LECTURE FIVE

DISTRIBUTIVITY, COLLECTIVITY AND CUMULATIVITY

5.1. THE LANGUAGE OF PLURALITY

In the language of plurality introduced in this lecture, we will not yet incorporate a full treatment of verbs. So the language (and the analysis of plurality, in this respect) is poorer than Scha's. For the moment, we won't have functional abstraction, and we will have only one-place verbs, which -again for the moment - we will treat in the same way as nouns: as sets. We do have some plurality operators that Scha doesn't have. In the next lecture, we will combine the present language of plurality with the language of events from lecture Two, to give a full treatment of verbs.

5.1.1. SYNTAX OF THE LANGUAGE OF PLURALITY

TYPES:
TYPE is the set {d, pow(d), t}
-d is the type of individuals
-pow(d) is the type of sets of individuals
-t is the type of truth values

EXPRESSIONS:
-We have constants and variables of type d.
-We have constants of type pow(d), nominal constants and verbal constants.
-We have the following special constants of type pow(d):
IND, GROUP, ATOM, SUM, SUMOFGROUP, D
These will get their obvious interpretation.

We define EXPa, the expressions of type a:
Constants and variables:
-CONa $\cup$ VARa $\subseteq$ EXPa

Connectives and identity:
-if $\phi,\psi \in$ t then $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi) \in$ t

-if $\alpha,\beta \in d$ then $(\alpha=\beta) \in t$

Quantification:
-if $x \in VARd$, $P \in pow(d)$ and $\phi \in t$
then $\forall x \in P{:}\phi$, $\exists x \in P{:}\phi \in t$
Set formation:
-if $x \in VARd$, $P \in pow(d)$ and $\phi \in t$
then $\{x \in P{:}\phi\} \in pow(d)$
Set application:
-if $\alpha \in d$ and $P \in pow(d)$ then $(\alpha \in P) \in t$

Plurality:
-If $\alpha,\beta \in d$ then $(\alpha \sqsubseteq \beta) \in t$
-if $\alpha,\beta \in d$ then $(\alpha \sqcup \beta) \in d$
-If $P \in pow(d)$ then $\sigma(P) \in d$
-If $P \in pow(d)$ then $\sqcup(P) \in d$
-if $\alpha \in d$ then $\uparrow\alpha, \downarrow\alpha \in d$
-If $\alpha \in d$ then $AT(\alpha) \in pow(d)$
-if $P \in pow(d)$ then $AT(P)$, ${}^{*}P$, ${}^{D}P \in pow(d)$

5.1.2. SEMANTICS FOR THE LANGUAGE OF PLURALITY

MODELS:
A **model for the language of plurality** is a tuple
$M = \langle \boldsymbol{D}, \perp, i \rangle$ where:
1. $\boldsymbol{D}$ is a domain of singular and plural individuals with groups.
2. $\perp$, the undefined object, is not in D.
3. i is the interpretation function for the constants:

Our domains are:
-$Dd = D \cup \{\perp\}$
-$Dpow(d) = pow(d)$
-$Dt = \{0,1\}$

The interpretation function i assigns to every constant of type a an interpretation in domain Da; i assigns all special constants their obvious interpretation.
Assignment functions are functions from VARd into Dd.

SEMANTICS:
We define $[\![\alpha]\!]M,g$, the interpretation of α in M relative to g:
Constants and variables:
-if $c \in CONd$ then $[\![c]\!]M,g = i(c)$
-if $x \in VARd$ then $[\![x]\!]M,g = g(x)$

Connectives and identity:

-$[\![\neg\phi]\!]^{M,g} = 1$ iff $[\![\phi]\!]^{M,g} = 0$; 0 otherwise.

-$[\![(\phi \wedge \psi)]\!]^{M,g} = 1$ iff $[\![\phi]\!]^{M,g}=1$ and $[\![\psi]\!]^{M,g} = 1$; 0 otherwise.

-$[\![(\phi \vee \psi)]\!]^{M,g} = 1$ iff $[\![\phi]\!]^{M,g} = 1$ or $[\![\psi]\!]^{M,g} = 1$; 0 otherwise.

-$[\![(\alpha=\beta)]\!]^{M,g} = 1$ iff $[\![\alpha]\!]^{M,g} = [\![\beta]\!]^{M,g}$ and $[\![\alpha]\!]^{M,g}, [\![\beta]\!]^{M,g} \neq \perp$; 0 otherwise.

