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# REVOLUTION BY STEALTH: REDEFINING UNIVERSITY MATHEMATICS

## 1. INTRODUCTION

Change, growth, and accountability dominate higher education at the dawn of the twenty-first century. According to delegates at UNESCO's recent World Conference on Higher Education, change is unrelenting, 'civilizational in scope,' affecting everything from the nature of work to the customs of society, from the role of government to the functioning of the economy. Growth in higher education has been equally dramatic, with worldwide enrolment rising from 13 million in 1960 to near 90 million today (World Conference on Higher Education, 1998).

Concomitant with change and growth is pressure from governments everywhere for greater accountability from professors and leaders of higher education, for evidence that, in a world of rapid change, universities are working effectively to address pressing needs of society. Autonomy, the prized possession of universities, presupposes accountability. The escalating pressures of global change, growth, and accountability will create, according to UNESCO Director Federico Mayor, "a radical transformation of the higher education landscape not only more but *different* learning opportunities" (Mayor, 1998).

Much of the upheaval in society and employment is a consequence of the truly revolutionary expansion of worldwide telecommunication, as is the stunning increase in demand for higher education. But this demand is predicated on the belief that universities properly anticipate signals from the changing world of work and create optimal linkages between students' studies and expectations of employers. Unfortunately, few universities have taken up this challenge, at least not unless pushed by external forces.

As the world changes rapidly and higher education grows explosively, universities evolve leisurely. Courses, curricula, and examinations remain steeped in tradition, some centuries old, while autonomy and academic freedom rule in the classroom. Few institutions of higher education readily embrace the culture of assessment that is required to ensure relevance and effectiveness of their curriculum. Under these circumstances it is only natural for political leaders to demand stronger connections between the classroom and the community, between the ivory tower and the industrial park.



*Derek Holton (Ed.), The Teaching and Learning of Mathematics at University Level: An ICMI Study,* 303—312.

## 2. UNDERGRADUATE MATHEMATICS

Demands for relevance and accountability are no strangers to undergraduate mathematics. Indeed, post-secondary mathematics can be viewed as higher education in microcosm. Growth in course enrolments has been enormous, paralleling the unprecedented penetration of mathematical methods into new areas of application. These new areas—ranging from biology to finance, from agriculture to neuroscience—have changed profoundly the profile of mathematical practice (see Odom, 1998). Yet for the most part these changes are invisible in the undergraduate mathematics curriculum, which still marches to the drumbeat of topics first developed in the eighteenth and nineteenth centuries.

It is, therefore, not at all surprising that the three themes identified at the UNESCO conference are presaged in the Discussion Document for this ICMI Study: the rapid growth in the number of students at the tertiary level; unprecedented changes in secondary school curricula, in teaching methods, and in technology; and increasing demand for public accountability (see Discussion Document, 1997). Worldwide demands for radical transformation of higher education bear on mathematics as much as on any other discipline.

Post-secondary students study mathematics for many different reasons. Some pursue clear professional goals in careers such as engineering or business where advanced mathematical thinking is directly useful. Some enrol in specialized mathematics courses that are required in programmes that prepare skilled workers such as nurses, automobile mechanics, or electronics technicians. Some study mathematics in order to teach mathematics to children, while others, far more numerous, study mathematics for much the same reason that students study literature or history: for critical thinking, for culture, and for intellectual breadth. Still others enrol in post-secondary courses designed to help older students master parts of secondary mathematics (especially algebra) that they never studied, never learned, or just forgot. (This latter group is especially numerous in countries such as the United States that provide relatively open access to tertiary education, see Phipps, 1998.)

In today's world, the majority of students who enrol in post-secondary education study some type of mathematics. Tomorrow, virtually all will. In the information age, mathematical competence is as essential for self-fulfilment as literacy has been in earlier eras. Both employment and citizenship now require that adults be comfortable with central mathematical notions such as numbers and symbols, graphs and geometry, formulas and equations, measurement and estimation, risks and data. More important, literate adults must be prepared to recognize and interpret mathematics embedded in different contexts, to think mathematically as naturally as they think in their native language (see Steen, 1997).

Since not all of this learning can possibly be accomplished in secondary education, much of it will take place in post-secondary contexts, either in traditional institutions of higher education (such as universities, four- and two-year colleges, polytechnics, or technical institutes) or, increasingly, in non-traditional settings such as the internet, corporate training centres, weekend short-courses, and for-profit universities. This profusion of post-secondary mathematics programmes at the end of the twentieth century contrasts sharply with the very limited forms of university mathematics education at the beginning of this century. The variety of forms, purposes, durations, degrees, and delivery systems of post-secondary mathematics reflects the changing character of society, of careers, and of student needs. Proliferation of choices is without doubt the most significant change that has taken place in tertiary mathematics education in the last one hundred years.

