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Verification of general relativity: strong fields and gravitational waves

At the time of the birth of general relativity (GR), experimental confirmation was almost a side issue. Einstein did calculate observable effects of general relativity, such as the deflection of light, which were tested, but compared to the inner consistency and elegance of the theory, he regarded such empirical questions as almost peripheral. But today, experimental gravitation is a major component of the field, characterized by continuing efforts to test the theory's predictions, to search for gravitational imprints of high-energy particle interactions, and to detect gravitational waves from astronomical sources.

The modern history of experimental relativity can be divided roughly into four periods: Genesis, Hibernation, a Golden Era, and the Quest for Strong Gravity. The Genesis (1887–1919) comprises the period of the two great experiments that were the foundation of relativistic physics – the Michelson–Morley experiment and the Eötvös experiment – and the two immediate confirmations of GR – the deflection of light and the perihelion advance of Mercury. Following this was a period of Hibernation (1920–1960) during which relatively few experiments were performed to test GR, and at the same time the field itself became sterile and stagnant, relegated to the backwaters of physics and astronomy.

But beginning around 1960, astronomical discoveries (quasars, pulsars, cosmic background radiation) and new experiments pushed GR to the forefront. Experimental gravitation experienced a Golden Era (1960–1980) during which a systematic, worldwide effort took place to understand the observable predictions of GR, to compare and contrast them with the predictions of alternative theories of gravity, and to perform new experiments to test them. The period began with an experiment to confirm the gravitational frequency shift of light (1960) and ended with the reported decrease in the orbital period of the binary pulsar at a rate consistent with the general relativity prediction of gravity-wave energy loss (1979). The results all supported GR, and most alternative theories of gravity fell by the wayside (for a popular review, see Will 1993a).

Since 1980, the field has moved toward what might be termed a Quest for Strong Gravity. The principal figure of merit that distinguishes strong from weak gravity is the quantity $\epsilon \sim GM/Rc^2$, where G is the Newtonian gravitational constant, M is the characteristic mass scale of the phenomenon, R is the characteristic distance scale, and c is the speed of light. Near the event horizon of a non-rotating black hole, or for the expanding Universe, $\epsilon \sim 0.5$; for neutron stars, $\epsilon \sim 0.2$. These are the regimes of strong gravity. For the solar system, $\epsilon < 10^{-5}$; this is the regime of weak gravity. Figure 1 displays these regimes and various phenomena of interest. Notice that the strong-gravity regime (near the diagonal line corresponding to $\epsilon = 0.5$) encompasses Planck-scale physics where quantum gravity and grand unification of the interactions are important, all the way to the observable universe.

Until the discovery of the binary pulsar, the empirical foundation of general relativity rested on tests in the weak-field regime. In Chapter 15, Kenneth Nordtvedt Jr. surveys experimental tests of general relativity primarily from the point of view of Solar System and laboratory experiments. While most of these focus on weak-gravity effects, some can be viewed as strong-gravity tests, primarily experiments to search for the relics of Planck-scale physics in fundamental interactions, as might show up in tests of the equivalence principle.

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Today, much of the focus has shifted to experiments which can probe the effects of strong gravitational fields (see Figure 1). At one extreme are the strong gravitational fields associated with Planck-scale physics. Will unification of the forces, or quantization of gravity at this scale leave observable effects accessible by experiment, even those that themselves are performed under weak-field conditions? Dramatically improved tests of the equivalence principle or of the "inverse square law" are being designed, to search for or to bound the imprinted effects of Planck-scale phenomena (see Chapter 15). At the other extreme are the strong fields associated with compact objects such as black holes or neutron stars or with the universe as a whole. Astrophysical observations and gravitational-wave detectors are being planned to explore and test GR in the strong-field, highly dynamical regime associated with the formation and dynamics of these objects.

In this chapter, we shall focus on tests of general relativity involving strong gravity and gravitational radiation. These tests mainly involve black holes, neutron stars, and cosmology, and make essential use of gravitational radiation as a carrier of information about strong gravity. Even though the gravitational waves that bathe the Earth are extraordinarily weak (and are themselves describable by the weak-field limit of general relativity), they often carry the imprints of strong-gravity phenomena occurring near black holes, neutron stars, or from the early Universe.

