

Contents

1. Introduction	1
1.1 What Is Chance, and Why Study It?	3
1.1.1 Chance vs. Determinism	3
1.1.2 Probability Problems in Optics	4
1.1.3 Statistical Problems in Optics	5
1.1.4 Historical Notes	5
2. The Axiomatic Approach	7
2.1 Notion of an Experiment; Events	7
2.1.1 Event Space; The Space Event	8
2.1.2 Disjoint Events	8
2.1.3 The Certain Event	9
2.2 Definition of Probability	9
2.3 Relation to Frequency of Occurrence	10
2.4 Some Elementary Consequences	10
2.4.1 Additivity Property	11
2.4.2 Normalization Property	11
2.5 Marginal Probability	12
2.6 The “Traditional” Definition of Probability	12
2.7 Illustrative Problem: A Dice Game	13
2.8 Illustrative Problem: Let’s (Try to) Take a Trip	14
2.9 Law of Large Numbers	15
2.10 Optical Objects and Images as Probability Laws	15
2.11 Conditional Probability	17
2.12 The Quantity of Information	18
2.13 Statistical Independence	20
2.13.1 Illustrative Problem: Let’s (Try to) Take a Trip (Continued)	21
2.14 Informationless Messages	22
2.15 A Definition of Noise	22
2.16 “Additivity” Property of Information	23
2.17 Partition Law	23
2.18 Illustrative Problem: Transmittance Through a Film	24
2.19 How to Correct a Success Rate for Guesses	25
2.20 Bayes’ Rule	26

2.21	Some Optical Applications	27
2.22	Information Theory Application	28
2.23	Application to Markov Events	28
2.24	Complex Number Events	29
2.25	What is the Probability of Winning a Lottery Jackpot?	30
2.26	What is the Probability of a Coincidence of Birthdays at a Party?	31
3.	Continuous Random Variables	39
3.1	Definition of a Random Variable	39
3.2	Probability Density Function, Basic Properties	39
3.3	Information Theory Application: Continuous Limit	41
3.4	Optical Application: Continuous Form of Imaging Law	41
3.5	Expected Values, Moments	42
3.6	Optical Application: Moments of the Slit Diffraction Pattern	43
3.7	Information Theory Application	44
3.8	Case of Statistical Independence	45
3.9	Mean of a Sum	45
3.10	Optical Application	46
3.11	Deterministic Limit; Representations of the Dirac δ -Function	47
3.12	Correspondence Between Discrete and Continuous Cases	48
3.13	Cumulative Probability	48
3.14	The Means of an Algebraic Expression: A Simplified Approach	49
3.15	A Potpourri of Probability Laws	50
3.15.1	Poisson	50
3.15.2	Binomial	51
3.15.3	Uniform	51
3.15.4	Exponential	52
3.15.5	Normal (One-Dimensional)	53
3.15.6	Normal (Two-Dimensional)	53
3.15.7	Normal (Multi-Dimensional)	55
3.15.8	Skewed Gaussian Case; Gram–Charlier Expansion	56
3.15.9	Optical Application	56
3.15.10	Geometric Law	58
3.15.11	Cauchy Law	58
3.15.12	sinc^2 Law	58
3.16	Derivation of Heisenberg Uncertainty Principle	68
3.16.1	Schwarz Inequality for Complex Functions	68
3.16.2	Fourier Relations	68
3.16.3	Uncertainty Product	69
3.17	Hirschman’s Form of the Uncertainty Principle	70
3.18	Measures of Information	70