Quantification:

-$[\![\forall x \in P{:}\phi]\!]^{M, g} = 1$ iff for all $d \in [\![P]\!]^{M,g}$: $[\![\phi]\!]^{M,g[x:d]} = 1$; 0 otherwise.

-$[\![\exists x \in P{:}\phi]\!]^{M,g} = 1$ iff for some $d \in [\![P]\!]^{M,g}$: $[\![\phi]\!]^{M,g[x:d]} = 1$; 0 otherwise.

Sets:

-$[\![\{x \in P{:}\phi\}]\!]^{M,g} = \{d \in [\![P]\!]^{M,g}: [\![\phi]\!]^{M,g[x:d]} = 1\}$

-$[\![(\alpha \in P)]\!]^{M,g} = 1$ iff $[\![\alpha]\!]^{M,g} \in [\![P]\!]^{M,g}$; 0 otherwise.

Plurality:

-$[\![\alpha \sqsubseteq \beta]\!]^{M,g} = 1$ iff $[\![\alpha]\!]^{M,g} \sqsubseteq [\![\beta]\!]^{M,g}$; 0 otherwise.

-$[\![\alpha \sqcup \beta]\!]^{M,g} = [\![\alpha]\!]^{M, g}\ [\![\beta]\!]^{M, g}$ if $[\![\alpha]\!]^{M, g} \neq \perp$, $[\![\beta]\!]^{M,g} \neq \perp$; $\perp$ otherwise

-$[\![\sigma(P)]\!]^{M,g} = \sqcup\ ([\![P]\!]^{M,g})$ if $\sqcup([\![P]\!]^{M,g}) \in [\![P]\!]^{M,g}$; $\perp$ otherwise

Note that σ does not have the same interpretation as it did in Scha's theory. σ is no longer the iota-operator, but Link's σ operator. Note: $\sigma(P)$ is written $\sigma x.P(x)$ in Link's papers and my earlier papers.

-$[\![\sqcup(P)]\!]^{M,g} = \sqcup([\![P]\!]^{M,g})$ if $[\![P]\!]^{M,g} \neq \emptyset$; $\perp$ otherwise

-$[\![\uparrow\alpha]\!]^{M,g} = \uparrow([\![\alpha]\!]^{M,g})$ if $[\![\alpha]\!]^{M,g} \in SUM$; $\perp$ otherwise

-$[\![\downarrow\alpha]\!]^{M,g} = \downarrow([\![\alpha]\!]^{M,g})$ if $[\![\alpha]\!]^{M,g} \in ATOM$; $\perp$ otherwise

Let $\alpha \in d$:

-$[\![AT(\alpha)]\!]^{M,g} = AT([\![\alpha]\!]^{M,g})$ if $[\![\alpha]\!]^{M,g} \neq \perp$; $\emptyset$ otherwise.

$AT(\alpha)$ can be defined as: $\{x \in AT: x \sqsubseteq \alpha\}$

-$[\![AT(P)]\!]^{M,g} = AT([\![P]\!]^{M,g})$

$AT(P)$ can be defined as: $\{x \in AT: x \in P\}$

-$[\![{}^{*}P]\!]^{M,g} = [[\![P]\!]^{M,g}]$

${}^{*}P$ denotes the i-join semilattice generated by $[\![P]\!]^{M,g}$: the **closure of** P under sum.

$[\![{}^{D}P]\!]^{M,g} = \{d \in D: \forall a \in AT(d): a \in [\![P]\!]^{M,g}\}$

${}^{D}P$ can be defined as: $\{x \in D: \forall y \in ATOM: y \sqsubseteq x \rightarrow y \in P\}$

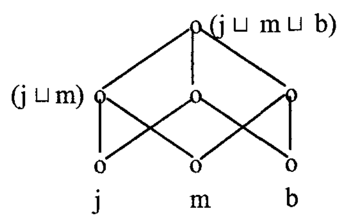
5.2. LINK'S THEORY OF PLURALITY

5.2.1. NOUN PHRASE CONJUNCTION AND DEFINITES

Link's theory incorporates noun phrase conjunction of the form: *John and Mary*, *The boys and the girls*. In Link's theory, *and* is interpreted as the **sum** operation, so:

John and Mary $\rightarrow$ $(j \sqcup m)$

In a picture:



Unlike Scha, Link does not assume that the singular *boy* and the plural *boys* have the same interpretation.

