

Preface

The primary objective of this monograph is to develop an elementary and self-contained approach to the mathematical theory of a viscous incompressible fluid in a domain Ω of the Euclidean space \mathbb{R}^n , described by the equations of Navier-Stokes.

The book is mainly directed to students familiar with basic functional analytic tools in Hilbert and Banach spaces. However, for readers' convenience, in the first two chapters we collect, without proof some fundamental properties of Sobolev spaces, distributions, operators, etc.

Another important objective is to formulate the theory for a completely general domain Ω . In particular, the theory applies to arbitrary unbounded, non-smooth domains. For this reason, in the nonlinear case, we have to restrict ourselves to space dimensions $n = 2, 3$ that are also most significant from the physical point of view. For mathematical generality, we will develop the linearized theory for all $n \geq 2$.

Although the functional-analytic approach developed here is, in principle, known to specialists, its systematic treatment is not available, and even the diverse aspects available are spread out in the literature. However, the literature is very wide, and I did not even try to include a full list of related papers, also because this could be confusing for the student. In this regard, I would like to apologize for not quoting all the works that, directly or indirectly, have inspired this monograph.

Nevertheless, there are some books, in particular, which I think can be useful for a more complete understanding of the subject. Specifically, for functional analysis I refer the reader to the book of Yosida, and to the books of Necăs and Adams for Sobolev spaces. Concerning the Navier-Stokes equations, the reader is referred to the monographs of Ladyzhenskaya, Temam and Galdi. In the latter in particular one can find more specific information on flow in domains with (smooth) bounded and unbounded boundaries.

I conveyed my research interest to the Navier-Stokes equations more than fifteen years ago, stimulated by my colleague Wolf von Wahl. Since then, we started a fruitful collaboration which lasted for several years.

I owe special thanks to Paolo Galdi and Christian Simader for collaboration, encouragement, motivation and good friendship.

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Last but not least, I would like to thank my dear wife Sigrid for keeping away from me all non-mathematical problems, and for giving me a quiet time to elaborate and complete this book.

Altenbeken-Buke, May 1999

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