Preface

This book deals with the symbiotic relationship between

I Quarkonial decompositions of functions,

on the one hand, and

II Sharp inequalities and embeddings in function spaces,

III Fractal elliptic operators,

IV Regularity theory for some semi-linear equations,

on the other hand.

Accordingly, the book has four chapters. In Chapter I we present the Weierstrassian approach to the theory of function spaces, which can be roughly described as follows. Let ψ be a non-negative C^∞ function in $\mathbb{R}^n$ with compact support such that $\{\psi(\cdot - m) : m \in \mathbb{Z}^n\}$ is a resolution of unity in $\mathbb{R}^n$. Let $\psi^\beta(x) = x^\beta \psi(x)$ where $x \in \mathbb{R}^n$ and $\beta \in \mathbb{N}_0^n$. One may ask under which circumstances functions and distributions f in $\mathbb{R}^n$ admit expansions

$$f(x) = \sum_{\beta \in \mathbb{N}_0^n} \sum_{j=0}^{\infty} \sum_{m \in \mathbb{Z}^n} \lambda_{jm}^\beta \psi^\beta(2^j x - m), \quad x \in \mathbb{R}^n, \tag{0.1}$$

with the coefficients $\lambda_{jm}^\beta \in \mathbb{C}$. This resembles, at least formally, the Weierstrassian approach to holomorphic functions (in the complex plane), combined with the wavelet philosophy: translations $x \mapsto x - m$ where $m \in \mathbb{Z}^n$ and dyadic dilations $x \mapsto 2^j x$ where $j \in \mathbb{N}_0$ in $\mathbb{R}^n$. Such representations pave the way to constructive definitions of function spaces. We are mainly interested in the two scales B_{pq}^s and F_{pq}^s with $s \in \mathbb{R}$, $0 < p \leq \infty$, $0 < q \leq \infty$, which cover many well-known classical spaces, such as (fractional) Sobolev spaces, Hölder-Zygmund spaces, Besov spaces and (inhomogeneous) Hardy spaces. The theory of these spaces has been developed systematically by many mathematicians since the early 1960s. The first chapter in [Triγ] is a historically-minded survey of this subject with many references covering the period up to 1990. In Chapter I of the present book we offer the indicated fresh constructive approach to these spaces on $\mathbb{R}^n$, domains, fractals and some manifolds. Chapters II, III, and IV deal with various applications of the results of Chapter I. In Chapter II we contribute to one of the main topics in the theory of function spaces: embeddings and inequalities. We are mostly interested in delicate limiting situations, asking for necessary and sufficient conditions. Chapter III deals with elliptic operators, preferably the Laplacian, in diverse fractal settings, such as fractal boundaries of underlying domains, measure-valued coefficients or potentials, etc. We wish to demonstrate the symbiotic relationship between some basic notation of fractal geometry and spectral theory. This chapter might be considered as the continuation of the Chapters IV and V in [Triδ]. Finally, in Chapter IV of the present book we study truncations in function spaces and we use these results, combined with the quarkonial decompositions indicated above, to develop a new regularity theory for some semi-linear integral and differential equations. Each chapter begins with a separate introduction, where we outline in somewhat greater detail what can be expected.

This book is mainly based on the results of the author and his co-workers obtained in the last few years. We tried to present the material in such a way that the main ideas can be understood independently of the existing literature. On the other hand, after proving in Chapter I that the function spaces introduced via quarkonial decompositions coincide with the well-established spaces B^s_{pq} and F^s_{pq} we feel free to use known results about these spaces, especially when we have nothing new to say about the assertions used. A reader who is mostly interested in the material presented in one of the Chapters II, III, or IV, which are largely independent of each other, may skip Chapter I, at the first glance. But most of the related proofs in these chapters depend substantially on the theory developed in the first chapter.

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