3.18.1	Kullback–Leibler Information	70
3.18.2	Renyi Information	71
3.18.3	Wootters Information	71
3.18.4	Hellinger Information	72
3.18.5	Tsallis Information	72
3.18.6	Fisher Information	72
3.19	Fisher Information Matrix	73
4.	Fourier Methods in Probability	79
4.1	Characteristic Function Defined	79
4.2	Use in Generating Moments	80
4.3	An Alternative to Describing RV x	80
4.4	On Optical Applications	80
4.5	Shift Theorem	81
4.6	Poisson Case	81
4.7	Binomial Case	82
4.8	Uniform Case	82
4.9	Exponential Case	82
4.10	Normal Case (One Dimension)	83
4.11	Multidimensional Cases	83
4.12	Normal Case (Two Dimensions)	83
4.13	Convolution Theorem, Transfer Theorem	83
4.14	Probability Law for the Sum of Two Independent RV's	84
4.15	Optical Applications	85
4.15.1	Imaging Equation as the Sum of Two Random Displacements	85
4.15.2	Unsharp Masking	85
4.16	Sum of n Independent RV's; The “Random Walk” Phenomenon	87
4.17	Resulting Mean and Variance: Normal, Poisson, and General Cases	89
4.18	Sum of n Dependent RV's	89
4.19	Case of Two Gaussian Bivariate RV's	90
4.20	Sampling Theorems for Probability	91
4.21	Case of Limited Range of x , Derivation	91
4.22	Discussion	92
4.23	Optical Application	93
4.24	Case of Limited Range of ω	94
4.25	Central Limit Theorem	94
4.26	Derivation	95
4.27	How Large Does n Have To Be?	97
4.28	Optical Applications	97
4.28.1	Cascaded Electro-Optical Systems	97
4.28.2	Laser Resonator	98
4.28.3	Atmospheric Turbulence	99

4.29	Generating Normally Distributed Numbers from Uniformly Random Numbers	100
4.30	The Error Function	102
5.	Functions of Random Variables	107
5.1	Case of a Single Random Variable	107
5.2	Unique Root	108
5.3	Application from Geometrical Optics	109
5.4	Multiple Roots	110
5.5	Illustrative Example	111
5.6	Case of n Random Variables, r Roots	111
5.7	Optical Applications	112
5.8	Statistical Modeling	112
5.9	Application of Transformation Theory to Laser Speckle	113
5.9.1	Physical Layout	113
5.9.2	Plan	114
5.9.3	Statistical Model	114
5.9.4	Marginal Probabilities for Light Amplitudes U_{re} , U_{im}	115
5.9.5	Correlation Between U_{re} and U_{im}	116
5.9.6	Joint Probability Law for U_{re} , U_{im}	117
5.9.7	Probability Laws for Intensity and Phase; Transformation of the RV's	117
5.9.8	Marginal Laws for Intensity and Phase	118
5.9.9	Signal-to-Noise (S/N) Ratio in the Speckle Image	118
5.10	Speckle Reduction by Use of a Scanning Aperture	119
5.10.1	Statistical Model	119
5.10.2	Probability Density for Output Intensity $p_I(v)$	120
5.10.3	Moments and S/N Ratio	121
5.10.4	Standard Form for the Chi-Square Distribution	122
5.11	Calculation of Spot Intensity Profiles Using Transformation Theory	123
5.11.1	Illustrative Example	124
5.11.2	Implementation by Ray-Trace	125
5.12	Application of Transformation Theory to a Satellite-Ground Communication Problem	126
5.13	Unequal Numbers of Input and Output Variables: "Helper Variables"	140
5.13.1	Probability Law for a Quotient of Random Variables	140
5.13.2	Probability Law for a Product of Independent Random Variables	141
5.13.3	More Complicated Transformation Problems	142
5.14	Use of an Invariance Principle to Find a Probability Law	142
5.15	Probability Law for Transformation of a Discrete Random Variable ..	144