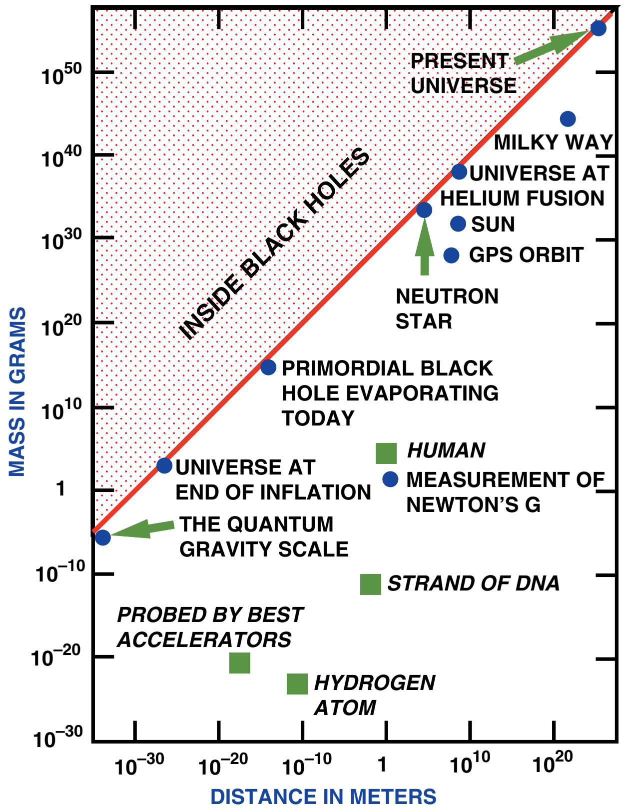

Figure 1 Strong *v.* weak gravity. Representative phenomena in gravitational physics are indicated by filled circles; other illustrative phenomena where gravitation plays little role are shown by filled squares and italics. Phenomena above the diagonal line are unobservable, because they take place inside black holes. Phenomena close to the diagonal line are in the strong-gravity regime.

We begin (Section 1) by defining the distinction between strong and weak gravity, and providing (Section 2) an introduction to the general relativistic description of systems involving compact relativistic objects and gravitational radiation. In Section 3 we focus on tests of gravitational theory using stellar binary systems of compact objects, the most famous of these being the binary pulsar PSR 1913+16. Section 4 focuses on the likely direct detection of gravitational radiation by Earth-based antennae in the first decade of the twenty-first century, and the possible tests of gravitational theory that could emerge. In Section 5 we briefly discuss other tests of strong gravity.

We use the standard "geometrized" units of general relativity textbooks (Misner *et al.* 1973), in which Newton's gravitational constant G and the speed of light c are unity. In these units, mass and energy have units of length, with the basic scale of length set by the solar mass: $M_\odot = 1.476\,\text{km}$, and velocities are dimensionless, representing fractions of the speed of light. In the few places where indices are used, Greek indices will run over the four spacetime dimensions, $\{0, 1, 2, 3\}$, with the 0 denoting time; while Roman indices will run only over spatial indices $\{1, 2, 3\}$.

Rather than provide complete references to work done in this field, we will refer the reader where possible to the appropriate review articles and monographs, specifically to *Theory and Experiment in Gravitational Physics* (Will 1993b), hereafter referred to as TEGP. References to TEGP will be by chapter or section, e.g. "TEGP 8.9".

1 STRONG-FIELD SYSTEMS IN GENERAL RELATIVITY

1.1 Defining weak and strong gravity

In the Solar System, gravity is weak, in the sense that the Newtonian gravitational potential

$$U(\mathbf{x}, t) \equiv \int \rho(\mathbf{x}', t)|\mathbf{x} - \mathbf{x}'|^{-1} d^3x' \tag{1}$$

is much smaller than unity (recall $G = c = 1$) everywhere. So, too, are the quantities v^2 and p/ρ, where v is a typical velocity of bodies in the Solar System, and where p and ρ are typical pressure and density, respectively, inside Solar System bodies. In fact $U \sim v^2 \sim p/\rho \sim \epsilon$. Throughout the Solar System, the metric of spacetime deviates only slightly from its flat-spacetime Minkowski form $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$. By expanding the metric about this form in powers of the small parameter ϵ, one can obtain the Newtonian limit of general relativity at lowest order, and the first "post-Newtonian", or "1PN", corrections to Newtonian gravity. The metric in such an expansion takes the schematic form

$$
\begin{aligned}
g_{00} &= -1 + 2U(\mathbf{x}, t) + O(\epsilon^2) \\
g_{0i} &= O(\epsilon^{3/2}) \\
g_{ij} &= \delta_{ij} + O(\epsilon)
\end{aligned}
\tag{2}
$$

