation	204
7.3 The Complexity of Some Real Functions	218
7.4 Complexity on Compact Sets	230
8. Some Extensions	237
8.1 Computable Metric Spaces	237
8.2 Degrees of Discontinuity	244
9. Other Approaches to Computable Analysis	249
9.1 Banach/Mazur Computability	249
9.2 Grzegorzczuk's Characterizations	250
9.3 The Pour-El/Richards Approach	252
9.4 Ko's Approach	254
9.5 Domain Theory	256
9.6 Markov's Approach	258
9.7 The real-RAM and Related Models	260
9.8 Comparison	266
References	269
Index	277

Computable Analysis

An Introduction

Weihrauch, K.

2000, X, 288 p. 45 illus., 1 illus. in color., Hardcover

ISBN: 978-3-540-66817-6