6.	Bernoulli Trials and Limiting Cases	147
6.1	Analysis	147
6.2	Illustrative Problems	149
6.2.1	Illustrative Problem: Let's (Try to) Take a Trip: The Last Word	149
6.2.2	Illustrative Problem: Mental Telepathy as a Communication Link?	150
6.3	Characteristic Function and Moments	152
6.4	Optical Application: Checkerboard Model of Granularity	152
6.5	The Poisson Limit	154
6.5.1	Analysis	154
6.5.2	Example of Degree of Approximation	155
6.5.3	Normal Limit of Poisson Law	156
6.6	Optical Application: The Shot Effect	157
6.7	Optical Application: Combined Sources	158
6.8	Poisson Joint Count for Two Detectors – Intensity Interferometry	158
6.9	The Normal Limit (De Moivre–Laplace Law)	162
6.9.1	Derivation	162
6.9.2	Conditions of Use	163
6.9.3	Use of the Error Function	164
7.	The Monte Carlo Calculation	175
7.1	Producing Random Numbers That Obey a Prescribed Probability Law	176
7.1.1	Illustrative Case	177
7.1.2	Normal Case	177
7.2	Analysis of the Photographic Emulsion by Monte Carlo Calculation	178
7.3	Application of the Monte Carlo Calculation to Remote Sensing	180
7.4	Monte Carlo Formation of Optical Images	181
7.4.1	An Example	182
7.5	Monte Carlo Simulation of Speckle Patterns	183
8.	Stochastic Processes	191
8.1	Definition of a Stochastic Process	191
8.2	Definition of Power Spectrum	192
8.2.1	Some Examples of Power Spectra	194
8.3	Definition of Autocorrelation Function; Kinds of Stationarity	194
8.4	Fourier Transform Theorem	195
8.5	Case of a “White” Power Spectrum	196
8.6	Application: Average Transfer Function Through Atmospheric Turbulence	197
8.6.1	Statistical Model for Phase Fluctuations	198

8.6.2	A Transfer Function for Turbulence	199
8.7	Transfer Theorems for Power Spectra	201
8.7.1	Determining the MTF Using Random Objects	201
8.7.2	Speckle Interferometry of Labeyrie	202
8.7.3	Resolution Limits of Speckle Interferometry	203
8.8	Transfer Theorem for Autocorrelation: The Knox–Thompson Method	208
8.9	Additive Noise	211
8.10	Random Noise	212
8.11	Ergodic Property	213
8.12	Optimum Restoring Filter	217
8.12.1	Definition of Restoring Filter	217
8.12.2	Model	218
8.12.3	Solution	219
8.13	Information Content in the Optical Image	221
8.13.1	Statistical Model	222
8.13.2	Analysis	223
8.13.3	Noise Entropy	223
8.13.4	Data Entropy	224
8.13.5	The Answer	225
8.13.6	Interpretation	225
8.14	Data Information and Its Ability to be Restored	226
8.15	Superposition Processes; the Shot Noise Process	227
8.15.1	Probability Law for i	229
8.15.2	Some Important Averages	229
8.15.3	Mean Value $\langle i(x_0) \rangle$	230
8.15.4	Shot Noise Case	231
8.15.5	Second Moment $\langle i^2(x_0) \rangle$	231
8.15.6	Variance $\sigma^2(x_0)$	232
8.15.7	Shot Noise Case	232
8.15.8	Signal-to-Noise (S/N) Ratio	233
8.15.9	Autocorrelation Function	234
8.15.10	Shot Noise Case	235
8.15.11	Application: An Overlapping Circular Grain Model for the Emulsion	236
8.15.12	Application: Light Fluctuations due to Randomly Tilted Waves, the “Swimming Pool” Effect . .	237
9.	Introduction to Statistical Methods: Estimating the Mean, Median, Variance, S/N, and Simple Probability	243
9.1	Estimating a Mean from a Finite Sample	244
9.2	Statistical Model	244
9.3	Analysis	245
9.4	Discussion	246