## 3. MATHEMATICAL PRACTICE

The primary purpose of mathematics programmes in higher education is to help students learn whatever mathematics they need, both for their immediate career goals and as preparation for life-long learning. Today's students expect institutions of higher education to offer mathematics courses that support a full range of educational and career goals, including:

- Agriculture
- Biological Sciences
- Business
- Computing
- Elementary Education
- Electronics
- Economics
- Engineering
- Finance
- Geography
- General Education
- Health Sciences
- Law
- Mathematical Research
- Management
- Medical Technology
- Physical Science
- Quantitative Literacy
- Remedial Mathematics
- Secondary Education
- Social Sciences
- Statistics
- Technical Mathematics
- Telecommunications

Even without exploring details of specific curricula or programmes, it should be obvious that the multiplicity of student career interests requires, if you will, multiple mathematics. Consider a few examples of how relatively simple mathematics is used in today's world of high performance work:

- Precision farming relies on satellite imaging data supplemented by soil samples to create terrain maps that reflect soil chemistry and moisture levels. These methods depend on geographic information systems that blend spreadsheet organization with a variety of algorithms for geometric projections (e.g., for rendering onto flat maps oblique satellite images of earth's curved surface).
- Technicians in semiconductor manufacturing plants, analyze real-time data from production processes in order to detect patterns of change that might signal an impending reduction in quality before it actually happens. These methods involve measurement strategies, graphical analyses, and tools of statistical quality control.
- Teams that design new commercial airplanes now engage designers, manufacturing personnel, maintenance workers, and operation managers in joint

planning with the goal of minimizing total costs of construction, maintenance, and operation over the life of the plane. This enterprise involves teamwork among individuals of quite different mathematical training as well as innovative methods of optimization.

- Emergency medical personnel need to interpret quickly and accurately dynamic graphs of heart action that record electrical potential, blood pressure, and other data. With experience, they learn to recognize both regular patterns and common pathologies. With understanding, they can also interpret uncommon signals.

These examples are not primarily about the relation of mathematical theory to applications—the traditional poles of curricular debate—but about something quite different: mathematical practice (see Denning, 1997). Behind each of these situations lurks much good mathematics (e.g., projection operators, optimization algorithms, fluid dynamics, statistical inference) that can be applied in these and many other circumstances. However, most students are not primarily motivated to learn this mathematics, but rather to increase crop yield, minimize manufacturing defects, reduce airplane costs, or stabilize heart patients. Although a mathematician will recognize these as mathematical goals—to increase, minimize, reduce, stabilize—neither students nor their teachers in agriculture, manufacturing, engineering, or medicine would recognize or describe their work in this way. To these individuals, the overwhelming majority of clients of post-secondary mathematics, mathematical methods are merely part of the routine practice of their profession.

Indeed, mathematics in the workplace is often so well hidden as to be invisible to everyone except a discerning observer. In the United States different industries have created skill standards for entry-level employees (e.g., electronics (American Electronics Association, 1994), photonics (Center for Occupational Research and Development, 1995), health care (FarWest Laboratory, 1995), and National Skills Standards Board, 1998). Virtually all of these standards include substantial uses of mathematics, but most such applications are embedded in routine job requirements without any visible hint of the underlying mathematics. Although mathematics is now ubiquitous in business and industry, the mathematics found there is often somewhat different from what students learn in school or college (see Davis, 1996 and Packer, 1997). Similarly, in the wider world of public policy, the gradual incursion of statistics and probability in measuring (and sometimes controlling) personal health, societal habits, and national economies has created whole new territories for students and professors to explore (see Bernstein, 1996, Porter, 1995 and Wise, 1995).

In sharp contrast to this profligate flowering of practical mathematics in diverse post-secondary settings, university mathematics—what mathematicians tend to think of as 'real mathematics'—matured in the last century as a tightly *disciplined* discipline led by professors of world-wide renown who held major chairs in leading universities and research institutes. However, this university mathematics, 'real mathematics' as practiced in real universities, now constitutes only a tiny fraction of post-secondary mathematics. One data point: in the United States, fewer than 15%

of traditional undergraduate mathematics enrolments are in courses above the level of calculus (Loftsgaarden, Rung, and Watkins, 1997). And this does not count non-traditional enrolments, where the variety of offerings is even greater. A realist might well argue that 'real mathematics' is found not in the traditional curriculum inherited from the past but in today's widely dispersed courses, where a multitude of students learn a cornucopia of mathematics in diverse situations for a plethora of purposes.

## 4. LEARNING MATHEMATICS

Where do students learn mathematics? Some take traditional mathematics courses such as calculus, geometry, and statistics. Some take courses specifically designed for certain professions—mathematics for nurses, statistics for lawyers, calculus for engineers—that are offered either by mathematics departments or by the professional programmes themselves. But many, perhaps even most, pick up mathematics invisibly and indirectly as they take regular courses and internships in their professional fields (e.g., in physiology, geographic information systems, or aircraft design).

Any university dean knows that statistics is more often taught outside of statistics programmes than inside them. The same is true of mathematics, but is not as widely recognized. Every professional programme, from one- and two-year certificates to four- and five-year engineering degrees, offers courses that provide students with mathematics (or statistics) in the context of specific professional practice. This is entirely natural, since most students find that they learn mathematics more readily, and are more likely to be able to use it when needed, if it is taught in a context that fits their career goals and in which the examples resonate with those that appear in their other professional courses.

The appeal of context-based mathematics is no surprise, nor is its widespread presence in university curricula. But what is somewhat new—and growing rapidly—is the extent to which good mathematics is unobtrusively embedded in routine courses in other subjects. Anywhere spreadsheets are used (which is almost everywhere) mathematics is learned. It is also learned in courses that deal with such diverse topics as image processing, environmental policy, and computer-aided manufacturing. From technicians to doctors, from managers to investors, most of the mathematics people use is learned not in a course called mathematics but in the actual practice of their craft. And in today's competitive world, where quantitative skills really count, embedded mathematical tools are often as sophisticated as the techniques of more traditional mathematics.

So tertiary mathematics now appears in three forms: as traditional mathematics courses (both pure and applied) taught primarily in departments of mathematics; as context-based mathematics courses taught in other departments; and as courses in other disciplines that employ significant (albeit often hidden) mathematical methods. I have no data to quantify the 'biomass' of mathematics taught through these three means, but to a first approximation I would conjecture that they are approximately equal.

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