

Preface

Line geometry in its original sense is concerned with the set of lines of three-dimensional space. This set of lines may be considered from different angles: there is the incidence geometry, i.e., projective geometry, of lines and linear line manifolds. Line space is a smooth manifold, whose simplest elements are ruled surfaces. There is the algebraic geometry of lines. Computing with lines requires coordinates and geometric models of line space. There are relations to mechanics and spatial kinematics, and therefore applications of line geometry in mechanism design and robotics.

When studying geometric problems of applied mathematics, in Computing and Visualization, line geometry often occurs naturally and leads to the most elegant and efficient solution. Our interest and research in this area showed us that line-geometric methods play an important role — not only do results of old geometers contribute to new problems, but new challenges typically inspire the theory behind the applications as well. Thus there is a very fruitful interplay between academic research and practical applications in this active area. It is one of the aims of this book to show these connections. For this purpose we have interpreted ‘line geometry’ in a broad sense.

This book is written for those graduate students and researchers in science and engineering, who want to learn more about geometry, geometric computing, and its applications. We assume that the reader is familiar with linear algebra, calculus, and elementary differential geometry. Material which requires more background and is not essential for the rest of the book (some remarks and one section) is marked with an asterisk.

In the first chapter we introduce projective geometry in some detail, and briefly consider projective differential geometry, some basic concepts concerning algebraic curves and surfaces, and elementary facts about Bézier and B-spline methods in Geometric Design. The core of this book is an exposition of line geometry both from the abstract point of view and the viewpoint of scientific computing. The presentation of the classical material is influenced by the work of Heinrich Brauner and Gunter Weiss and their consequent use of the Klein model.

The selection of the material has been governed by the applications we are aiming at: Computer Aided Design, Geometric Modeling, Scientific Visualization, Computer Aided Manufacturing and Robotics. We do not treat data structures and

do not perform analysis of algorithms. There is not much in the literature about the line-geometric side of these topics — the interested reader should consult [24].

We have included some recent and so far unpublished results from a research project on computational line geometry. We gratefully acknowledge the funding of this project by grant No. P13648-MAT of the Austrian Science Fund.

In the past years we have been encouraged by several people to write a book on ‘classical’ geometry and its applications in geometric computing. This task seemed too ambitious, so we have narrowed down the area a bit. However, as line geometry has so many connections with other areas which are addressed briefly as well, we hope that we can at least partially fulfill the wishes of our friends in the scientific community.

We should point out that the idea to write a book on ‘computational line geometry’ is not ours, but came from Bahram Ravani at the University of California in Davis, USA. We want to express our thanks to Bahram Ravani, especially for his continuing support of this project. He made it possible that the first author could enjoy the stimulating academic atmosphere at UC Davis for several times. Many discussions with Bahram Ravani, Rida Farouki, Kenneth Joy and Bernd Hamann had an important impact on the progress of the present book.

We have been lucky to find many people who have been willing to help us in various ways. Michael Hofer, Stefan Leopoldseder and Heidrun Mühlthaler have put much time and effort in reading the text carefully. They have eliminated numerous errors and inaccuracies, and their excellent suggestions have greatly improved the readability of the text. Any remaining errors are, of course, the authors’ sole responsibility.

Special emphasis has been laid on visualization, which is also expressed by the fact that this book has 264 figures. There would have been much less illustrations and much less elaborate ones, if we had not had help from Gershon Elber, Georg Glaeser, Bert Jüttler, Hannes Kaufmann, Stefan Leopoldseder, Martin Peterzell, Norbert Pfeifer, Norbert Pomaroli, Hellmuth Stachel, Tamás Várady, Michael Wagner, and Tony Wills. Most of the color plates have been created by Georg Glaeser and Hans-Peter Schröcker from the University of Applied Arts in Vienna — the software package ‘Open Geometry’ [64] turned out to be an excellent tool. Last, but by no means least, most of our figures and many examples have been created by Boris Odehnal, whose work was supported by grant No. P13648-MAT of the Austrian Science Fund. Many thanks again to all our friends who helped us with the present book.

Finally we are very grateful to Martin Peters and Ruth Allewelt from Springer-Verlag, Heidelberg. They supported this book project in the best possible way, and provided conditions where it has been a pleasure to work.

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