

KAREN KING, JOEL HILLEL AND MICHÈLE ARTIQUE

## TECHNOLOGY

### *A working group report*

#### 1. INTRODUCTION

The technology working group focused on the various ways in which technology can impact upon the teaching and learning of mathematics. As was already underlined in the Discussion Document for this ICMI Study, “Worldwide, increasing use is being made of computers and calculators in mathematics instruction. Much mathematical software and many teaching packages are available for a range of curriculum topics. This, of course, raises such issues as what such software and packages offer to the teaching and learning of the subject, and what potential problems for understanding and reasoning they might generate.” The Discussion Document proposed to identify and analyze innovative projects and research that are particularly fruitful for advancing our thinking in this domain.

Reflecting on the impact of information technologies on the teaching of mathematics is not new for an ICMI Study – ICMI had already launched a study in 1985 entitled “The influence of Computers and Informatics on Mathematics and its Teaching”. That ICMI Study touched all levels of instruction and underlined primarily the impact of computers on several areas, including:

- on mathematics itself; computers have prompted the revisiting of familiar notions such as number and elementary functions, the revitalizing of old problems, and the emergence of new domains. They have extended the range of applications of mathematics, and have blurred the boundaries between pure and applied mathematics;
- on the notion of proof in view of computer-assisted proofs;
- on the practice of mathematicians; computers have led to an increase in experimentation and the use of simulations. They afford new means of communication and accessing information that affect the way mathematicians carry on their professional lives.

This previous ICMI Study also recognized that despite an abundance of interesting experiences, the impact of technology on teaching was still globally weak, and that the introduction of computers in the classroom had not necessarily

led to any discernible improvements. The working group discussion focused on the present-day role of technology in teaching at the post-secondary level, on the perspectives envisaged for the future and on broader research questions that are affected by the use of technology. It centred mostly on the use of technological tools for supporting students' learning, particularly via visualization; computation, and programming. But, it also recognized the role of such tools for: demonstration by the teacher; presentation of lessons via distance learning; student assessment; and student drill.

## 2. TECHNOLOGY AS A MEANS FOR SUPPORTING STUDENTS' LEARNING

At the university level in general, and at the collegial level in particular, the introduction of technologies was seen as a means to renew pedagogical practices and to circumvent a style of teaching that was too formal or too algorithmic. It was intended to create better coherence between teaching practice and the constructivist approach to learning. Celia Hoyles, in her description of the potential contribution to post-secondary education of researches carried in the secondary level, has emphasized that:

“There is considerable evidence of the computer's potential to:

- foster more active learning using experimental approaches along with the possibility of helping students to forge connections between different forms of expression, e.g. visual, symbolic ;
- provoke constructionist approaches to learning mathematics where students learn by building, debugging and reflection, with the result that the structure of mathematics and the ways the pieces fit together are open to inspection ;
- motivate explanations in the face of “surprising” feedback : that is, start a process of argumentation which can (with due attention) be connected to formal proof ;
- foster cooperative work, encouraging discussion of different solutions and strategies ; computer work is more visible and more easily “conveyed” between lecturer and students ;
- open a window on to student thought processes : students hold different conceptions of mathematical ideas which are hard to access, even in the case of articulate adults. How students interact with the computer and respond to feedback can give insight into their conceptions and their beliefs about mathematics and the role of computers.”

Hoyles hastened to add that a successful integration of computers necessitates the rethinking of “the content and sequence of the mathematics courses given that students and mathematics have (or should have) changed in the light of the new technology [...] teaching approaches to take into account the broad range of

response inevitable in interacting with computers [...] and the relationship of 'computer maths' to paper and pencil maths" (Hoyles, 1999).

The question of what constitutes 'successful integration' of technology to the teaching and learning process was central to the working group discussion. Several presentations by participants on the way in which they have used technology to teach mathematics at the undergraduate level, helped to focus the discussion. These included presentations by: Karen King, on teaching differential equations; Ed Dubinsky, on programming using ISETL; Joel Hillel, on using Maple in teaching linear algebra; and, Rosalind Phang, on using statistical software.

### 2.1 Changes in the Nature of the Mathematics Taught

King's example illustrated the nature of the changes in teaching differential equations made possible by using a technology that graphs slope fields and direction fields. These enable students to engage in qualitative analyses of previously inaccessible differential equations rather than use traditional analytic techniques. Thus, the focus of a differential equations' course could shift from just finding the solution functions, to graphically organizing the space of solution functions using slope fields and bifurcation diagrams, and to examining the nature of the solution functions (see Rasmussen, 1999).

If one considers, for example, the differential equation

$$dy/dt = 0.3y(1-y/8)(y/3-1),$$

one could attempt to solve this using separation of variables but would not deduce a closed-form general solution. However, with a slope field as shown in Figure 1 derived from a TI-92 program written by King, a student can examine the types of solution functions and their general behaviours, given different initial conditions.

