

# Contents

<b>Preface</b>	v
<b>A Word on Notation</b>	ix
<b>Introduction</b>	1
<b>1. The First Main Theorem</b>	5
§1.1. The Poisson-Jensen Formula	5
§1.2. The Nevanlinna Functions	12
Counting Functions	12
Mean Proximity Functions	13
Height or Characteristic Functions	17
§1.3. The First Main Theorem	17
§1.4. Ramification and Wronskians	20
§1.5. Nevanlinna Functions for Sums and Products	23
§1.6. Nevanlinna Functions for Some Elementary Functions	24
§1.7. Growth Order and Maximum Modulus	27
§1.8. A Number Theoretic Digression: The Product Formula	28
§1.9. Some Differential Operators on the Plane	31
§1.10. Theorems of Stokes, Fubini, and Green-Jensen	33
§1.11. The Geometric Interpretation of the Ahlfors-Shimizu Characteristic	35
§1.12. Why $N$ and $\bar{T}$ are Used in Nevanlinna Theory Instead of $n$ and $A$	41
§1.13. Relationships Among the Nevanlinna Functions on Average	43
§1.14. Jensen's Inequality	47
<b>2. The Second Main Theorem via Negative Curvature</b>	49
§2.1. Khinchin Functions and Exceptional Sets	49
§2.2. The Nevanlinna Growth Lemma and the Height Transform	51
§2.3. Definitions and Notation	56
§2.4. The Ramification Theorem	57
§2.5. The Second Main Theorem	60
§2.6. A Simpler Error Term	71
§2.7. The Unintegrated Second Main Theorem	73

§2.8.	A Uniform Second Main Theorem . . . . .	74
§2.9.	The Spherical Isoperimetric Inequality . . . . .	78
<b>3.</b>	<b>Logarithmic Derivatives . . . . .</b>	<b>89</b>
§3.1.	Inequalities of Smirnov and Kolokolnikov . . . . .	89
§3.2.	The Gol'dberg-Grinshtein Estimate . . . . .	92
§3.3.	The Borel-Nevanlinna Growth Lemma . . . . .	98
§3.4.	The Logarithmic Derivative Lemma . . . . .	104
§3.5.	Functions of Finite Order . . . . .	106
<b>4.</b>	<b>The Second Main Theorem via Logarithmic Derivatives . . . . .</b>	<b>107</b>
§4.1.	Definitions and Notation . . . . .	107
§4.2.	The Second Main Theorem . . . . .	108
§4.3.	Functions of Finite Order . . . . .	117
<b>5.</b>	<b>Some Applications . . . . .</b>	<b>119</b>
§5.1.	Infinite Products . . . . .	120
§5.2.	Defect Relations . . . . .	123
§5.3.	Picard's Theorem . . . . .	128
§5.4.	Totally Ramified Values . . . . .	129
§5.5.	Meromorphic Solutions to Differential Equations . . . . .	129
§5.6.	Functions Sharing Values . . . . .	135
§5.7.	Bounding Radii of Discs . . . . .	136
§5.8.	Theorems of Landau and Schottky Type . . . . .	141
§5.9.	Slowly Moving Targets . . . . .	144
§5.10.	Fixed Points and Iteration . . . . .	146
<b>6.</b>	<b>A Further Digression into Number Theory: Theorems of Roth and Khinchin . . . . .</b>	<b>151</b>
§6.1.	Roth's Theorem and Vojta's Dictionary . . . . .	151
§6.2.	The Khinchin Convergence Condition . . . . .	158
<b>7.</b>	<b>More on the Error Term . . . . .</b>	<b>161</b>
§7.1.	Sharpness of the Second Main Theorem and the Logarithmic Derivative Lemma . . . . .	161
§7.2.	Better Error Terms for Functions with Controlled Growth . . . . .	170
§7.3.	Error Terms for Some Classical Special Functions . . . . .	177
	<b>Bibliography . . . . .</b>	<b>187</b>
	<b>Glossary of Notation . . . . .</b>	<b>193</b>
	<b>Index . . . . .</b>	<b>197</b>

Nevanlinna's Theory of Value Distribution  
The Second Main Theorem and its Error Terms  
Cherry, W.; Ye, Z.  
2001, XII, 203 p., Hardcover  
ISBN: 978-3-540-66416-1