9.5	Error in a Discrete, Linear Processor: Why Linear Methods Often Fail	246
9.6	Estimating a Probability: Derivation of the Law of Large Numbers	248
9.7	Variance of Error	249
9.8	Illustrative Uses of the Error Expression	250
9.8.1	Estimating Probabilities from Empirical Rates	250
9.8.2	Aperture Size for Required Accuracy in Transmittance Readings	251
9.9	Probability Law for the Estimated Probability; Confidence Limits	252
9.10	Calculation of the Sample Variance	253
9.10.1	Unbiased Estimate of the Variance	253
9.10.2	Expected Error in the Sample Variance	255
9.10.3	Illustrative Problems	256
9.11	Estimating the Signal-to-Noise Ratio; Student's Probability Law	258
9.11.1	Probability Law for SNR	258
9.11.2	Moments of SNR	259
9.11.3	Limit $c \rightarrow 0$; A Student Probability Law	261
9.12	Properties of a Median Window	261
9.13	Statistics of the Median	263
9.13.1	Probability Law for the Median	264
9.13.2	Laser Speckle Case: Exponential Probability Law	264
9.14	Dominance of the Cauchy Law in Diffraction	269
9.14.1	Estimating an Optical Slit Position: An Optical Central Limit Theorem	270
9.14.2	Analysis by Characteristic Function	270
9.14.3	Cauchy Limit, Showing Independence to Aberrations	272
9.14.4	Widening the Scope of the Optical Central Limit Theorem	273
10.	Introduction to Estimating Probability Laws	277
10.1	Estimating Probability Densities Using Orthogonal Expansions	278
10.2	Karhunen–Loeve Expansion	281
10.3	The Multinomial Probability Law	282
10.3.1	Derivation	282
10.3.2	Illustrative Example	283
10.4	Estimating an Empirical Occurrence Law as the Maximum Probable Answer	283
10.4.1	Principle of Maximum Probability (MP)	284
10.4.2	Maximum Entropy Estimate	285
10.4.3	The Search for “Maximum Prior Ignorance”	286
10.4.4	Other Types of Estimates (Summary)	287
10.4.5	Return to Maximum Entropy Estimation, Discrete Case	288
10.4.6	Transition to a Continuous Random Variable	289

10.4.7	Solution	290
10.4.8	Maximized H	290
10.4.9	Illustrative Example: Significance of the Normal Law	290
10.4.10	The Smoothness Property; Least Biased Aspect	291
10.4.11	A Well Known Distribution Derived	292
10.4.12	When Does the Maximum Entropy Estimate Equal the True Law?	293
10.4.13	Maximum Probable Estimates of Optical Objects	294
10.4.14	Case of Nearly Featureless Objects	296
11.	The Chi-Square Test of Significance	307
11.1	Forming the χ^2 Statistic	308
11.2	Probability Law for χ^2 Statistic	309
11.3	When is a Coin Fixed?	311
11.4	Equivalence of Chi-Square to Other Statistics; Sufficient Statistics	312
11.5	When Is a Vote Decisive?	313
11.6	Generalization to N Voters	314
11.7	Use as an Image Detector	315
12.	The Student t-Test on the Mean	321
12.1	Cases Where Data Accuracy is Unknown	322
12.2	Philosophy of the Approach: Statistical Inference	322
12.3	Forming the Statistic	323
12.4	Student's t -Distribution: Derivation	325
12.5	Some Properties of Student's t -Distribution	326
12.6	Application to the Problem; Student's t -Test	327
12.7	Illustrative Example	327
12.8	Other Applications	329
13.	The F-Test on Variance	333
13.1	Snedecor's F -Distribution; Derivation	333
13.2	Some Properties of Snedecor's F -Distribution	334
13.3	The F -Test	335
13.4	Illustrative Example	336
13.5	Application to Image Detection	336
14.	Least-Squares Curve Fitting – Regression Analysis	341
14.1	Summation Model for the Physical Effect	341
14.2	Linear Regression Model for the Noise	343
14.3	Equivalence of ML and Least-Squares Solutions	345
14.4	Solution	346
14.5	Return to Film Problem	347
14.6	“Significant” Factors; The R -Statistic	347
14.7	Example: Was T^2 an Insignificant Factor?	349