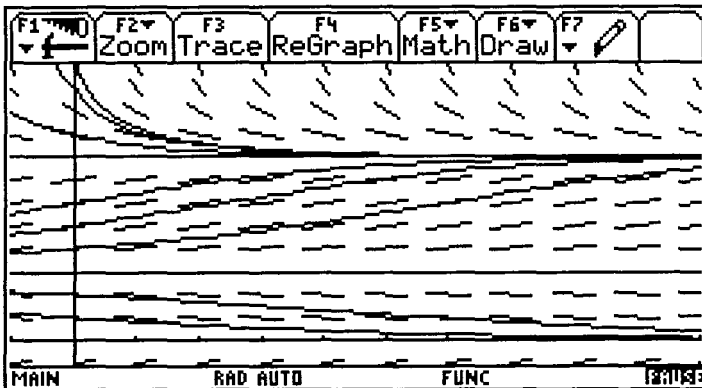


Figure 1 slope field program with slopes and several approximations  
 $dy/dt=0.3y(1-y/8)(y/3-1)$ ,  $y(0)=1, 2, 3, 4, 5, 6, 8, 9, 11, 12$

This example provides an instance where changes to an entire course can be made, including the order in which topics are taught and the mathematics with which the students engage (see Artigue, 1992, and Rasmussen and King, 1999). Such changes, in turn, lead to other changes in the curriculum. For example, the study of dynamical systems has been greatly impacted by the availability of computing technology and has resulted in an early focus on systems of differential equations in many courses. This is but one example where a particular mathematical discipline is changed by technology which, in turns, affects changes in the nature of how it is taught.

## *2.2 The role of professional tools*

Secondary schools tend, by and large, to use software products and calculators that have been specifically conceived for teaching. In contrast, universities mostly tend to use professional tools be they general symbolic manipulators (e.g. Maple, Mathematica, MuPad, Matlab, SciLab) or tools for specific domains such as Statistics (APSS, SASS), though some specific educational software such as Geometer's Sketchpad and Cabri are also relevant for instruction at the tertiary level. Faculty members are familiar with these professional tools since they use them in their own mathematical work and, consequently, they tend to be widely available on campus. There is an ever increasing number of texts that integrate the use of a software package, for example, "Calculus and Mathematica" (Uhl, 1999) or "Ordinary Differential Equations using MATLAB" (Polking, 1995). Individual universities have also written primers that bridge between the particular program or technology that they use and the mathematics texts in use in the department (see Colgan, 1999 for a discussion of such a primer from Australia).

Professional software tools are particularly powerful and, at first sight, seem to take full charge of what traditionally has been the mathematics work expected of students. They embody a tremendous amount of mathematical knowledge that, nevertheless, remains invisible and inaccessible to the users. The availability of these powerful tools raised the inevitable question in the working group regarding the necessary mathematical knowledge of users if they are to become efficient and in reasonable control of such tools. These tools also force us to both question and redefine the content of mathematical training, notably in sectors where mathematics is a service course. It prompts us to ask under what conditions can they become means for students to construct mathematical knowledge, over and above their role as powerful computational tools.

In response to the question regarding the necessary skills/concepts that students must possess before they can use a powerful CAS tool, Hillel suggested seizing the 'black box' feature as a learning opportunity. He presented an example on teaching the Cayley-Hamilton Theorem in linear algebra, where students are first asked to use Maple to build inductive evidence for that theorem. By using the software, students can compute the characteristic polynomial  $f(x)$  of a given matrix  $A$  and then compute  $f(A)$ . Among other things, it becomes apparent that the result of the computation is a square matrix, not a number, to which students must therefore attend and about which they must be explicit. Students also can explore these computations for

several matrices, focusing the process of computing  $f(x)$  rather than on the actual computations. Such activities can take place prior to introducing the theorem and its proof in class. This is a pedagogical choice, a kind of 'didactical inversion' which is made within the larger context of the instructor's course design (see Kent and Noss, 1999, and Noss, 1999).

In a slightly different vein, participants also recognized that there are computer applications designed for other purposes that could be mathematically exploited (e.g., Excel). This raised the questions of how one would characterize the difference (for instructors, for users, and for the types of tasks and interactions) in using educational mathematical tools, professional mathematical tools, and the mathematical usage of tools designed for other purposes.

### *2.3 The role of programming*

In the analysis of the potential of computers for mathematics learning, programming has always played an important role. In the early days of computers when tools for scientific calculations were very different from those of today, programming was essential. But even if software packages have evolved, programming can be seen as a means to change students' relation to algorithmic work, so important in mathematics, by putting the accent on the construction of algorithms rather than on their execution. This shift is seen as a way to give sense to both the algorithms and to the underlying concepts.

Dubinsky presented to the working group the use of programming in a function-based program language (ISETL) to facilitate students' learning about functions (see Dubinsky, 1999). Instead of having students use conventional programs, the students write their own. His work illustrates particularly well the conceptual gains that students make when they have to write mathematical constructions as programs. His approach is built on a theoretical model that looks at learning in terms of actions, processes, and objects. ISETL is particularly well adapted for mathematics, since it favours transforming actions into processes and encapsulation of processes as mathematical objects (see Dubinsky and MacDonald, this volume. pp. 275-282).

Programming activities could also be implemented via scripting which is an automatic execution of an often used sequence of commands. Scripting capabilities are now built into many applications, such as Excel. Whether one uses a programming or scripting language, it is important to pay attention to the kinds of instructional tasks that fit well with the language. Tasks that are appropriate to a function-based language such as ISETL, would not be so in other languages that do not operate the same way.

Finally, it was noted that programming can also play a large role in introducing students to the world of algorithms and the concomitant notions of complexity, validity, and efficiency.

The Teaching and Learning of Mathematics at  
University Level

An ICMI Study

Holton, D. (Ed.)

2001, VIII, 562 p., Softcover

ISBN: 978-1-4020-0072-0