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TRENDS IN CURRICULUM

A Working Group Report

1. INTRODUCTION

The Working Group: Trends in Curriculum, examined the various forces which act on a mathematics curriculum, and on curriculum trends, both at local and national levels. 'Curriculum' was considered in its widest sense to mean "matters pertaining to the purposes, goals and content of mathematics education" (Discussion Document for this ICMI Study, 1997), as well as the means for achieving curricular goals. Hence, the discussions in the Working Group touched on undergraduate programmes[1], specific courses, mathematical content, degree of rigour, modes of delivery and interaction, and assessment schemes. Inevitably, the issues discussed in this Working Group overlapped substantially with the other Working Groups of the conference.

2. BACKGROUND

2.1 Who are mathematics students?

Curricular issues are inextricably tied to the question: a mathematics curriculum for whom? The teaching of mathematics at universities and colleges is quite diverse in its organization hence there is a wide range of students populating mathematics courses.

Among those enrolling in mathematics courses, there are students for whom mathematics is the primary subject of their undergraduate studies, possibly coupled with another discipline such as statistics, physics, computer science, or economics. We will refer to this group as '(maths) programme students' so as to distinguish them from 'client students'. The latter come from client departments, traditionally the physical sciences and engineering departments, though nowadays, computer science has become a prominent client, replacing physics in many countries. Other client students come from departments such as social sciences, commerce and

[1] Any international gathering immediately points to the different senses attributed to words such as 'programme', 'course', 'module', or 'paper'. In this document we have adhered to the North American usage of 'programme' and 'course' whereby a programme is made up of a collection of compulsory and optional courses, and a (one semester) course constitutes about 40 hours of instruction.

Derek Holton (Ed.), The Teaching and Learning of Mathematics at University Level: An ICMI Study, 59—69.

economics, and psychology, who are increasingly requiring their students to take some mathematics. Thus, a somewhat facile distinction between the two groups is that programme students want to study mathematics while client students have to. In any case, it is usually expected that client students terminate their mathematics studies after a year or two of their undergraduate studies.

Future school mathematics teachers, particularly at the secondary level can be considered as either programme or client students depending on national criteria for the training of teachers. For example, in certain European countries, prospective secondary teachers have to complete a full 5-year undergraduate mathematics curriculum and hence are, in our terms, programme students. On the other hand, future teachers in North America generally take only a certain number of mathematics courses rather than a full mathematics programme.

2.2 Organization of undergraduate teaching

Many departments of Mathematics are responsible for teaching all programme and client students. In fact, the teaching of client students (the 'service role') is often the bread-and-butter component of departments' teaching and it justifies having a large department of mathematics. Other institutions have 'mini-departments' of mathematics housed in engineering (Polytechniques), finance, economics, or education and teach exclusively the students of their discipline.

Mathematics departments who teach both programme and client students do so in different ways. Some require all their students to take the same courses, say, calculus, differential equations, or linear algebra, resulting in classes with a heterogeneous group of students whose background preparation, career ambitions, and interest in mathematics are quite varied. This 'one-curriculum-for-all' approach inevitably raises a range of issues as to the appropriate level and emphasis, and as to the type and depth of applications. Other departments offer a variety of courses that are specifically geared for one client group or another, viz. 'calculus for engineers', 'calculus for chemists', or 'algebra for teachers'.

Focusing on the mathematics curriculum specifically targeted for programme students, there are also wide variations depending on the traditions of the universities involved. These traditions have to do with: admission standards, the juncture at which a student can choose a mathematics option, the length of study, the number of courses required, course choices (both in mathematics and outside the discipline), and whether or not there is a requirement for studying mathematics together with a cognate discipline. The intended goal(s) of a mathematics programme (even if not explicitly articulated) are also dissimilar. To take but one example, in most Canadian universities, programme students complete either a major or honours in some field of mathematics (e.g. pure, applied, statistics). A major programme is comprised of a certain concentration of core and elective mathematics courses (which can amount to as little as a third of the total number of courses necessary to obtain a bachelor's degree). Except for the first year of the major (in a 4-year programme) where there are several compulsory courses in the other sciences and computer science, students have nearly complete freedom to choose courses complementary to the major. The honours programme, on the other hand, tends to be a more selective programme with a substantially greater number of advanced courses, and may take possibly an extra year to complete. The goal of the honours programme is to train highly qualified persons who can continue doing graduate studies and research or be employed in demanding mathematical fields. On the other hand, the goals for the major programme are more modest, namely to graduate students who are mathematically literate, and who can function comfortably in work situations requiring quantitative, analytic, and mathematical problem solving skills. (For more details on a major and honours programme, see Hillel, this volume, pp. 179-184.)

3. FACTORS INFLUENCING CURRICULUM

3.1 Changes within mathematics

The undergraduate mathematical landscape is always in some state of flux mirroring the organic nature of mathematics. New theories and mathematical tools, sometimes supported by powerful computers, are being developed within mathematics or as applications in cognate disciplines such as physics, computer science, and engineering. Certain new subjects become highly visible (e.g., dynamical systems, computer algebra), others experience a renaissance (e.g., geometry, number theory, numerical analysis), and yet others become more marginalized (e.g., category theory). In fact, Steen has written that "strong departments find that they replace or change significantly half of their courses approximately once a decade" and "as new mathematics is continually created, so mathematics courses must be continually renewed" (Steen, 1992). These on-going updates to the curriculum can be regarded, in a sense, as 'deterministic' aspects of curriculum change, ones that do not put into question the purpose, goals, and means of undergraduate education.