14.8	Accuracy of the Estimated Coefficients	350
14.8.1	Absorptance of an Optical Fiber	350
14.8.2	Variance of Error in the General Case	351
14.8.3	Error in the Estimated Absorptance of an Optical Fiber . . .	353
15.	Principal Components Analysis	363
15.1	A Photographic Problem	363
15.2	Equivalent Eigenvalue Problem	364
15.3	The Eigenvalues as Sample Variances	366
15.4	The Data in Terms of Principal Components	366
15.5	Reduction in Data Dimensionality	367
15.6	Return to the H–D Problem	368
15.7	Application to Multispectral Imagery	368
15.8	Error Analysis	371
16.	The Controversy Between Bayesians and Classicists	375
16.1	Bayesian Approach to Confidence Limits for an Estimated Probability	376
16.1.1	Probability Law for the Unknown Probability	377
16.1.2	Assumption of a Uniform Prior	377
16.1.3	Irrelevance of Choice of Prior Statistic $p_0(x)$ if N is Large .	376
16.1.4	Limiting Form for N Large	379
16.1.5	Illustrative Problem	379
16.2	Laplace's Rule of Succession	380
16.2.1	Derivation	380
16.2.2	Role of the Prior	383
16.2.3	Bull Market, Bear Market	384
17.	Introduction to Estimation Methods	387
17.1	Deterministic Parameters: Likelihood Theory	388
17.1.1	Unbiased Estimators	388
17.1.2	Maximum Likelihood Estimators	389
17.1.3	Cramer–Rao Lower Bound on Error	391
17.1.4	Achieving the Lower Bound	393
17.1.5	Testing for Efficient Estimators	394
17.1.6	Can a Bound to the Error be Known if an Efficient Estimator <i>Does Not</i> Exist?	395
17.1.7	When can the Bhattacharyya Bound be Achieved?	397
17.2	Random Parameters: Bayesian Estimation Theory	398
17.2.1	Cost Functions	399
17.2.2	Risk	400
17.2.3	MAP Estimates	405
17.3	Exact Estimates of Probability Laws: The Principle of Extreme Physical Information	413
17.3.1	A Knowledge-Based View of Nature	415

17.3.2	Fisher Information as a Bridge Between Noumenon and Phenomenon	416
17.3.3	No Absolute Origins	419
17.3.4	Invariance of the Fisher Information Length to Unitary Transformation	420
17.3.5	Multidimensional Form of I	421
17.3.6	Lorentz Transformation of Special Relativity	422
17.3.7	What Constants Should be Regarded as Universal Physical Constants?	423
17.3.8	Transition to Complex Probability Amplitudes	424
17.3.9	Space-Time Measurement: Information Capacity in Fourier Space	424
17.3.10	Relation Among Energy, Mass and Momentum	425
17.3.11	Ultimate Resolution	426
17.3.12	Bound Information J and Efficiency Constant κ	426
17.3.13	Perturbing Effect of the Probe Particle	427
17.3.14	Equality of the Perturbed Informations	428
17.3.15	EPI Variational Principle, and Framework	431
17.3.16	The Measurement Process in Detail	433
17.3.17	Euler–Lagrange Solutions	435
17.3.18	Free-Field Klein–Gordon Equation	435
17.3.19	Dirac Equations	436
17.3.20	Schroedinger Wave Equation (SWE)	437
17.3.21	Dimensionality, and Plato’s Cave	438
17.3.22	Wheeler’s “Participatory Universe”	439
17.3.23	Exhaustivity Property, and Future Research	439
17.3.24	Can EPI be Used in a Design Mode?	440
17.3.25	EPI as a Knowledge Acquisition Game	440
17.3.26	Ultimate Uses of EPI	441
Appendix	451
Appendix A	451
Appendix B	453
Appendix C	455
Appendix D	456
Appendix E	459
Appendix F	460
Appendix G	464
References	469
Index	479

<http://www.springer.com/978-3-540-41708-8>

Probability, Statistical Optics, and Data Testing

A Problem Solving Approach

Frieden, R.

2001, XXIV, 494 p. 20 illus., 1 illus. in color., Softcover

ISBN: 978-3-540-41708-8