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KENNETH NORDTVEDT*

Verification of general relativity: tests in the Solar System

In 1900, Isaac Newton's worldview of gravity, space, and time still prevailed – that the gravitational force was a universal, direct, and instantaneous action-at-a-distance between the masses of the Universe, that bodies and light rays moved through an "absolute space, in its own nature, without anything external" whose geometric structure was rigidly Euclidean without end, and that the dynamics of physical law unfolded with respect to an "absolute, true, and mathematical time (flowing) equably without relation to anything external" (Newton 1687). Through the twentieth century that edifice was overthrown and replaced by Albert Einstein's general relativity (GR) perspective – that gravity is an interaction transmitted by a causal and dynamic field whose sources are all forms of energy, including its own contributions, and which then acts elsewhere upon the same; and that the metrical relations between the clocks, rulers, and signals throughout the cosmos are dynamic, non-Euclidean, locationally dependent, and established by the fields of gravity. The detailed structure of metric gravitational field components in the Solar System has in all cases been found to match the predictions of GR in a variety of experiments which primarily employed radar and laser ranging between Earth and other planets or spacecraft.

The first half of the twentieth century was occupied mainly by the construction and calculational exploration of Einstein's theory which he built upon the foundations of James Clerk Maxwell's electromagnetic field theory, his own special relativity theory, and then his Equivalence Principle. Further exploration and application of the theory to a variety of *gedanken* experiments, temporarily beyond the reach of experimental test, continued in subsequent decades. But when the post-World War II years brought forth a stream of new technical abilities to launch spacecraft and to send ranging signals out into the Solar System, and other supporting technologies, experiments were carried out with ever-increasing precision to confirm and quantitatively measure the full variety of novel phenomena in the Solar System predicted by GR.

Although the Newtonian model of Solar System dynamics had been quite successful when used to discover the planet Neptune from what seemed to be unexplained perturbations in the observed motions of the planet Uranus, and when it explained the Moon's numerous orbital "irregularities" which result from the competition of the Sun's gravitational influence with that of Earth on that satellite, the Newtonian system still faced problems, such as a robust anomaly found in the observed precession rate of the major axis of Mercury's orbit, and the failure of Albert Michelson and Edward Morley to detect any change in the speed of light passing through their interferometer as the Earth changed its velocity through the cosmos. Newtonian gravity and cosmology also faced theoretical challenges. This was brought into focus in the late nineteenth century by Ernst Mach, physicist and positivist critic of several concepts in physical law, who labeled the notion of absolute time "an idle metaphysical conception" (Mach 1893), and stressed that what is observed in nature and experiment is the behavior of clocks, not the unfolding of "time" as such. He also asserted that it was the relative feature of local motion with respect to the distant "fixed stars" of the Universe that was empirically meaningful, not the notion of absolute motion relative to "space" as such. Einstein acknowledged the influence of Mach's ideas in his formative years. Describing some consequences of his new theory – that the inertia of a

* Montana State University, Bozeman, MT, USA

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mass was increased by the proximity of other matter, and that accelerated or rotating matter induced corresponding accelerations or rotations of nearby inertial frames, Einstein (1922) pointed out that, "We must see in (these examples) a strong support for Mach's ideas as to the relativity of all inertial actions." A variety of such Machian effects indeed inspired some of the experimental tests of GR and alternative theories which were finally carried out in the late twentieth century. The result of these tests suggested by the Machian perspective is a more comprehensive empirical basis for theory. Einstein still had Mach in mind when he derived from his theory the possibility of a topologically closed Universe without spatial boundaries: "If we think these (Mach's) ideas consistently through to the end we must expect the whole inertia, that is, the whole (gravitational) $g_{\mu\nu}$-field, to be determined by the matter of the Universe, and not mainly by the boundary conditions at infinity."

In taking on the challenge of incorporating gravity into his special-relativity structure of physical law, Einstein worked to replace the instantaneous, action-at-a-distance force of Newtonian gravity with an interaction between separated matter which was carried by a dynamical field. The successful electromagnetic field theory of Maxwell was his guide. This view of the field-mediated interaction has triumphed throughout physical law in the twentieth century. In this paradigm, two bodies interact by a staged, causal process; one body is source of a dynamical field, the field then spreads out in space and time from its origins in accordance with its own dynamical laws, and finally another body located elsewhere is acted upon by the resulting field found at its location.