3.2 Changes in the pre-university math curriculum

Secondary school[2] mathematics curricula have undergone tremendous changes in the past 20 years. One most visible change in many countries is the reduction in the number of hours devoted to mathematics and science. For example, in France, up to 1994, secondary school (lycée) students had 15 hours of science teaching per week, of which 9 were in mathematics and physics. By 1999, there are only 8 hours a week in which to teach mathematics, physical sciences, biology and technology. Also in France, traditionally taught subjects like set theory and algebraic structures have been dropped, as well as the emphasis on definitions and proofs. Reports from other countries also allude to a de-emphasis on formal mathematics and on complicated

[2] Secondary school terminates after 11 years of schooling in some countries but lasts for up to 13 in others.

manipulations, the increased use of calculators with computer algebra systems capabilities, and the teaching of synthetic geometry, as well as a teaching approach which relies more on investigative project-oriented work. There are also increasing attempts to introduce quantitative, statistical and probabilistic reasoning in the secondary curriculum.

One should mention here that, quite often, changes in the secondary curriculum are brought about without coordination with the very universities and colleges that the students subsequently attend. Consequently, many university mathematicians are not always aware of the nature and extent of these changes nor of the pressures and constraints on the pre-university system that could explain, for example, why the number of hours devoted to mathematics is being reduced.

3.3 Changing clientele

Most countries have wisely abandoned the elitist view of university education in favour of a more open policy that makes university education accessible to a larger segment of the population. This policy has resulted in a great influx of students to universities (estimated to have increased 6-fold in the past 30 years, Steen, this volume, pp. 303-312), including a massive increase in the number of client students who are enrolled in agriculture, commerce, finance, social sciences, etc. These students tend to be heterogeneous in terms of their mathematical preparation, probably would rather not take mathematics at all if given the choice, and are not very interested in mathematical rigour and abstraction (nor even always convinced about the relevance of mathematics to their careers). At the same time, they constitute, in some universities, the main clientele of a mathematics department. Also, open immigration policies in some countries has resulted in an influx of students whose first language is not the language of instruction.

One general feature of incoming students is that they enter university having logged less hours of mathematics lessons because of the reduction of the number of hours devoted to mathematics at the secondary level. But even when a choice exists for taking more mathematics at the pre-university level, the trend is for students to forego this choice. For example, in England, there has been a significant drop in the number of students completing A-level mathematics. And, among those who do complete their A-level, the number of university mathematics candidates who have completed two A-levels in mathematics has dropped to about 1 in 10 in the 1990s whereas it was 1 in 3 in the 1960s (Simpson, 1998). A recent article indicates a drop of over 30% in students entering mathematics programmes in Germany (Jackson, 2000). Thus, the overall effect is that students' background preparation is not sufficient for meeting the rigours of traditional entry-level university courses in linear algebra and calculus, even for students who are relatively successful in their pre-university courses. (This point was also made by participants from Australia, Brazil, Canada, England, Japan, Malaysia, and the USA.)

Students' attitudes towards education, their study habits, and their expectations, are influenced by the traditions and values of the prevailing culture in which they live. There is also a sense that students nowadays are more career-oriented and thus interested in getting skills that lead directly to jobs. This explains why in some countries, the number of mathematics programme students has been dwindling dramatically and the ablest students are drawn to such fields as computer science, engineering, and finance, where career opportunities are more evident. Within mathematics, there is sharp increase in enrolment in actuarial mathematics, when this option is available (for example, in Australia, Canada - see Hillel, this volume, pp. 179-184, - and Switzerland - see Kirchgraber, this volume, pp. 185-190). There is also an increasing tendency for students to combine study and work, thus taking on part-time jobs to supplement their income.

Departments of mathematics are thus faced with the challenge of having to teach students whose background preparation, learning styles, study habits, and career ambitions are more and more at odds with the traditional lecture-style mathematical training with its Bourbaki-like curriculum, particularly, in pure mathematics. Furthermore, many departments are facing an increase in the number of client students and a decline in the number of programme students.

3.4 Resources

Certain countries have never had adequate resources for higher education; others are experiencing political and economic upheavals which greatly affect education, as well as all other aspects of life. Even more affluent and politically stable countries, are witnessing a continual erosion of the levels of government support for universities. Diminishing resources usually translate into less staff, larger classes, and pressures to be more efficient and financially accountable. In such instances, mathematics departments are finding themselves less able to offer specialized courses to a small number of students and so have redefined an appropriate core of an undergraduate mathematics programme.

3.5 Technology

Computers have impacted on the methods and results of several mathematical domains. Coupled with graphing software, computer algebra systems, dynamic geometry, or differential equations packages, computers and calculators pose interesting challenges to mathematics departments. They have led to questioning what mathematics content is central and what is redundant, as well as, how present-day learning, teaching and assessment practices, can be and ought to be changed.

3.6 External influences: Governments, Research Agencies and Business

There is a prevalent sentiment among mathematicians that they, as the professionals, are in the best position to define the undergraduate curriculum for their students. They view attempts to influence curricular choices by bodies external to the department as an unwarranted intrusion. However, governments who, in most cases, foot the universities' bills, have, in recent years, been much more vocal and http://www.springer.com/978-1-4020-0072-0

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