where the first terms beyond the Minkowski metric and the Newtonian potential at $O(\epsilon^2)$ in g_{00}, $O(\epsilon^{3/2})$ in g_{0i}, and $O(\epsilon)$ in g_{ij}, respectively, are the post-Newtonian terms. It turns out that, in a wide range of alternative theories of gravity, these post-Newtonian terms vary from one theory to the next only in the values of certain numerical coefficients. By inserting arbitrary dimensionless parameters in place of the numerical coefficients, one obtains a parametrized framework that encompasses many theories at once, and is a powerful tool for analyzing Solar System experiments. This "parametrized post-Newtonian" framework is described in more detail in Chapter 15; see also TEGP 4.

In strong-field systems, this simple 1PN approximation is no longer appropriate, for several reasons:

- The system may contain strongly relativistic objects, such as neutron stars or black holes, near and inside which $\epsilon \sim 1$, and the post-Newtonian approximation breaks down. Nevertheless, under some circumstances, the orbital motion may be such that the interbody potential and orbital velocities still satisfy $U \sim v^2 \ll 1$ so that a kind of post-Newtonian approximation for the orbital motion might work; however, the strong-field internal gravity of the bodies could (especially in alternative theories of gravity) leave imprints on the orbital motion.
- The evolution of the system may be affected by the emission of gravitational radiation. The 1PN approximation does not contain the effects of gravitational radiation back-reaction. In the expression for the metric given in eqns (2), radiation back-reaction effects do not occur until $O(\epsilon^{7/2})$ in g_{00}, $O(\epsilon^3)$ in g_{0i}, and $O(\epsilon^{5/2})$ in g_{ij}. Consequently, in order to describe such systems, one must carry out a solution of the equations substantially beyond 1PN order, sufficient to incorporate the leading radiation damping terms at 2.5PN order.
- The system may be highly relativistic in its orbital motion, so that $U \sim v^2 \sim 1$ even for the interbody field and orbital velocity. Systems like this include the late stage of the inspiral of binary systems of neutron stars or black holes, driven by gravitational radiation damping, prior to a merger and collapse to a final stationary state. Binary inspiral is one of the leading candidate sources for detection by a worldwide network of laser interferometric gravitational-wave observatories nearing completion. A proper description of such systems requires not only equations for the motion of the binary carried to extraordinarily high PN orders (at least 3.5PN), but also requires equations for the far-zone gravitational waveform measured at the detector, that are equally accurate to high PN orders beyond the leading "quadrupole" approximation.

Of course, some systems cannot be properly described by any post-Newtonian approximation because their behavior is fundamentally controlled by strong gravity. These include the imploding cores of supernovae, the final merger of two compact objects, the quasinormal-mode vibrations of neutron stars and black holes, the structure of rapidly rotating neutron stars, and so on. Phenomena such as these must be analyzed using different techniques. Chief among these is the full solution of Einstein's equations via numerical methods. This field of "numerical relativity" is a rapidly growing and evolving branch of gravitational physics, whose description is beyond the scope of this article (see, e.g., the articles in Marck and Lasota 1997).

1.2 Compact bodies and the strong equivalence principle

When dealing with the motion and gravitational-wave generation by orbiting bodies, one finds a remarkable simplification within general relativity. As long as the bodies are sufficiently well separated that one can ignore tidal interactions and other effects that depend upon the finite extent of the bodies (such as their quadrupole and higher multipole moments), then all aspects of their orbital behavior and gravitational-wave generation can be characterized by just two parameters: mass and angular momentum. Whether their internal structure is highly relativistic, as in black holes or neutron stars, or non-relativistic as in the Earth and Sun, only the mass and angular momentum are needed. Furthermore, both quantities are measurable in principle by examining the external gravitational field of the bodies, and make no reference whatsoever to their interiors.

Damour (1987) calls this the "effacement" of the bodies' internal structure. It is a consequence of the strong equivalence principle (SEP), which is satisfied by general relativity, but is violated by almost all other gravitational theories, including scalar–tensor gravity (Brans–Dicke theory and its generalizations). SEP states that (i) all bodies fall in an external gravitational field with the same acceleration (modulo tidal interactions), (ii) the outcome of any local test experiment is independent of the velocity of the (freely falling) apparatus, and (iii) the outcome of any local test experiment is independent of where and when in the Universe it is performed. The term "bodies" here includes everything from laboratory-sized objects to black holes; and the term "experiments" includes everything from electromagnetic experiments to measurements of Newton's G.