In the search for a gravitational field, Einstein was profoundly influenced by the empirical fact that gravity accelerates bodies at a rate which does not depend on their chemical composition or other internal properties – the rate is apparently universal. First noted and studied by Galileo and others four hundred years ago, tested by Newton to a one-part-in-a-thousand precision by comparing motions of differently composed pendulums, and tested in Einstein's time by Roland von Eötvös to precisions of a few parts in a billion with torsion balances supporting different substances in Earth's gravity, this strange and apparently exact proportionality between the inertia of bodies and the strength of gravitational force they experience led Einstein, still short of a theory, to make a grand hypothesis. Paraphrasing him: since a local laboratory falling freely in gravity "transforms gravity away" as far as the physics in that laboratory is concerned, the freely falling laboratory must in all phenomenological respects be locally equivalent to an inertial frame (even though it accelerates relative to the distant inertial frame). From this Equivalence Principle, Einstein was able to predict two novel phenomena: that light rays should deflect downward when passing through a gravitational field; and that clocks should tick slower the deeper they are in a gravitational potential, by a given fractional rate which is independent of their internal structure. Measuring these predictions of the Equivalence Principle, and checking the principle's foundational phenomena – the universality of gravitational free-fall rates – to the highest achievable precision continue to be at the core of the experimental program to test GR in the Solar System. In line with the field paradigm of modern physics, if the foundation or predictions of the Equivalence Principle were found to be violated, this would most likely signal the existence of a previously unseen interaction field in physical law which is not "transformed away" in freely falling laboratories. For the purposes of discovering any such inverse-square (or very long Yukawa range) interaction field whose strength of coupling to matter is very weak compared with the gravitational coupling strength, the contemporary space experiments designed to test the Equivalence Principle are unsurpassed instruments.

The universality of free fall also gave Einstein an important clue to the type of field which could be the basis of gravity and to the attribute of matter to which it must couple. From special relativity he learned that a body's inertia equals its total energy content: $E = mc^2$. If the gravitational force on bodies were to be in universal proportion to the bodies' inertial masses, then a gravitational field's coupling strength to bodies must also be proportional to the bodies' energy contents. This led to consideration of a single scalar field ϕ (tensor of rank 0) or a second-rank tensor field $g_{\mu\nu}$ of 10 potentials, being symmetric in its indices μ and ν which range over the four dimensions of space and time, as transmitters of the gravitational interaction; either field could couple rather naturally to the energy content of bodies. (By contrast, the field that transmits the electromagnetic interaction between charges is a first-rank tensor field A_μ of four potentials, and the other interactions of physics, nuclear and weak, have also been found to be based on multiplets of first-rank tensor fields.) Through the decade 1905–15, scalar and tensor gravity were both explored. For a variety of reasons – empirical implications and predictions, theoretical uniqueness, consistency, and completeness – the general theory of relativity emerged based on a pure second-rank tensor gravitational field $g_{\mu\nu}(\boldsymbol{r}, t)$ (Einstein 1916).

Special relativity, by itself, suggests that the motion of bodies interacting gravitationally should deviate in detail from Newtonian form. Since a body's momentum was now known to contain the speed-dependent modifications of special relativity, application of the law of motion in the presence of a force

$$\boldsymbol{f} = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m\,\boldsymbol{v}}{\sqrt{1 - v^2/c^2}}\right)$$

yields corrections of order v^2/c^2, even before considering the force side of the equation or further modifications to the momentum of the mass. But just like electromagnetism, GR's 10 gravitational potentials produce a total force between bodies which includes corrections from the static situation (with details depending on the motion of one or both of the interacting objects). A further novel feature of GR's interaction is its nonlinearity: the gravitational force due to a sum of sources is not simply the sum of the individual forces; additional forces come into play which are proportional to the product (and higher powers) of the source masses. All of these modifications, fractionally characterized by the factor v^2/c^2 or Gm/c^2r, are very small throughout the Solar System, each amounting to about 2.5×10^{-8} in the case of Mercury's orbit, for example.

General relativity is compatible with the possible existence of additional, very weakly coupled long-range fields which have so far escaped detection. The presence of such fields and their interactions with matter will produce phenomena which either violate the Equivalence Principle and metric foundations of GR, or which diverge from the predictions of pure tensor gravity at the post-Newtonian level. Testing GR can therefore also be viewed as the search for any "new" long-range interaction fields in physical law. This chapter reflects this interpretation of the twentieth century's achievements.