The singular *boy* denotes a set of atomic individuals:

boy $\rightarrow$ BOY where BOY $\subseteq$ ATOM

Pluralization corresponds to the *-operation:

boys $\rightarrow$ *BOY

*BOY is the closure of the singular predicate BOY under sum, i.e. it adds to the denotation of BOY all sums that you can form with elements of the denotation BOY.

Hence, for example:

If BOY = $\{j,b\}$, then *BOY = $\{j,b,j \sqcup b\}$
If BOY = $\{j,b,h\}$, then *BOY = $\{j,b,h,j \sqcup b, j \sqcup h, b \sqcup h, j \sqcup b \sqcup h\}$

Note that BOY $\subseteq$ *BOY. Hoeksema 1983 defines the plural *boys* as: *BOY-AT(BOY). Hence, in Hoeksema's analysis the plural predicate *boys* does not take atomic individuals in its extension. This would give a semantic account of why (1) is not wellformed:

(1) ?John are boys.

I will be following Link rather than Hoeksema here, for three reasons. In the first place, even if we think that Hoeksema's approach is better, it is much easier to develop the theory with Link's operation and afterwards change it to Hoeksema's. Thus, when we compare Link's approach and Hoeksema's, this difference is not a very big deal. Second, in the theory that I will develop later, the inclusion of the atoms **does** play a crucial role. As will become clear, trying to develop a Hoeksema-style variant of that theory, will be more than just cumbersome. Thirdly, as Lasersohn 1988 and Schwarzschild 1991 have pointed out, Hoeksema would have to have the determiner *no* reintroduce the atoms in the interpretation of a sentence like (2):

(2) No boys carried the piano upstairs.

On Hoeksema's account, the interpretation of (2) would be: no sum of two or more boys carried the piano upstairs. On Link's theory, it would be: no sum of boys carried the piano upstairs. Thus, on Link's theory, the quantifier *no boys* ranges over individuals and their sums, rather than non-individual sums, and this seems correct.

By giving the singular and the plural predicate a different interpretation, Link is able to assign the definite article *the* a single meaning, which can apply both to the singular noun and to the plural noun. *The* is interpreted as the σ operator:

the boy $\rightarrow \sigma(\text{BOY})$
the boys $\rightarrow \sigma({}^{*}\text{BOY})$

(Essentially the same analysis of *the* was developed in Sharvy 1980.) Moreover, Link can assume that - apart from a presuppositional factor which is particular to the definite - the definite article *the* is a generalization of NP conjunction *and*: just as *and* takes John and Bill together to form the plural entity $j \sqcup b$, *the* takes the boys, $b_1,\ldots,b_n,\ldots$ together and form the plural entity: $b_1 \sqcup \ldots \sqcup b_n \ldots$, which is the sum of all the boys.

But *the* differs presuppositionally from *and*. We capture this by interpreting *the* as σ rather than as $\sqcup$.

If P is a predicate, $\sigma(P)$ is interpreted as the sum of all the entities in P **if that sum is itself an entity in P**, otherwise it is undefined.

The boys is interpreted as $\sigma({}^{*}\text{BOY})$. $\sigma({}^{*}\text{BOY})$ is undefined if ${}^{*}\text{BOY}$, and hence BOY, is empty, so the use of the plural NP *the boys* presupposes that there are boys. If $\text{BOY}\neq\varnothing$, then ${}^{*}\text{BOY}$ is the closure of BOY under sum. This means that $\sqcup(\text{BOY}) \in {}^{*}\text{BOY}$, and hence if $\text{BOY}\neq\varnothing$, $\sigma({}^{*}\text{BOY}) = \sqcup(\text{BOY})$, the sum of all the boys.

The boy is interpreted as $\sigma(\text{BOY})$, where $\text{BOY} \subseteq \text{ATOM}$. If $\text{BOY} = \varnothing$, then similarly $\sigma(\text{BOY})$ is undefined. So the use of the singular NP *the boy* similarly presupposes that there are boys. However, if BOY has more than one element, then $\sqcup(\text{BOY}) \notin \text{BOY}$, since the sum of a non-singleton set of atoms is not itself an atom.

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