Preface

This handbook is addressed to students of technology institutes where a course on mathematical physics of relatively reduced volume is offered, as well as to engineers and scientists. The aim of the handbook is to treat (demonstrate) the basic methods for solving the simplest problems of classical mathematical physics.

The most basic among the methods considered here is the superposition method. It allows one, based on particular linearly independent solutions (solution "atoms"), to obtain the solution of a given problem. To that end the "supply" of solution atoms must be complete. This method is a development of the well-known method of particular solutions from the theory of ordinary differential equations. In contrast to the case of ordinary differential equations, where the number of linearly independent solutions is always finite, for a linear partial differential equation a complete "supply" of solution atoms is always infinite. This infinite set of solutions may be discrete (for example, for regular boundary value problems in a bounded domain), or form a continuum (for example, in the case of problems in the whole space). In the first case the superposition method reduces to the construction of a series in the indicated solution atoms with unknown coefficients, while in the second case the series is replaced by an integral with respect to the corresponding parameters (variables).

This first step leads us to the general solution of the associated homogeneous equation under the assumption that the set of solution atoms is "complete." The next step – like in the case of ordinary differential equations–is to find the requisite coefficients from the data of the problem.

Thus, the basic bricks of the superposition method are the solution "atoms." A main tool for finding such solutions is the method of separation of variables (assuming null boundary conditions). Unfortunately, this method is applicable only in the case of domains that possess a certain symmetry (for example, in the case of a disc, rectangle, cylinder, or ball). In the case of domains with a complicated structure finding such solution "atoms" is no longer possible. One can then resort to various approximate methods for solving boundary value problems, which is beyond the scope of the present handbook.

Another method discussed in the book is the method of conformal mappings, which allows one to reduce the solution of, say, the Dirichlet problem for the Laplace equation in a complicated domain, to the consideration of the Dirichlet problem in a simpler domain.

Alongside these methods we consider integral transform methods (Fourier, Laplace, Hankel) for nonstationary problems, which are also based on the linear superposition method.

We also devote place in the handbook to a group of problems connected with equations of 4th order. We consider boundary value problems for the biharmonic equation (in a disc and in a half-space) and for a nonhomogeneous equation of 4th order (in a disc).

To supplement our treatment of methods for solving elliptic, hyperbolic and parabolic problems, each chapter ends with problems for independent study and answers to them. We note that in chapters 2 and 3 we give problems also for hyperbolic and parabolic equations of 4th order, with hints for solution.

Let us emphasize that because this book is addressed to readers that are familiar only with differential and integral calculus and some methods for the integration of ordinary differential equations, we are not discussing questions pertaining to the existence of solutions and their membership in appropriate functional spaces.

The entire exposition is formal in nature; if desired, it can be readily given a rigorous mathematical meaning by enlisting the general theory of partial differential equations. The reader interested in a deeper study of the methods demonstrated here or other methods is referred to the bibliography, which contains a number of well-know collections of problems. In particular, as a handbook we recommend S. Farlow's book [22].

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