

LECTURE FOUR

SCHA'S THEORY OF PLURALITY

4.1. PLURALITY STRUCTURES

In this lecture and the next, I will discuss some approaches to plurality, in particular, the approaches by Remko Scha, Godehard Link, and myself. (Other proposals, like Craige Roberts', will be discussed in later lectures.) For reasons of comparison (and simplicity), I will reformulate the set theoretic proposals of Scha and myself in terms of lattice structures. I will start, then, by defining the plurality structures that I will use in formulating these proposals. I will be rather succinct here. Extensive exposition of these structures can be found in Landman 1991.

We will be interested in structures that can form the interpretation domain for singular and plural NPs. We will assume that such a domain is a domain of singular and plural individuals, which is **ordered** by an operation of **sum**, in such a way that plural individuals are sums of singular individuals; and which is ordered by a relation of **part of** (defined in terms of sum). The plural individual **john and bill** is the sum of singular individuals **john** and **bill**, and both **john** and **bill** are part of that plural individual. The difference between singular individuals and plural individuals is that singular individuals have only themselves as part; they are what is called **atoms** with respect to the relation of **part of**.

First, I introduce the notion of an **i-join semilattice**.

An **i-join semilattice** is a structure:

D = $\langle D, \sqcup \rangle$ where:

1. **D** is a non-empty set.
2. \sqcup is a function that assigns to every non-empty subset **X** of **D** an element of **D**: $\sqcup X$, the sum of **X**.
3. We define the relation \sqsubseteq by: $a \sqsubseteq b$ iff $\sqcup \{a, b\} = b$. We require \sqsubseteq to be a partial order on **D** (reflexive, transitive, and anti-symmetric).
4. For every non-empty $X \subseteq D$: $\sqcup X$ is the **join** of **X** under \sqsubseteq , where join is defined as follows:

Let X be a non-empty subset of D :

the **join of X under \sqsubseteq** is the unique element $\sqcup X \in D$

(if there is such a unique element) such that:

1. $\forall x \in X: x \sqsubseteq \sqcup X$
2. $\forall d \in D: \text{if } \forall x \in X: x \sqsubseteq d \text{ then } \sqcup X \sqsubseteq d.$

So the join of X is the smallest element that is bigger than (or equal to) every element in X . So, in an i-join semilattice, every non-empty subset of D has a join in D .

We write $(a \sqcup b)$ for $\sqcup \{a, b\}$.

As explained in Landman 1989a,b, 1991, not every i-join semilattice can function as a plurality domain (or any other semantic domain). Semantic domains of sums and parts are **part-of structures**.

First some notions:

Let $\langle D, \sqcup \rangle$ be an i-join semilattice:

a is the **minimum** (or zero) in $\langle D, \sqcup \rangle$ iff $\forall d \in D: a \sqsubseteq d$

So the minimum, if there is one, is smaller than or equal to everything. If there is a minimum, there is a unique one; we write it as 0.

a is a **minimal element** in $\langle D, \sqcup \rangle$ iff $\neg \exists d \in D: d \sqsubseteq a \text{ and } d \neq a$

So a minimal element has nothing smaller than it.

a is an **atom** in $\langle D, \sqcup \rangle$ iff a is a minimal element in $\langle D - \{0\}, \sqcup \rangle$

$\langle D, \sqcup \rangle$ is **atomic** iff $\forall d \in D: \text{if } d \neq 0 \text{ then there is an atom } a \in D: a \sqsubseteq d$

So D is atomic iff every non-zero element has an atom below it.

In the definitions of atom and atomic, we assume that D has more than one element. For the borderline case, $\langle \{0\}, \sqcup \rangle$, we stipulate that 0 is an atom and $\langle \{0\}, \sqcup \rangle$ is atomic.

a **overlaps** b , $a \circ b$, iff $\exists c \in D: c \sqsubseteq a \text{ and } c \sqsubseteq b$

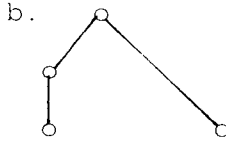
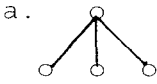
a overlaps b iff they have a common part. I write $a \oslash b$ for: a does not overlap b .

A **part-of structure** is an i-join semilattice $\langle D, \sqcup \rangle$ satisfying:

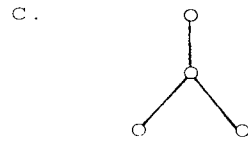
1. if D has a minimum 0 then $D = \{0\}$.
2. **distributivity**: if $a \sqsubseteq b \sqcup c$ then: either $a \sqsubseteq b$ or $a \sqsubseteq c$
or $\exists b' \sqsubseteq b \exists c' \sqsubseteq c: a = b' \sqcup c'$
3. **witness**: if $a \sqsubseteq b$ and $a \neq b$ then $\exists c \sqsubseteq b: a \oslash c$

Examples of i-join semilattices that are not part-of structures:

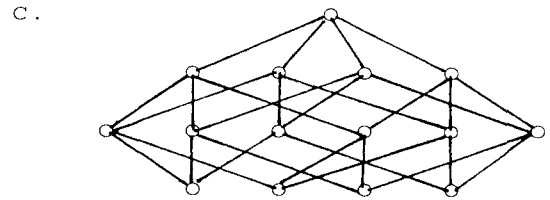
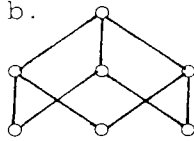
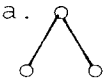
Non-distributive:



non witnessed:



Examples of part-of structures:



Theorem:

The part-of structures are up to isomorphism exactly the complete Boolean algebras with 0 deleted and the operations restricted to (generalized) \sqcup .

