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WHY THE PROFESSOR MUST BE A STIMULATING TEACHER

Towards a new paradigm of teaching mathematics at University level

1. INTRODUCTION

Mathematics at the University level is a complex field to explore. The diversity of institutions and social and cultural contexts, the variety of curricula and courses, the reforms taking place at present, etc., may induce us to believe that perhaps it makes no sense to talk about general or common aspects of our academic activities. But after many years of observing our own profession, of visiting so many places around the world and interacting with so many colleagues I have identified some problems and some challenges that may be of interest for mathematicians who love mathematics and love teaching. The aim of this presentation is to share some critical thoughts and to point out some constructive ideas on the educational goals of teaching mathematics at the university level.

2. SOME CRITICAL VIEWS ON EXISTING MYTHS AND PRACTICES IN UNIVERSITY TEACHING OF MATHEMATICS

In this section I would like to unmask some very general existing ‘myths’ (Kirwan, 1991) and practices in the teaching of mathematics at the undergraduate level that have a negative influence (Lewis, 1975) on the quality of mathematics teaching.

The researchers-always-make-good-teachers myth. This university myth says that ‘researchers are *ipso facto* good teachers ... therefore the key criteria for selection and promotion must be high quality research’. Following Kline (1977) we quote the statement that:

Hence appointment, promotion, tenure and salary are based entirely on status in research... but for most of the teaching that the universities are, or should be, offering, the research professor is useless.

This myth calls for a number of observations.

1. Sound knowledge does not necessarily mean active research;
2. The majority of mathematics courses do not include advanced results reached in recent decades;
3. Research takes place in thousands of different specialities, most of it in very narrow fields, and lines of research are often a matter of free choice and quite unrelated to teaching;
4. Unfortunately, research criteria are closely related to the Department's interests and rarely include research into mathematics education.

Let us remember here the critical words expressed in Kline (1977):

The mania for research has produced an invidious system of academic promotion, perversion of undergraduate education, and contempt for and flight from teaching.

While for graduate, doctoral and post-doctoral teaching activities there is no doubt that only the most up-to-date and active researchers can introduce students to the latest results, techniques and trends, this does not hold true for most undergraduate programmes (see Carrier et al, 1962).

The self-made-teacher tradition. This is another standard mathematical myth and is based upon the claim that excellence in university teaching does not require any specific training - it is just a matter of accumulated experience, clear presentation skills and a sound knowledge of the subject. This approach leaves room for a lot of creative freedom but at the same time it can lead to quite a lot of anxiety, especially for inexperienced young teachers, who will in general try to reproduce the models that they have been exposed to during their own education. This myth does not make provision for students who are exposed to various styles of teaching simultaneously and it also avoids the issue of critical input from colleagues as well as the positive training that one would expect from the institutions involved.

Some classical references on this topic come from the 70s (e.g. CTUM, 1979, EBLE, 1974, Rogers, 1975, Rosenberg, 1972, Wilson, 1974).

Clearly, teaching may benefit from training and this must be a compulsory activity for those who want to teach.

Context-free universal content. This idea justifies the content of many courses as 'basic skills and results which must be learned by everyone taking the course'. This myth generated classic courses that were given to almost everyone entering science or technological university studies. It is taken for granted that some elements of linear algebra, calculus, differential equations, discrete mathematics, probability, statistics, etc., constitute the 'core' curriculum of university mathematics. In particular this myth justifies the concept that teaching is context-free, i.e., independent of personal interests, of specific professional training, of cultural environment, of social circumstances, and so on. While this situation makes for a more flexible teaching organization (anyone can teach anything), it sacrifices students' interest and kills interdisciplinary approaches. This led to wide and even universal sales for some textbooks. We, however, believe that contents must be

related to interest, special needs, context, and the like (see COMAP, 1997, Howson, 1988, Pollack, 1988, Steen, 1989).

Deductive organization. In this case, 'teaching' is thought to be assimilated thanks to representations of deductive thinking. Topics are presented linearly, definitions-theorems-proofs are sequentially stated in their most general form. In particular this presentation leads to the need for constant proofs (the more formal the better) and leaves little room for discussion or historical remarks ... "How?" becomes more important than "Why?". (Freudenthal, 1991). Is deduction more important than induction? Is formal reasoning more important than plausible thinking? Clearly, deduction is only one component of mathematical thinking.

The top-down approach. This approach holds that by teaching mathematical topics in their most general form, students will be able to deal with any particular case, any example, any application. This gets rid of the problem of real data and the main elements of mathematics modelling. Learning is a bottom-up process, so teaching top-down is not an effective way of helping learners (see e.g. Begle, 1979).

The perfect-theory presentation. Mathematics courses present positive results, solved problems, bona fide models. Students become convinced that mathematics is almost complete, that theorem proving is just a deductive game, that errors, false trials, and zig-zag arguments, which play such a crucial role in human life, have no place in the mathematical world. Unfortunately, in some ways many textbooks have inherited the cold research-journal style. This style of presentation kidnaps the 'human nature' of mathematical discoveries, the mistakes that were made, the difficulties and the need for simplifications. In some cases (e.g. statistics) this gives the false idea that the 'real subject' is 'the mathematical model', when we know that mathematics may be a powerful tool but it needs to be used in combination with other disciplines or techniques. In addition, we are presented with the paradox that very often this perfect presentation implies only an instrumental understanding instead of a relational understanding. This perfect-theory presentation turns a living discipline into a dead garden.