General relativity satisfies SEP because it contains one, and only one, gravitational field, the spacetime metric $g_{\mu\nu}$. Consider the motion of a body in a binary system, whose size is small compared to the binary separation. Surround the body by a region that is large compared to the size of the body, yet small compared to the separation. Because of the general covariance of the theory, one can choose a freely falling coordinate system which co-moves with the body, whose spacetime metric takes the Minkowski form at its outer boundary (ignoring tidal effects generated by the companion). There is thus no evidence of the presence of the companion body, and the structure of the chosen body can be obtained using the field equations of GR in this coordinate system. Far from the chosen body, the metric is characterized by the mass and angular momentum (assuming that one ignores quadrupole and higher multipole moments of the body) as measured far from the body using orbiting test particles and gyroscopes. These asymptotically measured quantities are oblivious to the body's internal structure. A black hole of mass m and a planet of mass m would produce identical spacetimes in this outer region.

The geometry of this region surrounding the one body must be matched to the geometry provided by the companion body. Einstein's equations provide consistency conditions for this matching that yield constraints on the motion of the bodies. These are the equations of motion. As a result the motion of two planets of mass and angular momentum m_1, m_2, $\mathbf{J}_1$ and $\mathbf{J}_2$ is identical to that of two black holes of the same mass and angular momentum (again, ignoring tidal effects).

This effacement does not occur in an alternative gravitional theory like scalar–tensor gravity. There, in addition to the spacetime metric, a scalar field Φ is generated by the masses of the bodies, and controls the local value of the gravitational coupling constant (i.e., G is a function of Φ). Now, in the local frame surrounding one of the bodies in our binary system, while the metric can still be made Minkowskian far away, the scalar field will take on a value Φ_0 determined by the companion body. This can affect the value of G inside the chosen body, alter its internal structure (specifically its gravitational binding energy), and hence alter its mass. Effectively, each mass becomes several functions $m_A(\Phi)$ of the value of the scalar field at its location, and several distinct masses come into play; inertial mass, gravitational mass, "radiation" mass, and so on. The precise nature of the functions will depend on the body, specifically on its gravitational binding energy, and as a result, the motion and gravitational radiation may depend on the internal structure of each body. For compact bodies such as neutron stars and black holes, these internal structure effects could be large; for example, the gravitational binding energy of a neutron star can be 40% of its total mass.

At 1PN order, the leading manifestation of this effect is a violation of the equality of acceleration of massive bodies such as the Earth and the Moon. This effect, known as the Nordtvedt effect, has been tested by lunar laser ranging (see Chapter 15 or TEGP 8 for further discussion).

This is how the study of orbiting systems containing compact objects provides strong-field tests of general relativity. Even though the strong-field nature of the bodies is effaced in GR, it is not in other theories, thus any result in agreement with the predictions of GR constitutes a kind of "null" test of strong-field gravity.

2 MOTION AND GRAVITATIONAL RADIATION IN GENERAL RELATIVITY

2.1 Introduction

The motion of bodies and the generation of gravitational radiation are long-standing problems that date back to the first years following the publication of GR, when Einstein calculated the gravitational radiation emitted by a laboratory-scale object using the linearized version of GR, and de Sitter calculated N-body equations of motion for bodies in the 1PN approximation to GR. It has at times been controversial, with disputes over such issues as whether Einstein's equations alone imply equations of motion for bodies (Einstein, Infeld, and Hoffman demonstrated explicitly that they do, using a matching procedure similar to the one described in Section 1.2), whether gravitational waves are real or are artifacts of general covariance (Einstein waffled; Bondi and colleagues proved their reality rigorously in the 1950s), or even over algebraic errors (Einstein erred by a factor of two in his first radiation calculation; Eddington found the mistake). Shortly after the discovery of the binary pulsar PSR 1913 + 16 in 1974, questions were raised about the foundations of the "quadrupole formula" for gravitational radiation damping (and in some quarters, even about its quantitative validity). These questions were answered in part by theoretical work designed to shore up the foundations of the quadrupole approximation, and in part (perhaps mostly) by the agreement between the predictions of the quadrupole formula and the *observed* rate of damping of the pulsar's orbit (see Section 3.1). Damour (1987) gives a thorough review of this subject.

The problem of motion and radiation has received renewed interest since 1990, with proposals for the construction of large-scale laser interferometric gravitational-wave

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