

Table of Contents

Preface	v
Table of Contents	vii
1. Fundamentals	1
1.1 Real Projective Geometry	1
1.1.1 The Real Projective Plane	1
1.1.2 n -dimensional Projective Space	6
1.1.3 Projective Mappings	11
1.1.4 Projectivities, Cross Ratio and Harmonic Position	20
1.1.5 Polarities and Quadrics	28
1.1.6 Complex Extension and the Way from Projective to Euclidean Geometry	54
1.2 Basic Projective Differential Geometry	68
1.2.1 Curves	68
1.2.2 Surfaces	78
1.2.3 Duality	82
1.3 Elementary Concepts of Algebraic Geometry	86
1.3.1 Definitions and Algorithms	86
1.3.2 Geometric Properties of Varieties in Projective Space	99
1.3.3 Duality	104
1.4 Rational Curves and Surfaces in Geometric Design	105
1.4.1 Rational Bézier Curves	105
1.4.2 Dual Bézier Curves	121
1.4.3 Rational Bézier Surfaces	126
2. Models of Line Space	133
2.1 The Klein Model	133
2.1.1 Plücker Coordinates	133
2.1.2 Computing with Plücker Coordinates	137
2.1.3 The Klein Quadric	141
2.2 The Grassmann Algebra	144
2.3 The Study Sphere	154

3. Linear Complexes	159
3.1 The Structure of a Linear Complex	159
3.1.1 Linear Complexes and Null Polarities in Projective Space	159
3.1.2 Linear Complexes and Helical Motions in Euclidean Space	163
3.1.3 Linear Complexes in the Klein Model	168
3.2 Linear Manifolds of Complexes	171
3.2.1 Pencils of Linear Line Complexes	172
3.2.2 Euclidean Properties of Pencils of Linear Complexes	178
3.3 Reguli and Bundles of Linear Complexes	181
3.4 Applications	185
3.4.1 Spatial Kinematics	185
3.4.2 Statics and Screw Theory	191
4. Approximation in Line Space	195
4.1 Fitting Linear Complexes	195
4.2 Kinematic Surfaces	202
4.3 Approximation via Local Mappings into Euclidean 4-Space	211
4.4 Approximation in the Set of Line Segments	221
5. Ruled Surfaces	223
5.1 Projective Differential Geometry of Ruled Surfaces	223
5.1.1 Infinitesimal Properties of First Order	225
5.1.2 Infinitesimal Properties of Higher Order	234
5.2 Algebraic Ruled Surfaces	238
5.2.1 Rational Ruled Surfaces	242
5.2.2 The Bézier Representation of Rational Ruled Surfaces	247
5.2.3 Skew Cubic Surfaces	252
5.3 Euclidean Geometry of Ruled Surfaces	261
5.3.1 First Order Properties	263
5.3.2 A Complete System of Euclidean Invariants	270
5.4 Numerical Geometry of Ruled Surfaces	282
5.4.1 Discrete Models and Difference Geometry	282
5.4.2 Interpolation and Approximation Algorithms	291
5.4.3 Variational Design	296
5.4.4 Offset Surfaces and their Applications	303
5.4.5 Intersection of Ruled Surfaces	309
Color Plates	311
6. Developable Surfaces	327
6.1 Differential Geometry of Developable Surfaces	327
6.2 Dual Representation	334
6.2.1 Differential Geometry of the Dual Surface	334
6.2.2 Developable Bézier and B-Spline Surfaces	343

6.2.3	Interpolation and Approximation Algorithms with Developable Surfaces	352
6.3	Developable Surfaces of Constant Slope and Applications	358
6.3.1	Basics	359
6.3.2	The Cyclographic Mapping and its Applications	366
6.3.3	Rational Developable Surfaces of Constant Slope and Rational Pythagorean-Hodograph Curves	383
6.4	Connecting Developables and Applications	396
6.4.1	Basics	396
6.4.2	Convex Hulls and Binder Surfaces	400
6.4.3	Geometric Tolerancing	405
6.4.4	Two-Dimensional Normed Spaces and Minkowski Offsets	410
6.5	Developable Surfaces with Creases	416
7.	Line Congruences and Line Complexes	423
7.1	Line Congruences	423
7.1.1	Projective Differential Geometry of Congruences	423
7.1.2	Rational Congruences and Trivariate Bézier Representations	428
7.1.3	Euclidean Differential Geometry of Line Congruences	434
7.1.4	Normal Congruences and Geometrical Optics	446
7.1.5	Singularities of Motions Constrained by Contacting Sur- faces and Applications in Sculptured Surface Machining	452
7.1.6	Numerical Geometry of Line Congruences	465
7.1.7	Projection via Line Congruences	469
7.2	Line Complexes	474
7.2.1	Differential Geometry of Line Complexes	474
7.2.2	Algebraic Complexes and Congruences	480
7.2.3	Special Quadratic Complexes	487
8.	Linear Line Mappings — Computational Kinematics	497
8.1	Linear Line Mappings and Visualization of the Klein Model	497
8.1.1	Linear Line Mappings into P^2	498
8.1.2	Linear Line Mappings into P^3	511
8.1.3	Visualization of the Klein Image	519
8.2	Kinematic Mappings	522
8.2.1	Quaternions	523
8.2.2	The Spherical Kinematic Mapping	527
8.2.3	Other Kinematic Mappings	535
8.3	Motion Design	538
	References	547
	List of Symbols	556
	Index	557

Computational Line Geometry

Pottmann, H.; Wallner, J.

2001, X, 564 p. 264 illus., 17 illus. in color., Hardcover

ISBN: 978-3-540-42058-3