The 'master class'/formal lecture paradigm. Teaching has frequently been oriented towards 'communicating' mathematical knowledge. Typically, a class for undergraduates would consist of a large group of students sitting, listening and writing in a classroom where a professor delivers several hours per week of spoken-written presentation before a blackboard, see Bligh (1972). After the lectures, students are supposed to study the delivered content by reading notes, the textbook and by solving *ad hoc* exercises proposed for each chapter-talk. This reduces 'teaching' to lecturing, and 'learning' to an individual after-class activity of assimilating results and practising techniques. In particular, as noted by Clements (1998), students spend a lot of time inefficiently or unproductively:

... a considerable part of the time is devoted to the transference from the notes of the lecturer to the notepads of the students of relatively straightforward factual material.

While ‘master classes’/formal lectures are fine when truly ‘masterly’, they could nevertheless be combined with other techniques of communicating and working.

The mature students myth. At the freshman level, this myth assumes that during the few weeks between high school and university registration, students have grown in such a way that their integration into the new university atmosphere does not require any special attention. In particular, students going into scientific or technical courses are assumed to be already motivated and aware of the relevance of mathematics to their training, and students going into other studies are assumed to constitute a low-interest class. The diversity of backgrounds is often ignored. The high school curriculum may often be unknown. Clearly, the transition from secondary schools to universities needs special attention.

The routine individual-written assessment. This presents the final test, or a written examination mixing questions and exercises, as an ideal method of marking, i.e., of gauging how well students master the content delivered in lectures. The method focuses on individual preparation and rarely opens doors to project work, group activities, open questions, etc. In its most rigorous form, this assessment is reduced to a final exam to be marked and rarely integrates other activities or information attained during the course into the student’s progress. More flexible assessment resources should be considered (see e.g. Dossey, 1998).

The non-emotional audience. This tries to present students enrolled in a course as an audience at a movie show or a theatre. The main goal ‘for all’ is simply mathematics. Individual problems, emotional difficulties, personality features do not belong to the teaching and learning of mathematics. Tuition is for solving technical doubts or clarifying previous lectures. Outside the classroom or the scheduled office hours there is no place for further human interaction. The university walls keep human nature out. To sum up, let me quote Krantz (1993):

I don’t think that it is healthy for a mathematics teacher to worry about math anxiety.
Your job is to teach mathematics. Go do it.

That’s a terrible mistake. The ‘audience’ is a group of people in which each individual needs attention.

We, as mathematics educators working at university level, need to destroy the above myths, practices and considerations by taking some positive steps towards another way of teaching (see Howson, 1994).

3. TOWARDS A NEW PARADIGM OF TEACHING MATHEMATICS AT UNIVERSITY LEVEL

In this section we will identify some changes to be considered, some questions which need to be faced urgently and some goals for our future as mathematicians and mathematics educators.

There is a need to redefine mathematical research as a university activity, combining it with a soundly based teaching excellence. The critical pressure of research has evolved into a crazy rolling snowball: publishing as many papers as possible, going into citation and impact indices, attending an increasing number of congresses. It is time to sit down and think about what the main goals of universities today are. It is just possible that good teaching, fine multimedia and educational materials, virtual projects, community work, etc. are becoming more relevant to administrators and society than subscriptions to journals, abstract announcements and department reports. This does not mean a change from the research-realm to the teaching-paradigm. The 'either-research-or-teaching' polarity is false. With a little wisdom both activities can be (and should be) combined. Research also means writing expository papers, critiques of trends, historical perspectives, good texts, analyses of pedagogical materials, improvement of proofs, suggestions as to new approaches or interdisciplinary applications. Institutions and authorities should recognise and stimulate scholarship and research. And there is no need to say that the creation of exclusive research institutions is to be welcomed. But universities cannot close their eyes to their teaching ends. It is not just a question of achieving one annual award or medal for academic distinction but rather it is a matter of continuously controlling and stimulating the quality of education. Good teaching is according to a classic definition: "building understanding, communicating, engaging, problem solving, nurturing and organizing for learning", a complete task that merits special attention and preparation (see Krantz, 1993).

Research into mathematics education at tertiary level may be itself an interesting field of research and may give rise to useful results for all teachers for application to their teaching. Research into mathematics education is a growing scientific discipline (see Niss, 1998, Thurston, 1990). Nowadays it involves many researchers focusing on a wide range of topics and levels. However, there is clearly still a rich agenda for research on teaching and learning problems at university level. It would be marvellous if in the years to come this university research attracted well qualified mathematics specialists. If institutions wish or need to pay more attention to their educational goals, then mathematics education may – or indeed is certain to – play an increasingly important role in people's *vitae*. Though non-educational research has been a priority in people's careers until now, it could well be healthier if future mathematics specialists combined research with more educational aims. Moreover, research into mathematics education gives rise to useful results which should be disseminated and used, so that all mathematics teaching staff may benefit from an up-to-date knowledge of this field (see Niss, 1998).



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