In terms of the notion of a part-of structure, we define the notion of a **domain of singular and plural entities**.

A **domain of singular and plural individuals** is a structure $\mathbf{D} = \langle D, \sqcup, AT \rangle$ where: $\langle D, \sqcup \rangle$ is an **atomic** part-of structure with set of atoms AT .

It follows from the above theorem that the domains of plural and singular individuals are precisely the complete atomic boolean algebras with 0 cut out and operations restricted to (generalized) \sqcup . From this another theorem follows:

Theorem:

The domains of singular and plural individuals are up to isomorphism exactly the structures: $\langle \text{pow}(D) - \{\emptyset\}, \cup \rangle$, where D is any set.

Some more definitions.

Let $\langle D, \sqcup \rangle$ be an atomic i-join semilattice with set of atoms AT .

D is **atomistic** iff every element of D is the sum of atoms
(i.e. for every $d \in D$ there is an $X \subseteq AT$: $d = \sqcup X$)

If $d \in D$ then $AT(d) = \{a \in AT: a \sqsubseteq d\}$
 $AT(d)$ is the set of atoms below d .

If $X \subseteq D$ then $AT(X) = X \cap AT$
 $AT(X)$ is the set of atoms in X

If $d \in D$ then $|d| = |\{a \in AT: a \sqsubseteq d\}|$
 $|d|$, the cardinality of d , is the number of atoms below d .

For our purposes the following facts about part-of structures motivate our restriction to these structures:

Let $\langle D, \sqcup, AT \rangle$ be an atomic part of structure.

Theorem:

D is **atomistic**, more precisely: for every $d \in D$: $d = \sqcup AT(d)$

Theorem:

D satisfies **distinctness**: if $X, Y \subseteq AT$ and $X \neq Y$ then $\sqcup X \neq \sqcup Y$

One more definition:

Let $\langle D, \sqcup \rangle$ be an i-join semilattice and $X \subseteq D$ (X non-empty), then:
 $[X]$, the i-join semilattice **generated** by X , is the closure of X under \sqcup
 (i.e. $[X] = \{y \in D: \exists Y \subseteq X: y = \sqcup Y\}$)

Theorem:

$[X]$ is a **sub-i-join semilattice** of D , in fact, the smallest sub-i-join semilattice of D containing X .

(note: we let $[\emptyset] = \emptyset$ and call it the i-join semilattice generated by \emptyset , though this is a misuse of terminology, because \emptyset is not an i-join semilattice.)

With this notion we can generalize the above theorem about distinctness to part-of structures in general:

Theorem:

Let $\langle D, \sqcup \rangle$ be a part-of structure and X a set of mutually non-overlapping elements of D . Then $[X]$ is itself an atomic part of structure with X as set of atoms.

As discussed in Landman 1991, these distinctness properties are essential for any semantic domain of sums and parts, because they reflect the identity conditions for objects and their parts: They guarantee that if THE BOYS, THE GIRLS, and THE LINGUISTS are plural individuals that have no parts in common, then THE BOYS \sqcup THE GIRLS and THE BOYS \sqcup THE LINGUISTS and THE GIRLS \sqcup THE LINGUISTS are three distinct plural individuals.

Similarly in other semantic domains: if I have three buckets of water, then the stuff that I get when I put the first two together is different from the stuff that I get when I put the first and the third together, yet different from what I get when I put the second and third together.

On this approach, the difference between the count (plurality) domain and the mass domain is that the count domain is atomic, while the mass domain is atomless.

We turn a domain of singular and plural individuals into a domain of singular and plural individuals **with groups** in the following way:

- We **sort** the set of atoms into two sorts: individual atoms and group atoms.
- We add an operation of **group formation** from sums of individual atoms into atoms, which maps each non-atomic sum of individuals onto a group atom, and which maps each individual atom onto itself. Group formation turns THE BOYS, as the sum of the individual boys, into THE BOYS, AS A GROUP, a collective entity in its own right, which is no longer strictly determined by its part-of structure.
- We add an operation of **membership specification** from atoms into sums of individual atoms, which maps a group atom onto the sum of its members, and an individual atom onto itself.

Formally:

A domain of singular and plural individuals with groups is a structure:

$\mathbf{D} = \langle D, \sqcup, AT, IND, GROUP, \uparrow, \downarrow \rangle$ where:

1. $\langle D, \sqcup, AT \rangle$ is a domain of singular and plural individuals.
2. $AT = IND \cup GROUP$ and $IND \cap GROUP = \emptyset$

Let $SUM = [IND]$

3. \uparrow is a **one-one** function from SUM into $ATOM$ such that:
 1. $\forall d \in SUM-IND: \uparrow(d) \in GROUP$
 2. $\forall d \in IND: \uparrow(d) = d$
4. \downarrow is a function from $ATOM$ onto SUM such that:
 1. $\forall d \in SUM: \downarrow(\uparrow(d)) = d$
 2. $\forall d \in IND: \downarrow(d) = d$

SUM is the set of sums of **individuals**. $SUMOFGROUP$ is the set of sums of **groups**. Note that $IND \subseteq SUM$; if we want to refer to the domain of non-singular sums, this domain is $SUM-IND$.

These structures differ a bit from the structures I introduced in Landman 1989a. The notion of group formation is not iterative (I don't have groups of groups here) and group formation is identity for individual atoms. Both of these changes are motivated by the work of Roger Schwarzschild (Schwarzschild 1991, 1992). I will discuss their rationale later.



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