THE EARLY FOUNDATIONAL YEARS

The Equivalence Principle

Soon after formulating his theory of special relativity in 1905, Einstein turned to the task of properly incorporating gravity, the other known force of that time, into physical law. His goal was to replace Newton's instantaneous action-at-a-distance with a causal field theory of gravity, analagous to the electromagnetic theory of interaction between charged particles transmitted by the Maxwell fields. He was also profoundly influenced by the apparent fact that the gravitational forces on bodies were in universal proportion to the bodies' inertial masses. Because of the resulting identity of free-fall rates at a given location, a localised laboratory freely falling in gravity would appear as an inertial frame, though accelerating relative to other distant inertial frames. He elevated this well-confirmed feature of gravity to a grand hypothesis – his *Equivalence Principle*:

> All local phenomena seen in a laboratory freely falling in gravity are equivalent to phenomena in a gravity-free inertial frame, or conversely, phenomena present in a gravity-free but accelerated frame of reference and understood from the basic kinematics of that frame must also be found to occur in a local gravitational field of equivalent acceleration. (Einstein 1907)

As one example of Einstein's ingenious reasoning which followed from this principle, consider two identically constructed clocks freely falling in gravity and separated by a small vertical distance. Light signals triggered by each tick of the higher clock are sent down to the lower clock which, because of its gravitational acceleration and the finite velocity of light, is always receiving the signals at a Doppler-shifted rate which is lower than the transmitted rate. But by the Equivalence Principle, the observed phenomena should be identical to what occurs when two separated clocks at rest in inertial space exchange signals – the frequency of signal transmission recorded by the transmitting clock equals the frequency of signal as recorded by the receiving clock. This will occur only for the gravitationally free-falling clocks if the lower clock ticks slower than the upper clock by an amount needed to compensate for the Doppler shift.

Unlike Newton, who had no underlying theory for the origins and magnitudes of mass (beyond its simple additivity), and who therefore on empirical grounds could simply adopt the equality of inertial and gravitational mass for all objects, Einstein had found from his special theory of relativity that a body's inertial mass was equal to its total energy content, $E = mc^2$, and he was therefore led to seek a theory of gravity in which the gravitational field's coupling strength to a body was also naturally and generally proportional to the body's total energy content. Achieving this took him over a decade. But more immediately, using only his Equivalence Principle, he predicted two novel phenomena. Using the argument outlined in the previous paragraph, he predicted that the rate of any laboratory clock located in a gravitational potential $U(\boldsymbol{r})$ will differ from the rate of an otherwise identical clock located elsewhere at gravitational potential $U(\boldsymbol{r}')$:

$$\frac{f'-f}{f} \cong \frac{U(\boldsymbol{r})-U(\boldsymbol{r}')}{c^2}, \qquad U(\boldsymbol{r}) = G\sum_i \frac{m_i}{|\boldsymbol{r}-\boldsymbol{r}_i|} \tag{1}$$

where c is the speed of light, G is Newton's gravitational constant, and m_i are the masses responsible for the gravitational potential. Using similar forms of argument, he also concluded that a light ray's propagation direction $\hat{c}$ will be deflected by the transverse part of any gravitational acceleration field $\boldsymbol{g}$ through which it propagates:

$$\frac{\mathrm{d}\hat{c}}{\mathrm{d}s} \cong \frac{\boldsymbol{g}-\boldsymbol{g}\cdot\hat{c}\hat{c}}{c^2}, \qquad \boldsymbol{g}(\boldsymbol{r}) = \nabla U(\boldsymbol{r}) \tag{2}$$

These effects are actually closely related. If the locally measured speed of light is to be a universal constant, but clock rates are universally diminished in gravitational potentials, then the globally viewed speed of light must also diminish in those gravitational potentials:

$$c(\boldsymbol{r}) \cong c_\infty(1 - U(\boldsymbol{r})/c^2)$$

with c_∞ being the speed of light where the gravitational potential of local bodies is negligible. As in any medium with an inhomogeneous speed of wave propagation, this speed of light function then results in the downward deflection of light wavefronts at the rate indicated by eqn (2).

