
Contents

Preface	V
1. Discretization Techniques Based on Domain Decomposition	1
1.1 Introduction to Mortar Finite Element Methods	3
1.2 Mortar Methods with Alternative Lagrange Multiplier Spaces ..	11
1.2.1 An Approximation Property	15
1.2.2 The Consistency Error	19
1.2.3 Discrete Inf-sup Conditions	24
1.2.4 Examples of Lagrange Multiplier Spaces	27
1.2.4.1 The First Order Case in 2D	29
1.2.4.2 The First Order Case in 3D	33
1.2.4.3 The Second Order Case in 2D	36
1.3 Discretization Techniques Based on the Product Space	37
1.3.1 A Dirichlet–Neumann Formulation	40
1.3.2 Variational Formulations	43
1.3.3 Algebraic Formulations	47
1.4 Examples for Special Mortar Finite Element Discretizations ...	50
1.4.1 The Coupling of Primal and Dual Finite Elements	51
1.4.2 An Equivalent Nonconforming Formulation	54
1.4.3 Crouzeix–Raviart Finite Elements	58
1.5 Numerical Results	61
1.5.1 Influence of the Lagrange Multiplier Spaces	64
1.5.2 A Non-optimal Mortar Method	71
1.5.3 Influence of the Choice of the Mortar Side	74
1.5.4 Influence of the Jump of the Coefficients	83
2. Iterative Solvers Based on Domain Decomposition	85
2.1 Abstract Schwarz Theory	87
2.1.1 Additive Schwarz Methods	88
2.1.2 Multiplicative Schwarz Methods	95
2.1.3 Multigrid Methods	97
2.2 Vector Field Discretizations	99
2.2.1 Raviart–Thomas Finite Elements	101
2.2.2 An Iterative Substructuring Method	103

2.2.2.1	An Interpolation Operator onto \mathbf{V}_H	105
2.2.2.2	An Extension Operator onto \mathbf{V}_F	107
2.2.2.3	Quasi-optimal Bounds	113
2.2.3	A Hierarchical Basis Method	114
2.2.3.1	Horizontal Decomposition	115
2.2.3.2	Vertical Decomposition	118
2.2.4	Numerical Results	121
2.2.4.1	The 2D Case	122
2.2.4.2	The 3D Case	123
2.3	A Multigrid Method for the Mortar Product Space Formulation	125
2.3.1	Bilinear Forms	126
2.3.2	An Approximation Property	129
2.3.3	Smoothing and Stability Properties	131
2.3.4	Implementation of the Smoothing Step	136
2.3.5	Numerical Results in 2D and 3D	137
2.3.6	Extensions to Linear Elasticity	142
2.3.6.1	Uniform Ellipticity	145
2.3.6.2	Numerical Results	149
2.3.6.3	A Weaker Interface Condition	151
2.4	A Dirichlet–Neumann Type Method	155
2.4.1	The Algorithm	155
2.4.2	Numerical Results	158
2.5	A Multigrid Method for the Mortar Saddle Point Formulation	162
2.5.1	An Approximation Property	164
2.5.2	Smoothing and Stability Properties	167
2.5.2.1	A Block Diagonal Smoother	168
2.5.2.2	An Indefinite Smoother	171
2.5.3	Numerical Results	174
	Bibliography	177
	List of Figures	187
	List of Tables	189
	Notations	191
	Index	195

Discretization Methods and Iterative Solvers Based on
Domain Decomposition

Wohlmuth, B.I.

2001, X, 199 p. 5 illus., Softcover

ISBN: 978-3-540-41083-6