

Contents

Part I. Beginnings

0. Two Starting Examples	3
0.1 A Blend of Probability, Logic and Combinatorics	3
0.2 A Random Unary Predicate	8
0.3 Some Comments on References	11
1. Preliminaries	13
1.1 What is the Random Graph $G(n, p)$?	13
1.1.1 The Erdős-Rényi Evolution	14
1.1.2 The Appearance of Small Subgraphs	15
1.2 What is a First Order Theory?	15
1.3 Extension Statements and Rooted Graphs	17
1.4 What is a Zero-One Law?	18
1.5 Almost Sure Theories and Complete Theories	20
1.6 Countable Models	20
2. The Ehrenfeucht Game	23
2.1 The Rules of the Game	23
2.2 Equivalence Classes and Ehrenfeucht Value	27
2.3 Connection to First Order Sentences	31
2.4 Inside-Outside Strategies	33
2.5 The Bridge to Zero-One Laws	37
2.6 Other Structures	39
2.6.1 General First Order Structures	39
2.6.2 The Simple Case of Total Order	40
2.6.3 k -Similar Neighborhoods	42

Part II. Random Graphs

3. Very Sparse Graphs	49
3.1 The Void	50
3.2 On the k -th Day	50
3.3 On Day ω	51

3.3.1	An Excursion into Rooted Trees	51
3.3.2	Two Consequences	55
3.4	Past the Double Jump	56
3.5	Beyond Connectivity	57
3.6	Limiting Probabilities	58
3.6.1	A General Result on Limiting Probabilities	58
3.6.2	In the Beginning	59
3.6.3	On the k -th Day	60
3.6.4	At the Threshold of Connectivity	61
3.7	The Double Jump in the First Order World	62
3.7.1	Poisson Childbearing	63
3.7.2	Almost Completing the Almost Sure Theory	65
4.	The Combinatorics of Rooted Graphs	69
4.1	Sparse, Dense, Rigid, Safe	69
4.2	The t -Closure	73
4.3	The Finite Closure Theorem	74
5.	The Janson Inequality	79
5.1	Extension Statements	80
5.2	Counting Extensions	82
5.3	Generic Extension	85
6.	The Main Theorem	87
6.1	The Look-Ahead Strategy	87
6.1.1	The Final Move	88
6.1.2	The Core Argument (Middle Moves)	88
6.1.3	The First Move	89
6.2	The Original Argument	90
7.	Countable Models	93
7.1	An Axiomatization for T_α	93
7.1.1	The Schema	93
7.1.2	Completeness Proof	93
7.1.3	The Truth Game	95
7.2	Countable Models	97
7.2.1	Construction	97
7.2.2	Uniqueness of the Model	99
7.2.3	NonUniqueness of the Model	100
7.3	A Continuum of Complete Theories	102
8.	Near Rational Powers of n	103
8.1	Infinitely Many Ups and Downs	103
8.1.1	In the Second Order World	103
8.1.2	Replacing Second Order by First Order	105

8.1.3	Are First Order Properties Natural?	107
8.2	Existence of Finite Models	108
8.3	NonSeparability and NonConvergence	109
8.3.1	Representing All Finite Graphs	110
8.3.2	NonSeparability	111
8.3.3	Arithmetization	112
8.3.4	NonConvergence	113
8.4	The Last Threshold	115
8.4.1	Just Past $n^{-\alpha}$: The Theory T_{α}^{-}	115
8.4.2	Just Past $n^{-\alpha}$: A Zero-One Law	117

Part III. Extras

9.	A Dynamic View	121
9.1	More Zero-One Laws	121
9.1.1	Near Irrational Powers	121
9.1.2	Dense Random Graphs	122
9.2	The Limit Function	122
9.2.1	Definition	122
9.2.2	Look-Ahead Functions	123
9.2.3	Well Ordered Discontinuities	124
9.2.4	Underapproximation sequences	125
9.2.5	Determination in PH	127
10.	Strings	131
10.1	Models and Language	131
10.2	Ehrenfeucht Redux	132
10.2.1	The Rules	132
10.2.2	The Semigroup	133
10.2.3	Long Strings	133
10.3	Persistent and Transient	134
10.4	Persistent Strings	136
10.5	Random Strings	137
10.6	Circular Strings	138
10.7	Sparse Unary Predicate	140
11.	Stronger Logics	145
11.1	Ordered Graphs	145
11.1.1	Arithmetization	145
11.1.2	Dance Marathon	147
11.1.3	Slow Oscillation	148
11.2	Existential Monadic	149

12. Three Final Examples	153
12.1 Random Functions	153
12.2 Distance Random $G(n, \bar{p})$	155
12.2.1 Without Order	155
12.2.2 With Order	157
12.3 Random Lifts	160
Bibliography	165
Index	167



<http://www.springer.com/978-3-540-41654-8>

The Strange Logic of Random Graphs

Spencer, J.

2001, X, 168 p., Hardcover

ISBN: 978-3-540-41654-8