Einstein soon realized that his prediction of light deflection could be experimentally tested. If the path of light between a distant star and an observing telescope on Earth passes the Sun at distance of closest approach D, the integrated deflection angle will be approximately (Einstein 1911)

$$\delta\Theta \cong \frac{2GM}{c^2 D} \cong 0.84 \text{ arcsec for a grazing ray}$$

The angular locations of such star images would therefore move away from the Sun's location and closer to the images from the less distorted regions of the starfield whose light did not pass so close to the Sun. Erwin Freundlich led an expedition to southern Russia in the late summer of 1914 to measure the light deflection by photographing the starfield during an eclipse of the Sun. But World War I was breaking out, and the expedition personnel were arrested and detained by the Russian authorities. Such an experiment was not to be carried out until the occurrence of the next post-war eclipse in 1919, a delay which permitted completion of Einstein's theory and a change in the prediction.

As early as 1907, Einstein was aware of the outstanding anomaly in Newtonian Solar System dynamics, and he sought to account for the discrepancy in his new theory. The accumulated astronomical observations of the previous century had shown that the secular precession rate of the planet Mercury's orbital major axis – the advance of its perihelion – amounted to 574 arcsec/century. This was a quality observation aided by the relatively large eccentricity of Mercury's orbit ($e \cong 0.2$). But only 531 arcsec/century of this precession could be accounted for by the perturbing gravitational accelerations from the other planets in the Solar System, and no other explanations within Newtonian gravity emerged which remained plausible. It seems likely that the failure during the decade 1905–15 of several preliminary versions of a gravitational field theory to naturally account for this excess 43 arcsec/century precession led Einstein to the further considerations from which general relativity was formulated.

General relativity and Mercury's anomalous perihelion advance

Guided by the physical insights gained from his Equivalence Principle and special relativity theory, and with formal help from mathematician and friend Marcel Grossmann, Einstein ultimately built his theory of gravity upon a dynamical second-rank tensor field of 10 potentials – $g_{\mu\nu}(\boldsymbol{r}, t)$ – symmetric in its indices μ and ν, which each range over the four spacetime coordinates. Finding physically acceptable and consistent field equations for these gravitational potentials, including their coupling to matter, consumed several years of labor. In the finished form of the theory, the gravitational potentials fulfill a set of second-order, nonlinear partial differential field equations for which the entire stress–energy–momentum tensor of laboratory matter $T_{\mu\nu}$ is the source of gravity:

$$G_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

where $G_{\mu\nu}$ is a tensor constructed from the ten gravitational potentials, the square of their first partial derivatives, and their second partial derivatives with respect to the space and time coordinates. This tensor is found to be unique by the necessity to be mathematically consistent with the property that its source tensor $T_{\mu\nu}$ fulfills the traditional conservation laws of energy, momentum, and angular momentum under the appropriate local conditions (Einstein 1916). The theory also specified how matter and other fields respond to the gravitational fields; for example, an atom or object of negligible size will move between two spacetime locations on the trajectory $\boldsymbol{r}(t)$, which gives an "extremal" value to the action integral

$$S = -\int \sqrt{g_{\mu\nu}(\boldsymbol{r}, t)\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu} \qquad (3)$$

performed along that trajectory; here $\mathrm{d}x^\mu = c\,\mathrm{d}t, \mathrm{d}\boldsymbol{r}$ for $\mu = 0, 1, 2, 3$. The specific mass of the atom is absent from this action integral because of the identity of gravitational and inertial mass; a geometrical interpretation of these trajectories as the extremal paths in a curved Riemannian spacetime geometry established by the field potentials $g_{\mu\nu}$ can then straightforwardly follow.

In late 1915, just weeks before arriving at the final form of his theory, Einstein succeeded in using a slightly incomplete version of his gravitational field equations to calculate the static, spherically symmetric tensor gravitational field in the empty space surrounding the Sun; he then used the equation of motion which results from the action integral given by eqn (3) to obtain the relativistic motion of the planet Mercury in that gravity field. Having no free theoretical parameters to adjust, his new theory nevertheless explained the anomalous 43 arcsec/century perihelion advance with good accuracy. Owing to the theory's intrinsic nonlinearity, the Sun's gravitational field included an important correction to the Newtonian inverse-square field: it varied as the inverse third power of distance and was

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