

Contents

Introduction	1
I Finite dimension	19
1 Kolmogorov equations in \mathbb{R}^d with unbounded coefficients	21
1.1 Assumptions	23
1.2 The solution of the associated SDE	25
1.3 Estimates for the derivatives of the solution	34
1.3.1 First derivative	37
1.3.2 Higher order derivatives	39
1.3.3 Conclusion	44
1.4 The transition semigroup	46
1.5 The derivatives of the semigroup	48
1.6 Existence and uniqueness of solutions	52
1.6.1 The parabolic case	52
1.6.2 The elliptic case	53
1.7 Schauder estimates	56
2 Asymptotic behaviour of solutions	65
2.1 Notations and preliminary results	66
2.2 Existence	70
2.3 Uniqueness	71
2.3.1 Strong Feller property	72
2.3.2 Irreducibility	74
2.4 Absolute continuity	77
2.5 The strongly dissipative case	77
3 Analyticity of the semigroup in a degenerate case	81
3.1 Assumptions	83
3.2 Some properties of the solution of the SDE	85
3.3 A generalization of the Bismut-Elworthy formula	91

3.4	The transition semigroup	94
3.5	The generation result	99
II	Infinite dimension	103
4	Smooth dependence on data for the SPDE: the Lipschitz case	105
4.1	Notations and assumptions	107
4.1.1	The operator A	107
4.1.2	The operator Q and the stochastic convolution $w^A(t)$	110
4.1.3	The Nemytskii operator	112
4.2	Differential dependence on initial data	113
4.2.1	First derivative	114
4.2.2	Higher order derivatives	118
4.3	The transition semigroup	130
4.4	Differentiability of the transition semigroup	132
5	Kolmogorov equations in Hilbert spaces	143
5.1	Assumptions	145
5.2	The trace-class property of $D^2(P_t\varphi)QQ^*$	148
5.3	The parabolic problem	154
5.4	The elliptic problem	164
5.5	Schauder estimates	168
6	Smooth dependence on data for the SPDE: the non-Lipschitz case	
(I)		171
6.1	Assumptions and preliminary results	173
6.1.1	The Nemytskii operator	179
6.1.2	The approximating Nemytskii operators	181
6.1.3	Functional spaces	184
6.2	Some a priori estimates for the solution	185
6.2.1	The approximating problem	190
6.3	Differential dependence on initial data	191
6.4	Further properties of the derivatives of the solution	196
6.5	Smoothing properties of the transition semigroup	200
7	Smooth dependence on data for the SPDE: the non-Lipschitz case	
(II)		205
7.1	The transition semigroup	206
7.2	Some approximation results	210
7.3	Smoothing property of the transition semigroup	216

8	Ergodicity	221
8.1	Assumptions	222
8.2	Existence	223
8.3	Uniqueness	228
8.3.1	Strong Feller Property	229
8.3.2	Irreducibility	230
9	Hamilton-Jacobi-Bellman equations in Hilbert spaces	237
9.1	The state equation	239
9.2	The first variation equation	246
9.3	The approximating transition semigroups	250
9.4	The parabolic Hamilton-Jacobi-Bellman equation	253
9.4.1	An a priori estimate	258
9.4.2	Proof of the Theorem 9.4.2	265
9.5	The elliptic Hamilton-Jacobi-Bellman equation	267
9.5.1	Lipschitz hamiltonian K	269
9.5.2	Locally Lipschitz hamiltonian K	276
10	Application to stochastic optimal control problems	281
10.1	The finite horizon case	284
10.2	The infinite horizon case	289
10.3	Existence of the optimal control in the one dimensional case	297
	Appendices	301
A	Dissipative mappings	301
A.1	Subdifferential of the norm	301
A.2	Dissipative mappings	303
B	Weakly continuous semigroups	305
B.1	Definition and main properties	305
B.2	Differentiability of weakly continuous semigroups	309
C	Theorem of contractions depending on parameters	313
	Bibliography	319



<http://www.springer.com/978-3-540-42136-8>

Second Order PDE's in Finite and Infinite Dimension

A Probabilistic Approach

Cerrai, S.

2001, XII, 332 p., Softcover

ISBN: 978-3-540-42136-8